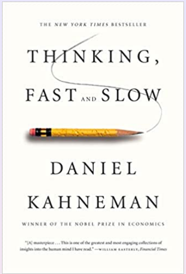
THE UNIVERSITY OF ARIZONA

PROGRAM IN APPLIED MATHEMATICS

Physics Informed and Data Driven Approaches to Managing Energy Systems at Scales & Under Uncertainty

Misha Chertkov Applied Math @ UArizona, Chair

Sep 6, 2023 – NREL, Golden



System 1 & 2 in DL and AI

- "From System 1 Deep Learning to System 2 Deep Learning" – Yoshua Bengio, NeurIPS 2019

Modern ('21) Applied Mathematics as System 2 ... Harvesting 20's Applied Math + (System 1.8) Data & Model Revolution (System 1.2)

- System 1 – operates automatically & quickly [Deep Learning, empowered by Automatic Differentiation]
- System 2 – allocates attention to effortfull mental activities [**Physics Informed AI** – Explainable, Generalizable, Generative]

Outline

Physics-Informed, AI-enabled Reduced Models

AI/Machine Learning (e.g. Neural Network, Graph Models, etc)

- will make Energy System Computations
  - faster (efficient)
  - possible even when data/measurements incomplete
- requires ground-truth data
  - actual measurements (Phasor Measurement Units, pressures, temperature in the room, etc)
  - energy/gas flow solvers (microscopic simulations) – reliable, possibly heavy
- can be energy-system "informed" (System 2) vs "agnostic" (System 1)
- contemporary Applied Mathematics methods/options are many
  - should be gauged to available data, level of uncertainty, etc

Incomplete Review: Brief, Recent, Biased

AI/ML in Power Systems (System 1, System 2 & juxtaposition)

- **Structure Learning**, Sparse Measurements, Graphical Models, Focus on Power Distribution: Deka, et al [2016-2019]
- **Learning ODE**: Power Transmission, Dynamic Coefficients in Swing Equations, Deterministic and Stochastic, Lokhov, et al [2017]
- Real-time **Faulted Line Localization** and PMU Placement in Power Transmission through CNN: Li, et al [2018]
- **Collocation Point Neural ODE** for Power Systems: Misuris, et al [2018]
- Learning a **Generator Model** from Terminal Bus Data: many ML schemes, tradeoffs, ranking models according to regimes, Stulov et al [2019]
- Learning from power system data stream, **phasor-detective** approach, Escobar et al [2019]

Incomplete Review: Brief, Recent, Biased

AI/ML in Power Systems (System 1, System 2 & juxtaposition)

- **Physics-Informed** Graphical Neural Network for **Parameter & State Estimations** in Power Systems https://arxiv.org/abs/2102.06349 (Pagnier & MC))
- **Embedding Power Flow** into Machine Learning for Parameter and State Estimation https://arxiv.org/abs/2103.14251 (Pagnier & MC)
- **Which Neural Network to Choose** for Post-Fault Localization, Dynamic State Estimation and Optimal Measurement Placement in Power Systems? https://arxiv.org/abs/2104.03115 (Afonin & MC))
- Towards **Model Reduction** for Power System Transients with Physics-Informed PDE, IEEE Access 2022, https://ieeexplore.ieee.org/document/9796532 (Pagnier, Fritzsch, Jacquod & MC)
- Physics-Informed **Machine Learning for Electricity Markets**: A NYISO Case Study, under review at IEEE Transactions on Energy Markets, Policy, and Regulation, https://arxiv.org/abs/2304.00062 (Ferrando, Pagnier, Mieth, Liang, Dvorkin, Bienstock & MC)

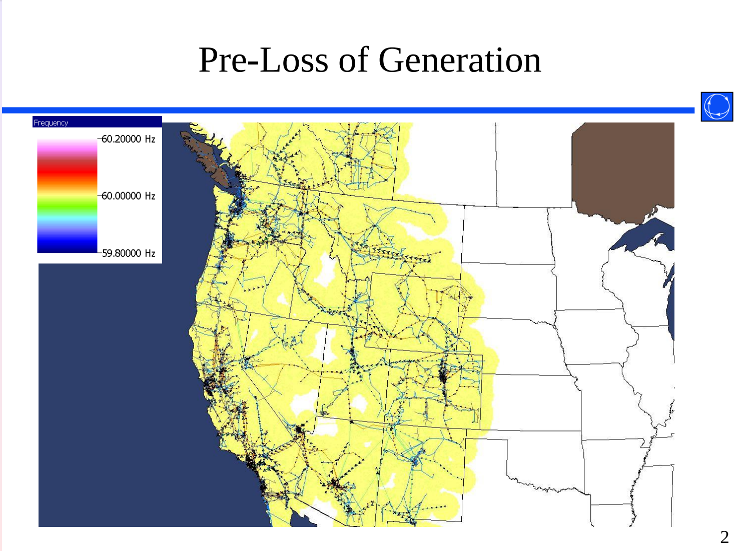
- L. Pagnier, J. Fritzsch, P. Jacquod and M. Chertkov, "Toward Model Reduction for Power System Transients With Physics-Informed PDE", in IEEE Access, vol. 10, pp. 65118-65125, 2022, doi: 10.1109/ACCESS.2022.3183336.
- + work in progress

