SORTA SOLVING THE OPF BY NOT SOLVING THE OPF: DAE Control Theory and the Price of Realtime Regulation

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Talk Outline

- Introduction and Motivations
- Traditional AC-OPF
- Power Systems Differential-Algebraic Equation Model
- No-OPF OPF
- Numerical Case Studies
- Concluding Remarks

Control Layers in Power Systems

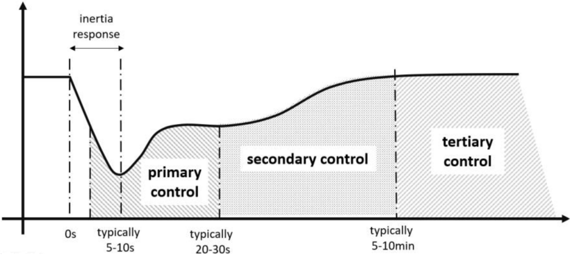

Figure: System frequency response and control layers in power system

- Primary control regulates frequency dynamics and contains AVR and PSS etc.
- Secondary control layer removes steady-state error via AGC
- Tertiary control is used for economic dispatch via running AC-OPF

Summary of AC-OPF

- AC-OPF can be defined as computing cost-optimal generators setpoints while satisfying key system constraints

$$\textbf{OPF:}\ \ \text{minimize}\ \ f(\boldsymbol{x})\ \ \text{s.t.}\ \ \boldsymbol{g}(\boldsymbol{x}) \leq \boldsymbol{0}\ \ \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0}$$

- $\boldsymbol{x}$ defines many variables
- $\boldsymbol{f}(\boldsymbol{x})$ represents the total cost of generation from fuel-based power plants
- $\boldsymbol{g}(\boldsymbol{x})$ lumps inequality constraints such as thermal line, voltages, and generation limits
- $\boldsymbol{h}(\boldsymbol{x})$ denotes the system power balance equation—a nonlinear non-convex constraint
- Most solved engineering optimization problem?

Literature

To solve the AC-OPF, academics often resort to one of these four approaches

- Assume DC power flow and eliminate some variables, resulting in convex quadratic programs [Taylor (2015); Momoh et al. (1999)]
- Derive SDP relaxations of OPF appended with methods to recover an optimal solution [Andersen et al. (2014); Louca et al. (2013).]
- Design global optimization methods with some performance guarantees under various relaxations of nonconvex OPF [Lu et al. (2018); Lee et al. (2020)]
- Obtain machine learning-based algorithms that learn solutions to OPF [Baker (2019); Huang et al. (2022)]

Literature (Cont'd)

- AC-OPF generator setpoints are *control- and dynamics-unaware*
- The provided setpoints might not even be cost-optimal anymore
- ...due to future power grid with high uncertainty and fluctuations
- **Need for realtime and dynamics-constrained AC-OPF**
- ...that goes beyond markets and *cares more for stability*
- This is *not* new, lots of studies to augment OPF with dynamics

Relevant Work and Research Objectives

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- Real-Time Optimal Power Flow [Yan and Xu (2020); Tang et al. (2017)]
- Approaches where dynamic stability or optimal control metrics are appended to the OPF [Bazrafshan et al. (2019); Li et al. (2016); Dorfler et al. (2016)]
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  - Jointly approximate AC-OPF solution while performing frequency regulation and realtime control
  - No more AC-OPF centered optimization
  - Move problem to control theory
  - Still satisfy nonconvex OPF constraints

Part 1:
AC-OPF Formulation

AC-OPF: A Nonconvex Optimization Problem

$$\underbrace{\underset{\boldsymbol{P}_{\mathrm{G}},\boldsymbol{Q}_{\mathrm{G}},\boldsymbol{\theta},\boldsymbol{v}}{\text{minimize}}}_{\text{variables}}$$

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$$\text{subject to} \quad \left.\begin{aligned} & P_{\mathrm{G}i} + P_{\mathrm{R}i} + P_{\mathrm{L}i} = \\ & v_i \textstyle\sum_j v_j (G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) \\ & Q_{\mathrm{G}i} + Q_{\mathrm{R}i} + Q_{\mathrm{L}i} = \\ & v_i \textstyle\sum_j v_j (G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) \end{aligned}\right\} \text{Power balance equations}$$

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$$v_i^{\min} \leq v_i \leq v_i^{\max} \quad \text{Voltage limits}$$

$$\left.\begin{aligned} & S_{f_i} \leq F_{\max} \\ & S_{t_i} \leq F_{\max} \end{aligned}\right\} \text{Line flow limits}$$

AC-OPF (Cont'd)

- The AC-OPF is usually solved every 5–10 minutes, although the frequency at which its solved depends on various factors
- Ideally, a system operator would have all of the constraints satisfied at each time step t, and one would solve a realtime AC-OPF
- ...as realtime predictions of loads/renewables become available
- Does not take into account the power system differential equations and uncertainties vector $\boldsymbol{w}$ (loads/renewables)

Part 2:
Dynamic-Algebraic Power System Modeling

Differential Equations of Multi-Machine Power systems

- **System's set-up**:
  - N number of buses
  - Modeled as $(\mathcal{N}, \mathcal{E})$ where $\mathcal{N} = \{1, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$
  - $\mathcal{N} = \mathcal{G} \cup \mathcal{L} \cup \mathcal{R}$
  - $\mathcal{N}_M \subseteq \mathcal{N} \rightarrow$ buses with PMUs
- **System dynamics** $(i \in \mathcal{G} \cup \mathcal{R} \cup \mathcal{L})$:

$$\dot{\delta}_i = \cdots$$
$$\dot{\omega}_i = \cdots$$
$$\dot{E}'_{qi} = \cdots$$
$$\vdots$$

- System dynamics can contain higher-order generator dynamics along with power-electronics-based solar, wind, and load dynamical models
- Framework accomodates a lot more variations

Algebraic Equations of Multi-Machine Power Systems

- **Generator real and reactive power equations**

$$P_{\mathrm{G}i} = \frac{1}{x'_{\mathrm{d}i}} E'_{qi} v_i \sin(\delta_i - \theta_i) - \frac{x_{\mathrm{q}i} - x'_{\mathrm{d}i}}{2x'_{\mathrm{d}i} x_{\mathrm{q}i}} v_i^2 \sin(2(\delta_i - \theta_i))$$

$$Q_{\mathrm{G}i} = \frac{1}{x'_{\mathrm{d}i}} E'_{qi} v_i \cos(\delta_i - \theta_i) - \frac{x'_{\mathrm{d}i} + x_{\mathrm{q}i}}{2x'_{\mathrm{d}i} x_{\mathrm{q}i}} v_i^2 - \frac{x_{\mathrm{q}i} - x'_{\mathrm{d}i}}{2x'_{\mathrm{d}i} x_{\mathrm{q}i}} v_i^2 \cos(2(\delta_i - \theta_i))$$

- **Power balance equations**

$$P_{\mathrm{G}i} + P_{\mathrm{R}i} - P_{\mathrm{L}i} = \sum_j v_i v_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij})$$

$$Q_{\mathrm{G}i} + Q_{\mathrm{R}i} - Q_{\mathrm{L}i} = \sum_j v_i v_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij})$$

- Power balance equations can be written as current balance equations

Power System State Space Representation

Power systems NDAE model can be written as:

$$\begin{aligned} \textit{nonlinear generator ODEs} \quad \dot{\boldsymbol{x}}_d &= \boldsymbol{A}_d \boldsymbol{x}_d + \boldsymbol{f}_d(\boldsymbol{x}_d, \boldsymbol{x}_a) + \boldsymbol{B}_d \boldsymbol{u} \\ \textit{nonlinear power flow} \quad \boldsymbol{0} &= \boldsymbol{A}_a \boldsymbol{x}_a + \boldsymbol{f}_a(\boldsymbol{x}_d, \boldsymbol{x}_a) + \boldsymbol{B}_a \boldsymbol{w} \end{aligned}$$

- $\boldsymbol{x}_d$ lumps dynamics states of generator, renewables, and loads
- $\boldsymbol{x}_a$ defines algebraic power network states: $\boldsymbol{P}_G$, $\boldsymbol{Q}_G$, $\boldsymbol{v}$, $\boldsymbol{\theta}$
- $\boldsymbol{u}$ lumps all the control inputs for both generators and renewables
- Lump $\boldsymbol{x}_d$ and $\boldsymbol{x}_a$ into $\boldsymbol{x}$

$\Rightarrow$ dynamics can be written as nonlinear differential algberaic equation (NDAE):

$$\boxed{\boldsymbol{E}\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{B}_w \boldsymbol{w}}$$