Laurent Pagnier

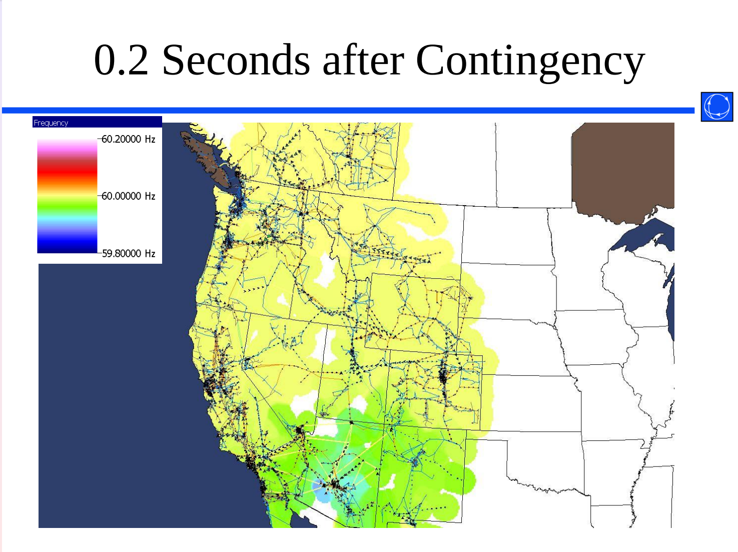
Julian Fritzsch

Philippe Jacquod

Physics = Electro-Mechanical Waves



Physics = Electro-Mechanical Waves



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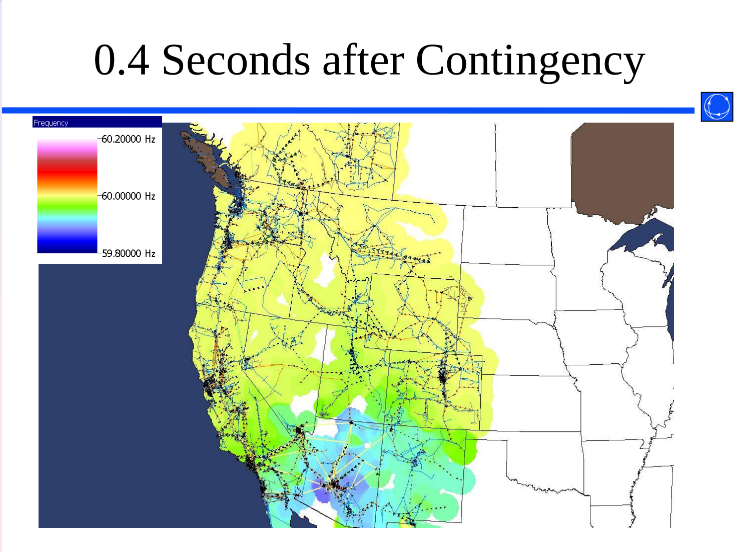

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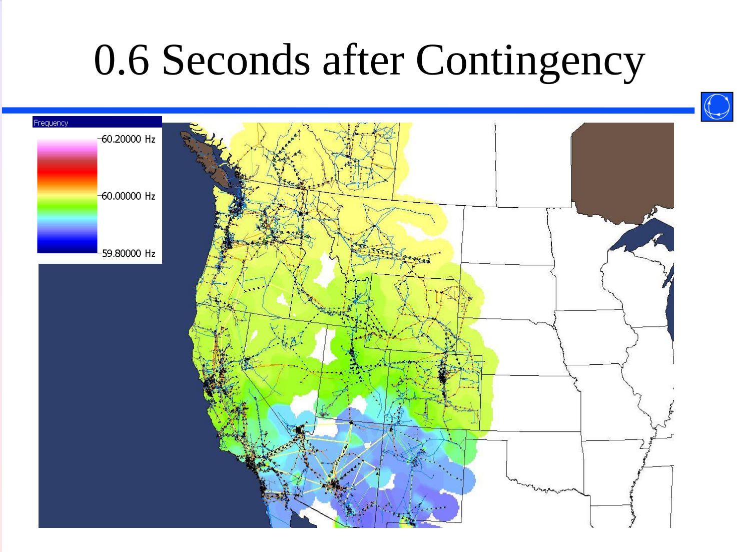

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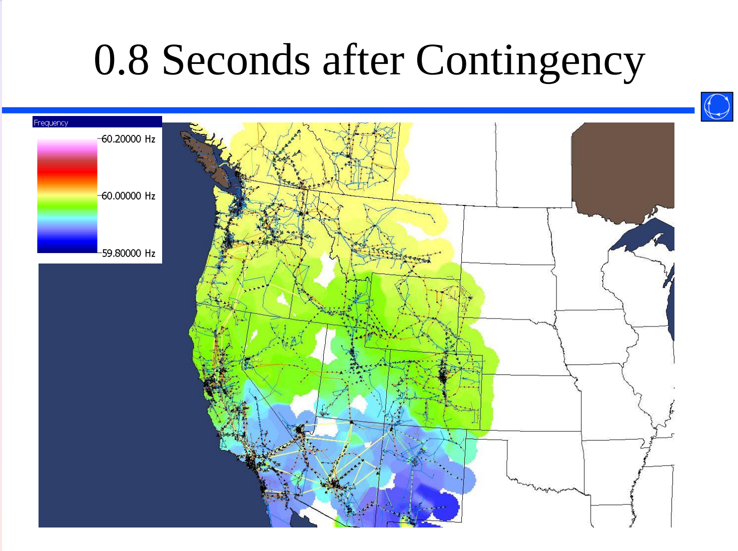

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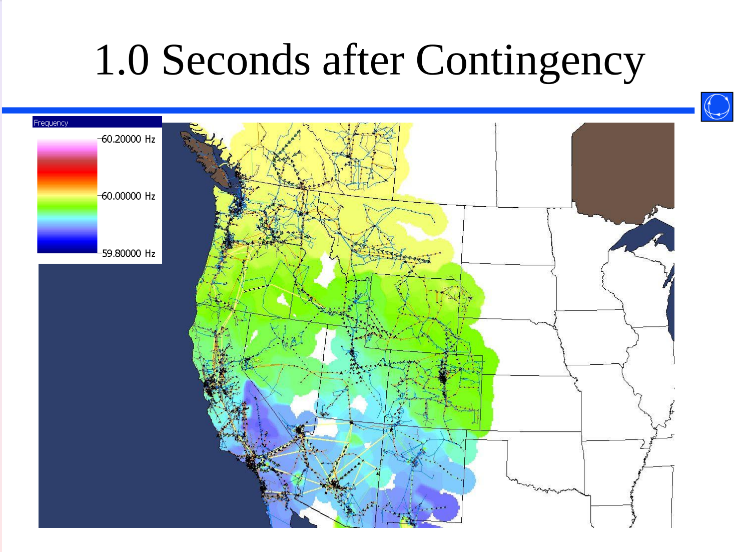
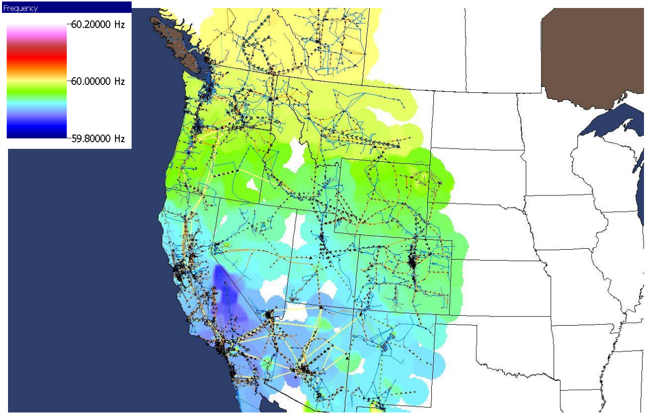