Part 3: Feedback Controller Design and NO-OPF Formulation

No-OPF Control Formulation

- First, let us assume we have realtime information of $\boldsymbol{x}(t)$
- And let's consider a control law as

$$\boxed{\boldsymbol{u}(t) = \boldsymbol{u}_0 + \boldsymbol{K}\left(\boldsymbol{x}(t) - \boldsymbol{x}_0\right)}$$

  where

  - $\boldsymbol{u}_0$ is the reference input; such as setpoints of field voltage $\boldsymbol{E}_{fd}$ and governor $\boldsymbol{T}_r$ in case of 4^{th}-order system
  - $\boldsymbol{x}_0$ is the steady state value of the state vector
- $\boldsymbol{K}$ is a design variable

No-OPF (Cont'd)

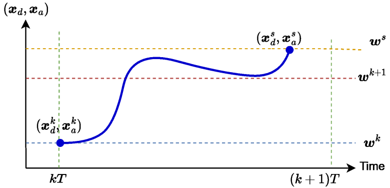
- Now let us write the perturbed closed loop dynamics as:

$$\begin{aligned} \boldsymbol{E}\dot{\boldsymbol{x}} &= (\boldsymbol{A} + \boldsymbol{B}\boldsymbol{K})\boldsymbol{x} + \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{B}_w\boldsymbol{w} \\ \boldsymbol{z} &= (\boldsymbol{C} + \boldsymbol{D}\boldsymbol{K})\boldsymbol{x} \end{aligned}$$

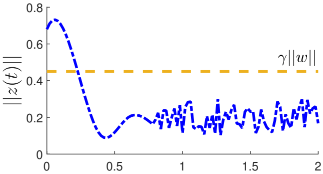
- $z(t)$ is the performance index—*can model costs or frequency violations*
- The main objective is to design control gain matrix $\boldsymbol{K}$ which can hedge against disturbance $\boldsymbol{w}$ and make system stable
- To consider disturbance $\boldsymbol{w}$ in the controller architecture one can use the robust $\mathcal{H}_2$, $\mathcal{H}_\infty$ or $\mathcal{L}_\infty$ stability notion

$\mathcal{H}_\infty$ Notion and WAC Design

- Design $\boldsymbol{K}$ such that $\|\boldsymbol{z}\| < \gamma\|\boldsymbol{w}\|$ with γ as performance index
- Doing so the controller minimizes the impact of disturbance $\boldsymbol{w}$
- Thus the controller will stabilize the system at the post-fault equilibrium



(a)



(b)

Figure: (a) Stabilization of power system at post-fault equilibrium (b) Visualization of $\mathcal{H}_\infty$ notion

Relation to the AC-OPF Formulation

- Compute $\boldsymbol{K}$ such that it explicitly encodes the algebraic constraints along with differential equations
- Then $\boldsymbol{K}$ will inherently satisfy key AC-OPF constraints
- The constraints related to generators' capacity limits can be encoded via saturation dynamics in the differential equations
- Other constraints such as thermal limits of lines cannot be modeled in this approach
- How to compute $\boldsymbol{K}$? Theoretical properties?

Centralized $\boldsymbol{K}$ that Satisfy Key AC-OPF Constraints

Main Result

Given any unknown disturbance $\boldsymbol{w}(t)$, solving the following optimization problem

$$(\textbf{Centeralized Control-OPF}) \quad \underset{\boldsymbol{K},\gamma}{\text{minimize}} \quad \gamma$$
$$\text{subject to} \quad \text{LMI}(\boldsymbol{K},\gamma) > 0$$
$$\boldsymbol{K} \in \mathcal{K}$$

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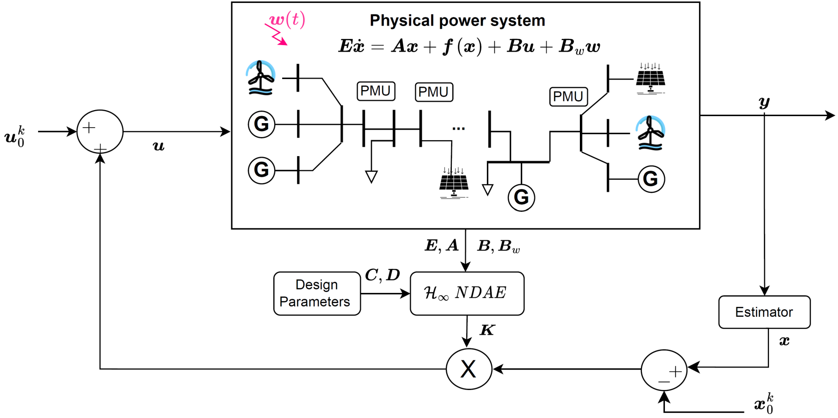
4. designed SDP is a convex semi-definite optimization problem