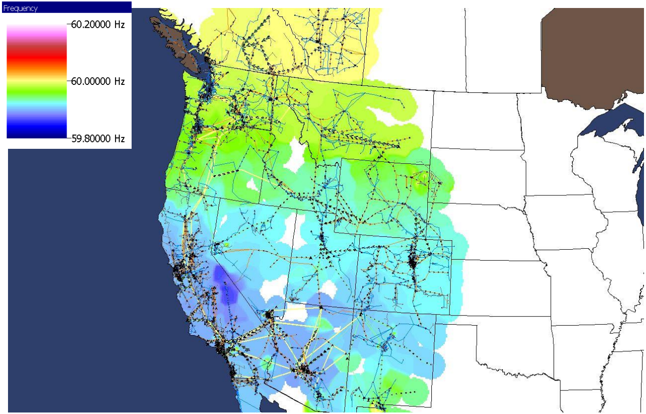
Physics = Electro-Mechanical Waves

1.2 Seconds after Contingency



Physics = Electro-Mechanical Waves

1.4 Seconds after Contingency



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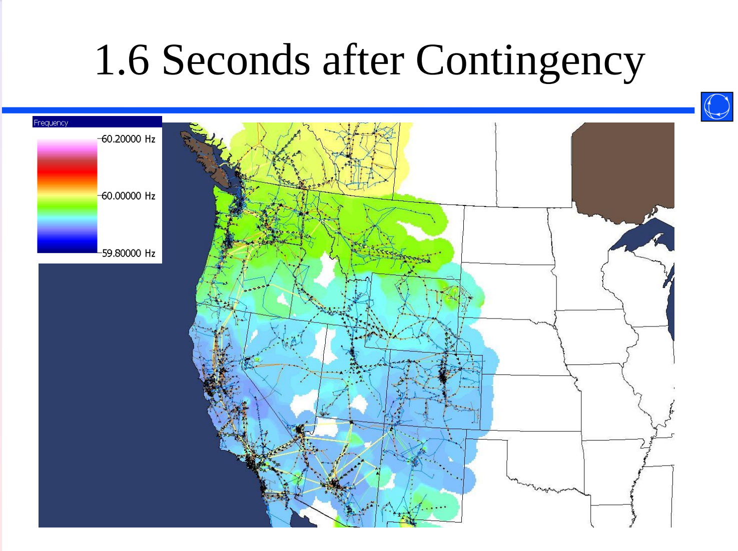
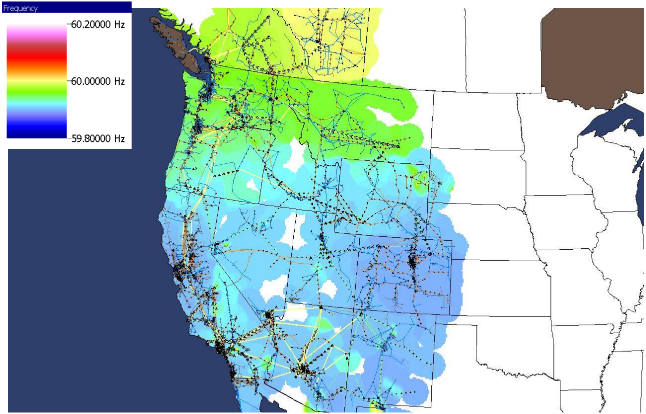


Physics = Electro-Mechanical Waves

1.8 Seconds after Contingency



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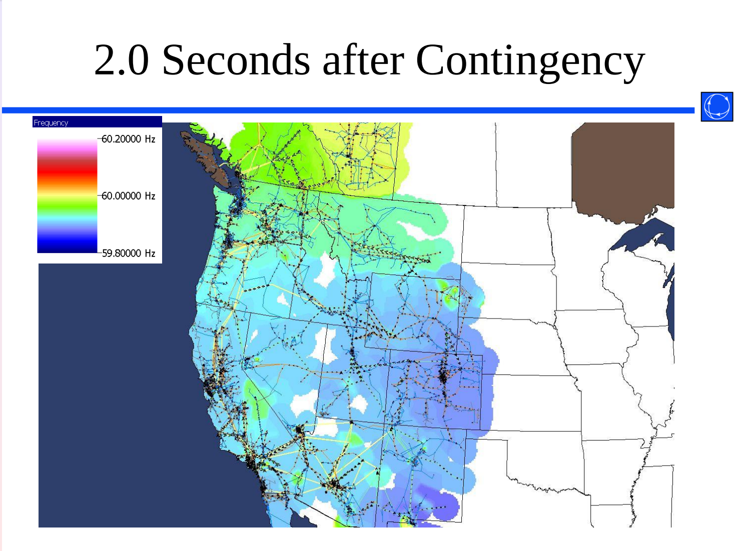

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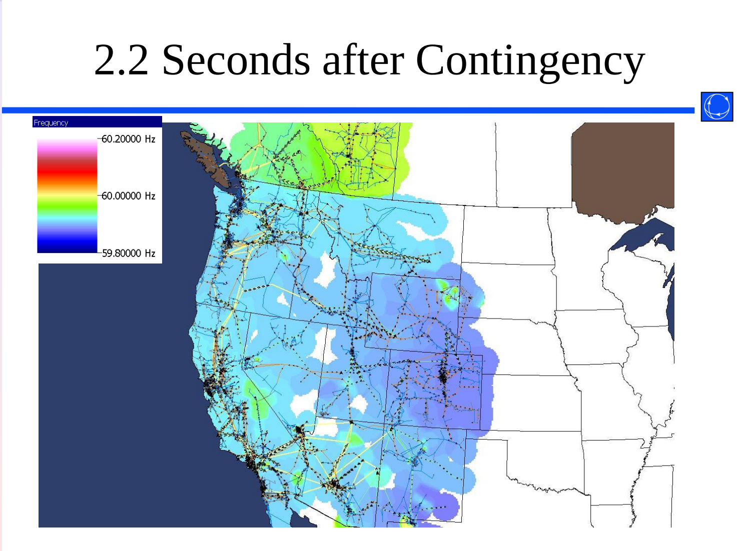

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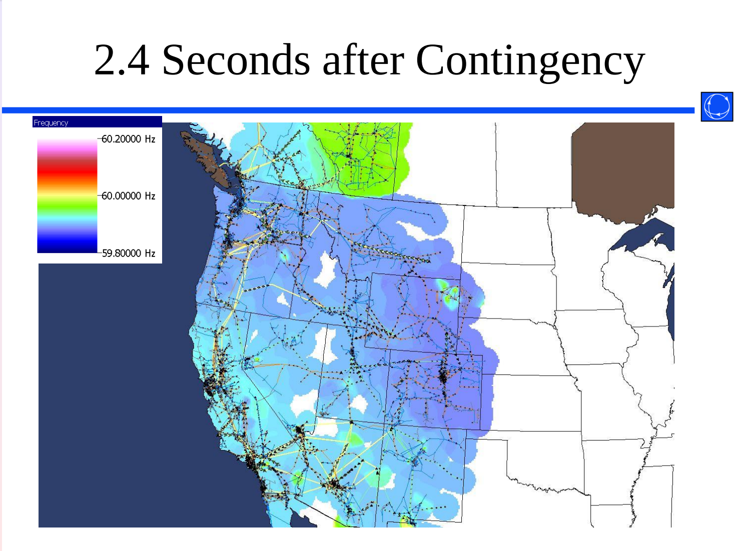