Integrated Framework



Discussion on the Control-OPF Formulation

- Regardless of the computation technique used (i.e., centralized or decentralized) the gain $\boldsymbol{K}$ is computed offline and only depends on the constant system matrices
- Fully abides by some of the key AC-OPF constraints
- Can be implemented in realtime using measurements received from the PMUs
- Can seemingly integrate the detailed dynamics of the generator and renewables
- Deals with the uncertainty in renewables, loads, and parameters in a control-theoretic way
- Robust to some topological changes
- Not dependent on a linearization point

Remarks Regarding Control-OPF

- Does not provide any theoretical guarantees regarding optimally of the system cost after a large disturbance
- Does not explicitly account for the other AC-OPF constraints but only the power/current balance equations
- Requires knowledge of system matrices, although feedback controllers are known to be robust against small parametric uncertainty in the system

Part 4:
Numerical Case Studies

Case Studies

- Various numerical simulations performed under random disturbances in load and renewables
- Since control-OPF provides time-varying vectors of $\boldsymbol{P}_G$ and $\boldsymbol{Q}_G$, average system cost is computed as:

$$J_{\mathrm{OPF}}(\boldsymbol{P}_G) = \frac{1}{T}\sum_{t=1}^{T}\sum_{i\in\mathcal{G}} a_i P_{\mathrm{G}i}^2(t) + b_i P_{\mathrm{G}i}(t) + c_i$$

- This seems to be the *fair* way of comparing costs

AC-OPF and control-OPF Power Set-points

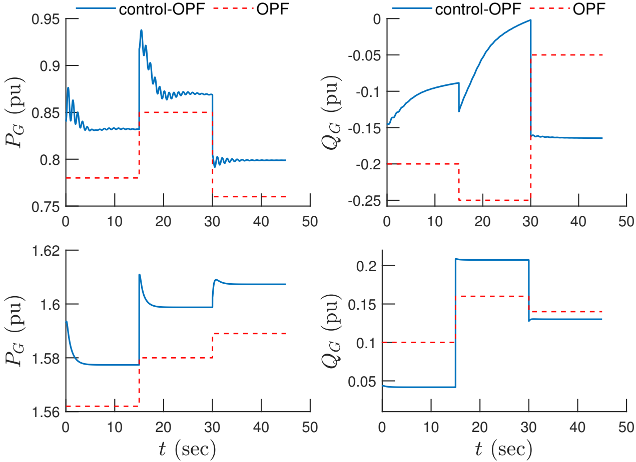


Figure: Time-varying power set-points by control-OPF and static set-points from AC-OPF for three random step disturbances in load demand; case 39 (above) and case 9 (below)

Power, Voltages, Line Flows and their Limits

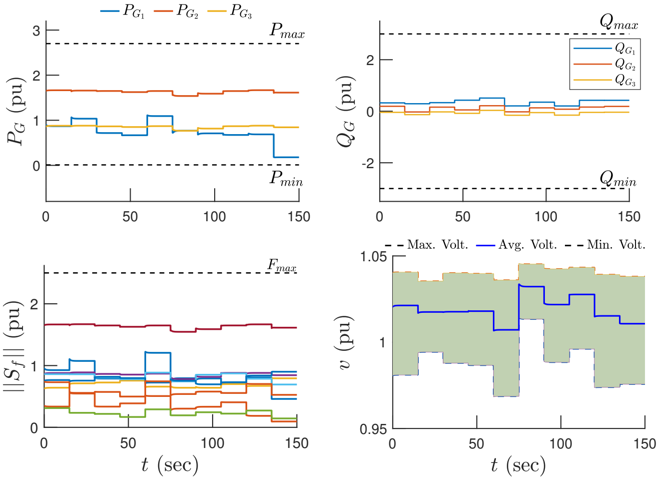


Figure: Active and reactive power generated by the all the generators and their respective limits, line flows and their maximum rating, and the overall modulus of all bus voltages for case 9 bus test system.

System Cost Comparison

Table: Cost comparison for the control-OPF and AC-OPF.

System	Method	Total system cost $\times 10^3$ \$	Percentage difference from AC-OPF
Case 9	AC-OPF	5.4188	—
	control-OPF	5.5805	3.001
Case 14	AC-OPF	8.4591	—
	control-OPF	9.3522	14.251
Case 39	AC-OPF	41.819	—
	control-OPF	46.105	10.243
Case 57	AC-OPF	42.791	—
	control-OPF	48.002	10.894

Constraints Violations?