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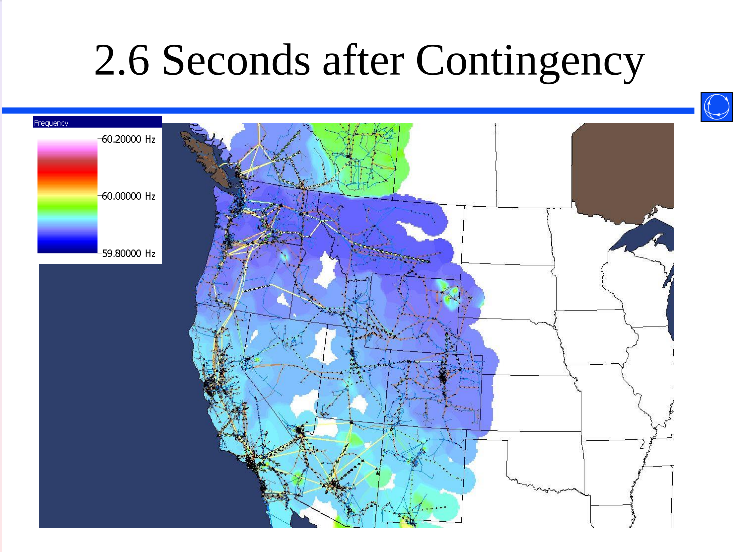

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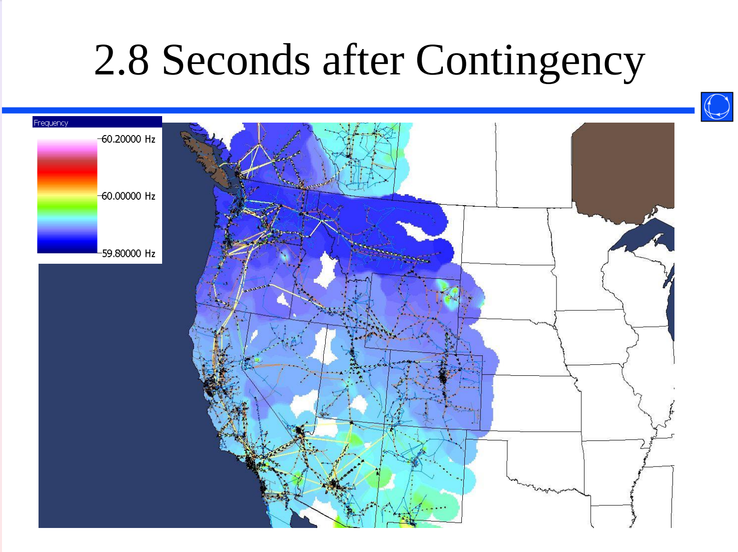

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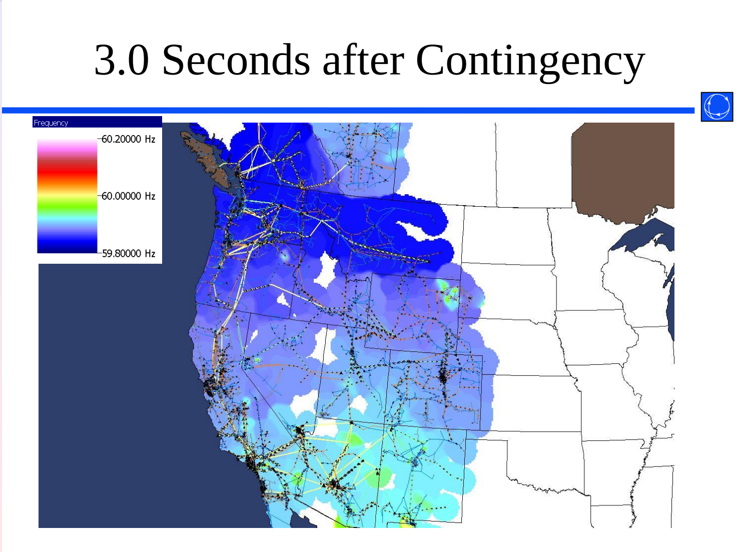
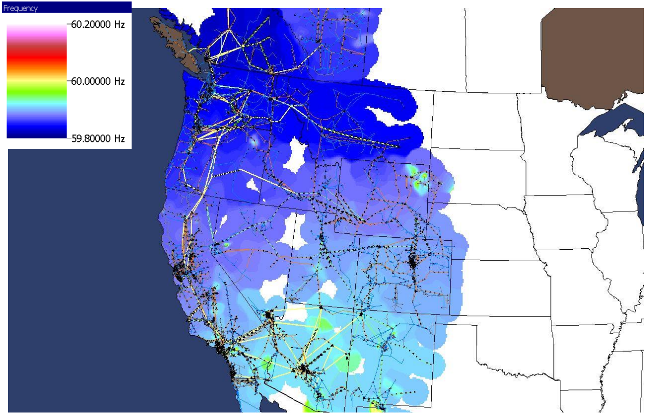


Physics = Electro-Mechanical Waves

3.2 Seconds after Contingency



Model Reduction

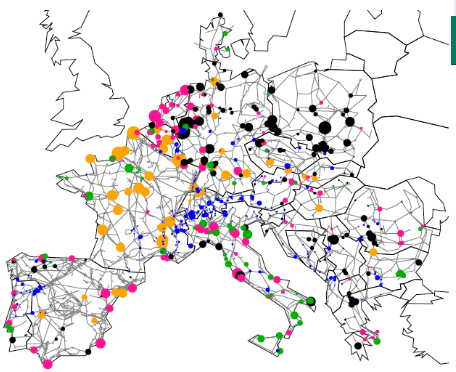
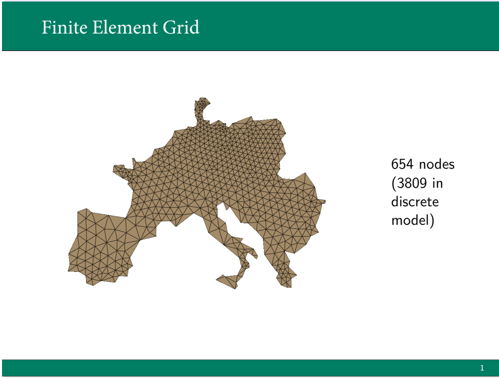


Fig 4. Inertia parameters of generators in our model of the synchronous grid of continental Europe. The disk size is proportional to m_i and the colors label hydro (blue), nuclear (orange), gas (pink), coal (black) and other (green) power plants.

https://doi.org/10.1371/journal.pone.0213550.g004



How does model reduction work?

- **Ground Truth** – reliable but computations "heavy" $\Rightarrow$
- Reduced Model – lighter computations-wise, loosing some accuracy (but hopefully not too much)

Transient (seconds) Dynamics of the grid

- Swing Equation: $m_i\ddot{\theta}_i + d_i\dot{\theta}_i = p_i - \sum_j v_i v_j b_{ij} \sin(\theta_i - \theta_j) \Rightarrow$
- Reduced Model Options?

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PDE as the Reduced Model

- $m(\boldsymbol{x})\frac{\partial^2}{\partial t^2}\theta(t;\boldsymbol{x}) + d(\boldsymbol{x})\frac{\partial}{\partial t}\theta(t;\boldsymbol{x}) = p(t;\boldsymbol{x}) + \sum_{\alpha,\beta=1,2} \partial_{r_\alpha} b_{\alpha\beta}(\boldsymbol{x})\partial_{r_\beta}\theta(t;\boldsymbol{x})$

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- Why is Partial Differential Equation modeling a sound option for model reduction?

Why is PDE a sound option for model reduction?

Approximating the swing ODEs by a PDE? Really?

- Naively: increases # degrees of freedom

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... but thinking a bit more (system 2) it has a sense because

- Solutions of linear 2+1 dimensional PDE assume spatial regularization via a 2d grid with fewer # grid points
- Operations are much more efficient over a regular lattice
- # physical parameters can be reduced dramatically via coarsening – fewer & large-scale harmonics

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Inspired by 1+1 PDE modeling of PS:

- A. Semlyen, 1974.
- J. S. Thorp, C. E. Seyler, and A. G. Phadke, 1998.
- M. Parashar, J. S. Thorp, and C. E. Seyler, 2004.
- I. Stolbova, S. Backhaus, M. Chertkov, 2015.

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How can we make it work?

From Swing Model to PDE Model

- From Swing Equation:
  $m_i\ddot{\theta}_i + d_i\dot{\theta}_i = p_i - \sum_j v_i v_j b_{ij}\sin(\theta_i - \theta_j)$
- To PDE as the Reduced Model $m(\mathbf{x})\frac{\partial^2}{\partial t^2}\theta(t;\mathbf{x}) + d(\mathbf{x})\frac{\partial}{\partial t}\theta(t;\mathbf{x}) = p(t;\mathbf{x}) + \sum\limits_{\alpha,\beta=1,2} \partial_{r_\alpha} b_{\alpha\beta}(\mathbf{x})\partial_{r_\beta}\theta(t;\mathbf{x})$
- $\forall i: \quad \theta_i(t)\to\theta(t;\mathbf{x}),\ m_i\to m(\mathbf{x}),\ d_i\to d(\mathbf{x}),\ p_i(t)\to p(t;\mathbf{x}),\ b_{ij}\to b_{\alpha\beta}(\mathbf{x}),\ \forall\alpha,\beta=1,2.$

Neumann Boundary Conditions:

- Vanishing normal derivative of the angle field on the domain boundary $\partial\Omega$:
  $$\forall t,\ \forall \mathbf{x}\in\partial\Omega: \quad \sum_{\alpha,\beta=1,2} n_\alpha(\mathbf{x}) b_{\alpha\beta}(\mathbf{x})\partial_{r_\beta}\theta(t;\mathbf{x}) = 0$$
- e.g. guaranteeing equilibration to the same frequency

Learning the PDE

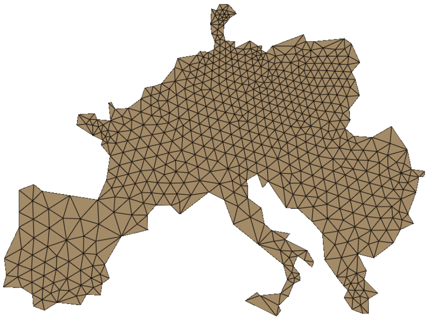
Learning (work in progress)

$$m(\mathbf{x})\ddot{\theta}(\mathbf{x}) + d(\mathbf{x})\dot{\theta}(\mathbf{x}) = p(\mathbf{x}) + \nabla\left(\begin{bmatrix} b_x(\mathbf{x}) & 0 \\ 0 & b_y(\mathbf{x}) \end{bmatrix}\nabla\theta(\mathbf{x})\right) \quad (1)$$

1. Switched to finite element method
2. b_x and b_y are now proper fields
3. We want to learn susceptances from steady state solutions
4. We want to learn m and d from dynamical simulations

Learning the PDE

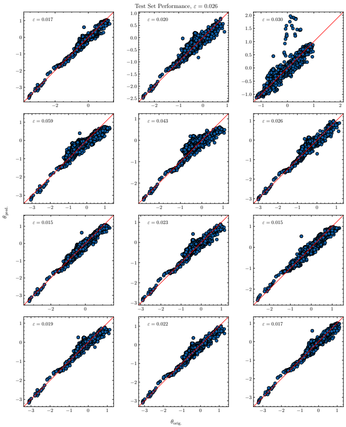
Finite Element Grid



654 nodes (3809 in discrete model)

Learning the PDE

Training in Steps: Steady State First

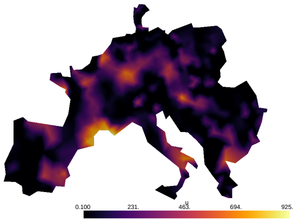


Trained on 48 different dispatches

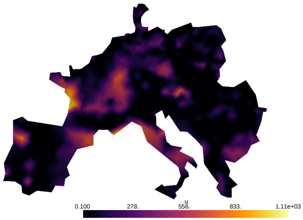
Learning the PDE

Steady State Training (Results)

$b_x(\mathbf{x})$

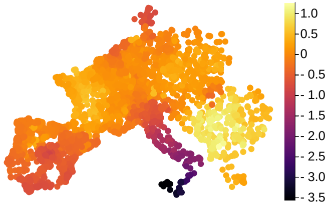


$b_y(\mathbf{x})$



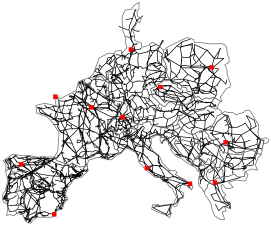
Learning the PDE

Steady State (solution)



Learning the PDE

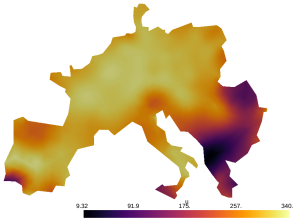
Training Dynamical Parameters

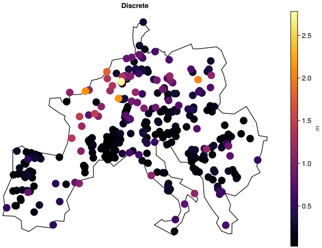


- We use 900 MW faults on 12 generators
- Integrate dynamics for 25 seconds
- The frequency repsonse is compared on 509 nodes homogeneously spread over the grid
- $m(\mathbf{x})$ and $d(\mathbf{x})$ are expressed as linear combination of the first 130 eigenvectors of the grid Laplacian
- We learn the coefficients of the eigenvectors