Table: Summary of AC-OPF constraints for different test system with control-OPF. The results indicate **no** constraint violations for flows, maximum active/reactive powers.

Test System	$\Delta_{\max}\boldsymbol{S}_f(t)$	$\Delta_{\max}\boldsymbol{S}_t$	$\Delta_{\max}\boldsymbol{P}_g(t)$	$\Delta_{\min}\boldsymbol{Q}_g(t)$	$\Delta_{\max}\boldsymbol{Q}_g(t)$
Case 9	-0.5612	-0.4570	-1.0626	3.2456	-1.1414
Case 14	-0.4297	-0.3910	-0.6606	0.4726	-0.0046
Case 39	-0.6762	-0.6675	-0.0778	3.1338	-0.0464
Case 57	-0.2391	-0.8312	-0.0014	2.0121	-0.0396

where

- $\Delta_{\max}\boldsymbol{X}(t) = \max_t(\boldsymbol{X}(t) - \boldsymbol{X}_{\max})$
- $\Delta_{\min}\boldsymbol{X}(t) = \max_t(\boldsymbol{X}(t) - \boldsymbol{X}_{\min})$

Frequencies under Load and Renewable Uncertainty

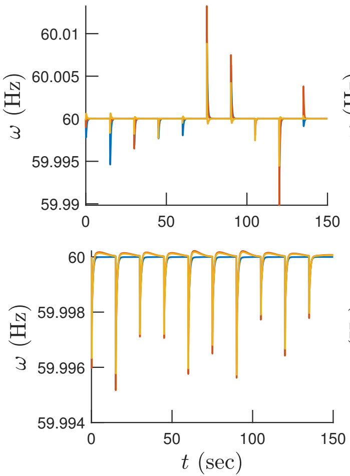


Figure: Generator frequencies under ten random disturbances in load and renewables for case 9, case 14, case 39, and case 57 test systems respectively.

Comparison with LQR Control

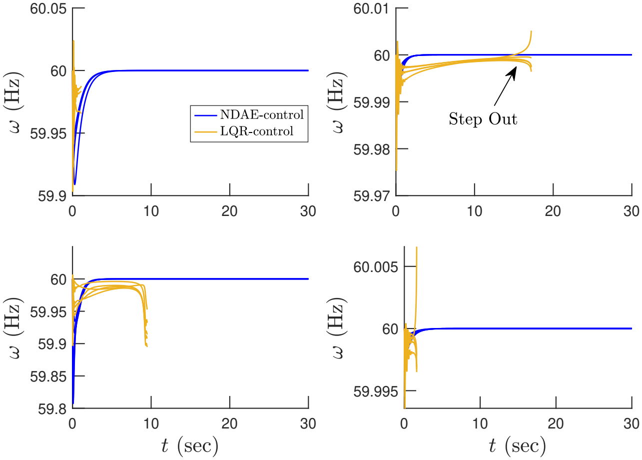


Figure: The generator frequencies for 9-bus (top-left), 14-bus (top-right), 39-bus (bottom-left), and 57-bus (bottom-right) test systems, for disturbance in load demand and renewable power.

Concluding Remarks

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- cost(Control-OPF) $>$ cost(AC-OPF)
- Control-OPF results in no constraint violations of flows, limits, ...
- Control-OPF produces realtime regulation of grid's voltages $+$ frequencies

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- Control-OPF results in no constraint violations of flows, limits, ...
- Control-OPF produces realtime regulation of grid's voltages + frequencies
- One could consider the 2–15% increase in the system cost as a *regulation cost*
- Comparisons are somewhat unfair to Control-OPF. Why?
- AC-OPF knows exact values for all uncertain loads and renewables, the control-OPF is truly uncertainty-unaware

Concluding Remarks

- cost(Control-OPF) $>$ cost(AC-OPF)
- Control-OPF results in no constraint violations of flows, limits, ...
- Control-OPF produces realtime regulation of grid's voltages + frequencies
- One could consider the 2–15% increase in the system cost as a *regulation cost*
- Comparisons are somewhat unfair to Control-OPF. Why?
- AC-OPF knows exact values for all uncertain loads and renewables, the control-OPF is truly uncertainty-unaware
- SDPs are slow but it's an offline computation

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- New concept of realtime pricing (LMPs extracted from ODEs)

Moving Forward

- How would this be applied to more detailed models with renewables?
- Can make this apporoach PMU-based
- Include an estimator in the feedback loop
- Compare with robust optimization approaches
- Embed generation cost curves within the robust control

Thank You!

Please email me for questions/discussions

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