Learning the PDE

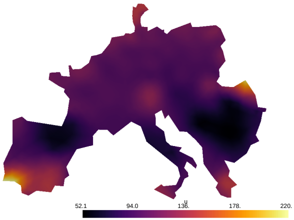
Dynamical Parameters

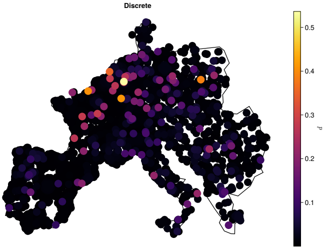




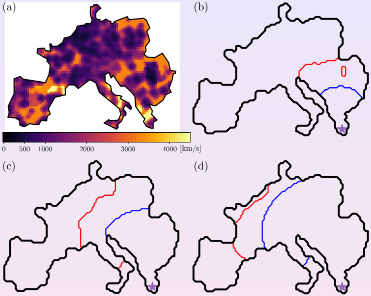
Learning the PDE

Dynamical Parameters





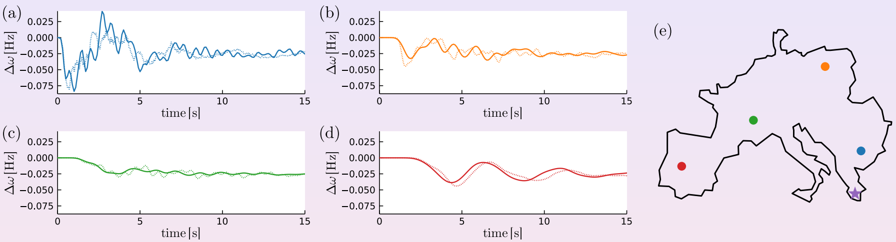
Physics Test: Speed of EM waves: Inhomogeneous Map



- PanTaGruEl model: 3809 buses, 618 generators and 4944 lines. (3221 nodes in the "full" discretization of our PDE model.)

- (a) Assessment of the local propagation speed as $c(\boldsymbol{x}) = \sqrt{b(\boldsymbol{x})/m(\boldsymbol{x})}$.
- (b)-(d) Fronts of the perturbation at incremental time intervals of $\Delta t = 0.6$s, after a fault in Greece (violet star), for inhomogeneous (red) and average parameters (blue) – slower.

Physics Test: Frequency Response of Generators



PDE vs Ground Truth (ODE)

- Response in (a) Bulgaria, (b) Poland, (c) France, and (d) Spain to a 900 MW loss of power in Greece.
- dotted – PDE, solid – Ground Truth (ODEs)

Summary & Path Forward

What did we achieve so far?

- **Construction** of the **reduced** PDE model (of the ODE/swing equations). Included:
- **Validation** via **Static** and **Dynamic** Tests – reduced PDE vs Ground Truth (ODEs)
- **Observation:** — PDE offers significant gain in efficiency. Need further development.

Work in Progress: Towards Physics (System 2) Informed ML

- Improving warm start and functional maps for $m(\boldsymbol{x}), d(\boldsymbol{x})$ and $b_{\alpha\beta}(\boldsymbol{x})$
- Adaptive grids, towards control

Goal: Efficient & Accurate Evaluation of Multiple Scenarios

- Automatic & much faster than real-time dynamics & control

Outline

1. Physics Helps to Build Reduced Models [Power]
   - Physics = Electro-Mechanical Waves
   - From ODE to PDE for Model Reduction
   - Power System Transients With Physics-Informed PDE
2. Towards Control Under Insults & Uncertainty [Gas]
   - Use Case of Israel Natural Gas System
   - Modeling: Gas Flow. Staggered Grid Method
   - Insults, Uncertainty & Control
3. Predict & Prevent Against Rare Events [Heat]
   - Multiplicative Noise
   - Thermal Control of Buildings
   - Fat (Algebraic) Tails & Synthesis

Natural Gas System: Setting

Control of Linepack in Natural Gas System: Balancing Limited Resources Under Uncertainty

Criston Hyett, Laurent Pagnier, Jean Alisse, Lilah Saban, Igal Goldshtein, Misha Chertkov
University of Arizona & NOGA Israel

May 17, 2023

PSIG Pipeline Simulation Interest Group

Natural Gas System: Setting

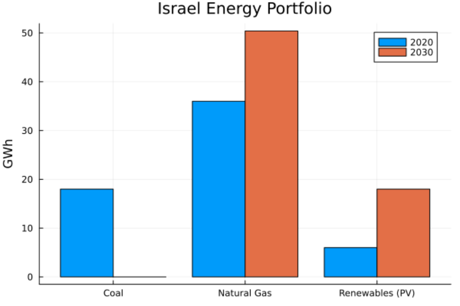
Israel Natural Gas

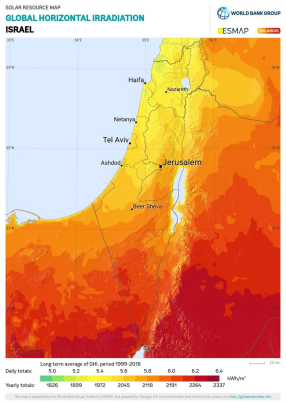
- Starting ~2010, large natural gas (NG) reserves were discovered off the coast of Israel
- These supplies transitioned Israel from an energy importer to an exporter of NG, and set NG as main fuel for electricity production.
- Following the global agreement at the Paris Climate Accords in 2015, Israel plans to convert remaining coal-fired plants to NG.

Natural Gas System: Setting

Israel Natural Gas

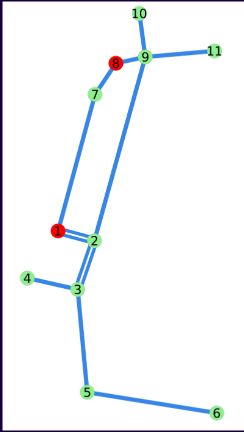
- Simultaneously, Israel is committed to increasing renewables (mainly PV), with the goal of 30% production by 2030





© 2020 The World Bank, Source: Global Solar Atlas 2.0, Solar resource data: Solargis.

Reduced Model of Israel Natural Gas



Control of Linepack in Natural Gas System

Effective Gas Flow Equations

Under reasonable assumptions, the system of PDEs governing gas flow is

$$\partial_t \rho + \partial_x \phi = 0$$
$$\partial_t \phi + \partial_x p = -\beta \frac{\phi|\phi|}{\rho}$$

Supplemented with initial

$$\rho(x,0) = \rho_0(x)$$
$$\phi(x,0) = \phi_0(x)$$

And boundary conditions at each node

$$\rho_i(t) \; or \; \phi_i(t)$$

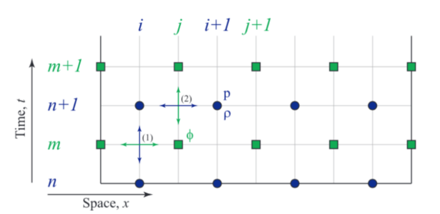
And an equation of state relating pressure and density – we use CNGA

$$p(\rho) = Z(p,T)RT\rho$$

PSIG

Control of Linepack in Natural Gas System

Staggered-Grid Method



Gyrya, Vitaliy, and Anatoly Zlotnik. "An explicit staggered-grid method for numerical simulation of large-scale natural gas pipeline networks." *Applied Mathematical Modelling* 65 (2019): 34-51.

- Explicit, 2^{nd} order, centered finite difference method
- Solves conservation of mass and momentum on staggered grids
- Conserves mass to numerical precision
- Stable given condition is satisfied
  - $\sqrt{p'(\rho)}\,\frac{\Delta t}{\Delta x} \leq 1$

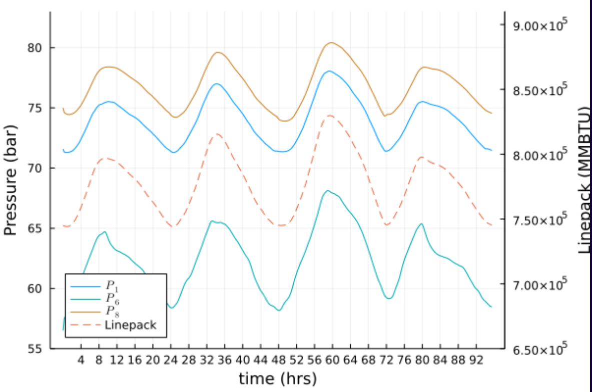
Insults, Uncertainty & Control of Natural Gas System

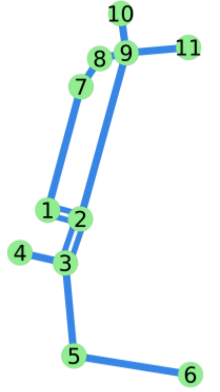
Scenarios

Scenario #	Description	Features
1	A reference week in August	Pressure variation in flow-control regime
2	Scenario #1 with empirical noise added to demand curves, supplies unchanged	Linepack and pressure drift when using flow control and uncertain demand
3	Scenario #2 with insult at node 1	Introduce the notion of survival time, and set baseline without any controls.
4	Scenario #3 with insult time change to trough of linepack timeseries.	Illustrate that survival times change with timing of insult.
5	Scenario #4 with step-wise supply increase from node # 8.	Survival times lengthen, but become less certain.
6	Scenario #5 with step-wise curtailing of demand.	No low pressure crossings are found. The high pressure at node # 8 shows need for finer control.

Insults, Uncertainty & Control of Natural Gas System

Results: Scenario 1





Nominal week in August

Insults, Uncertainty & Control of Natural Gas System

Uncertainty

Moderate uncertainty at demand nodes represented via substitution of stochastic process for boundary condition

$$d_i(t) \to X_i(t)$$

where

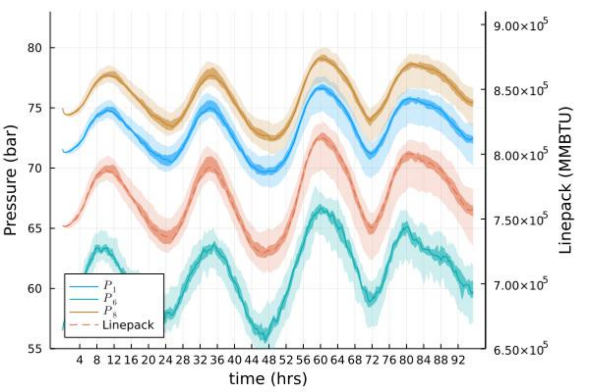
$$dX_i(t) = \alpha\big(d_i(t) - X_i(t)\big) + \gamma dW$$

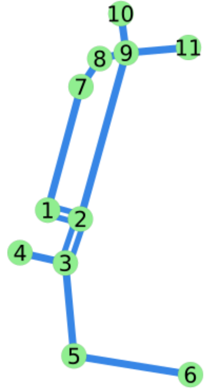
Is an Ornstein-Uhlenbeck process

- $E[X_i(t)] = d_i(t)$
- $Var\big(X_i(t)\big) = \frac{\gamma}{2\alpha}(1 - e^{2\alpha t})$
- The parameters were tuned heuristically to ensure the mean was respected, and the variance approaches
$$Var\big(X_i(t)\big) \approx 0.01\mu_i^2$$
with μ_i being the mean withdrawal of node i.

Insults, Uncertainty & Control of Natural Gas System

Results: Scenario 2

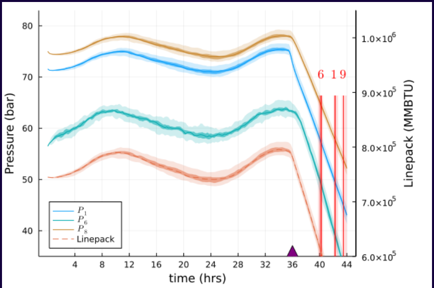


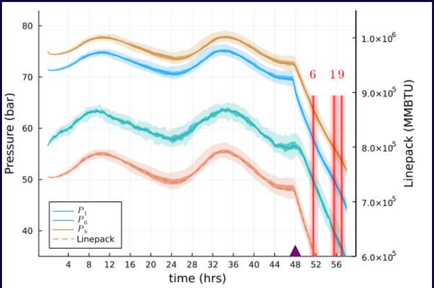


Distributions of linepack and pressures for random perturbations added to August week

Insults, Uncertainty & Control of Natural Gas System

Results: Scenario 3 & 4





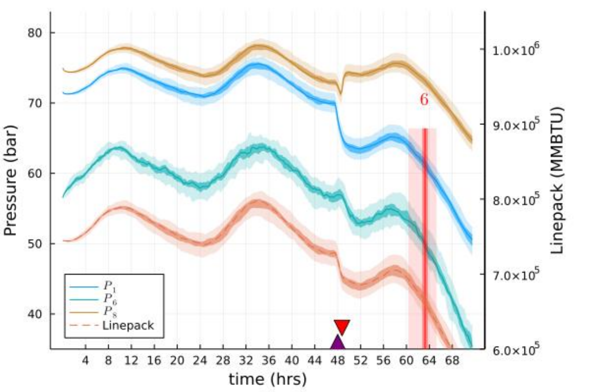
Linepack and pressures responding to loss of supply at node 1. (Left) shows the insult at a peak of intraday linepack, and (right) shows the same insult at the trough.

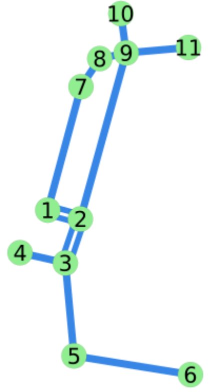
$\tau = 4.13 \pm 0.38$ hrs $\qquad \tau = 3.58 \pm 0.89$ hrs

PSIG

Insults, Uncertainty & Control of Natural Gas System

Results: Scenario 5





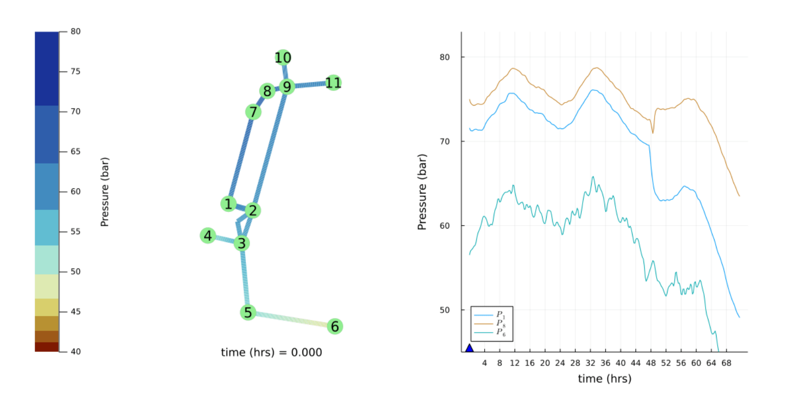
Insult at hour 48, implementing a max-flow control on the remaining supply at node 8

$$\tau = 14.17 \pm 4.07$$

PSIG

Insults, Uncertainty & Control of Natural Gas System

Results: Scenario 5

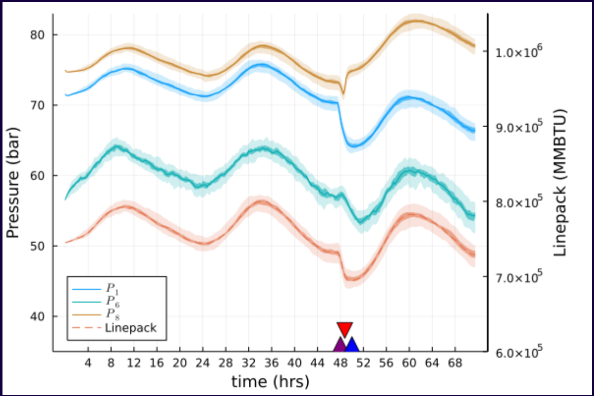


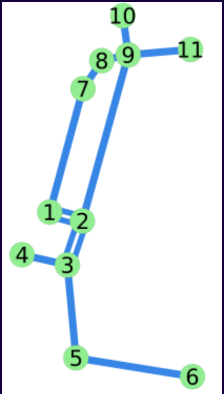
Insult at hour 48, implementing a max-flow control on the remaining supply at node 8

$$\tau = 14.17 \pm 4.07$$

Insults, Uncertainty & Control of Natural Gas System

Results: Scenario 6





Insult at hour 48, implementing a max flow control at node 8, and curtailing demand at hour 50.

Summary & Path Forward

What did we achieve so far?

- Built Reduced Model of Israel Gas System. Considered Realistic Insult(s) and Uncertainty Scenarios.
- Started to work on implementing (for gas and gas+power systems) and developing new Tools for Sensitivity Analysis – what if ... with Julia, Automatic Differentiation of adjoints

Work in Progress:

- Real-time Modeling: insults at any time, broader set of uncertain scenarios
- Effect on Power Systems – Emergency Transition to Secondary Fuel
- New Tools for New Problems – under umbrella of Physics-Informed & Data-Driven Learning & Control

Outline

"Universality and Control of Fat Tails" MC

- IEEE Control System Letters & CDC 2023, `https://ieeexplore.ieee.org/document/10131981`
- + reinforcement learning extension(s): work in progress with S. Konkimalla & L. Pagnier

Linear System Driven by Multiplicative Noise

$\frac{dx_i}{dt} \quad \sum_j \left(m_{ij} + \sigma_{ij}(t)\right) x_j(t) + \xi_i(t) + u_i(t)$

- $\boldsymbol{m} = (m_{ij} : \forall i, j = 1, \cdots, d)$=const
- $\boldsymbol{\sigma}(t) = (\sigma_{ij}(t) : \forall i, j)$ – zero-mean stochastic
- $\boldsymbol{\xi}(t) = (\xi_i(t) : \forall i)$ – zero-mean white-Gaussian
- $\boldsymbol{u}(t) = (u_i(t) : \forall i)$ – vector of control

$\boldsymbol{\sigma}(t)$ Multiplicative Stochastic

- $\frac{d}{dt}\boldsymbol{W} = \boldsymbol{\sigma}\boldsymbol{W}$, $\boldsymbol{W}(t)$ – $T\exp$
- Oseledets theorem: at $t \to \infty, \log(\boldsymbol{W}^+\boldsymbol{W})/t \to$ const
    - $\boldsymbol{W}\boldsymbol{f}_i = c_i \boldsymbol{f}_i$, $\lambda_i = \log|\boldsymbol{W}\boldsymbol{f}_i|/t$, $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_d$
    - $P(\lambda_1, \cdots, \lambda_d|t) \propto \exp\left(-tS(\lambda_1, \cdots, \lambda_d)\right)$
    - $S(\cdots)$ – Crámer function

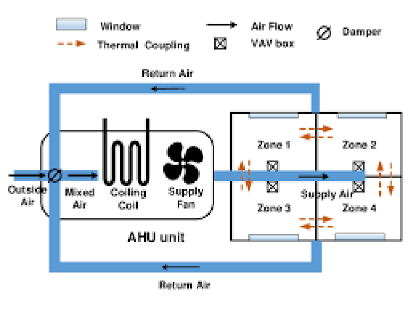
Active & Passive Swimmers

$$-\alpha\left(\frac{d\boldsymbol{r}}{dt} - \boldsymbol{\sigma}(t)\boldsymbol{r}\right) \qquad \boldsymbol{u}(t) + \xi(t)$$

- $\boldsymbol{u}$ – control exerted by an active swimmer:
  - keep in-sight
- $\boldsymbol{\sigma}(t)$ – fluctuating velocity gradient in "Batchelor" flow

Dynamics of Temperature in Multi-Zone Buildings

$$\frac{dT}{dt} \quad -c_o(T-T_o)-c_s(T-T_s)u(t)+\xi(t)$$ [as seen from a zone]



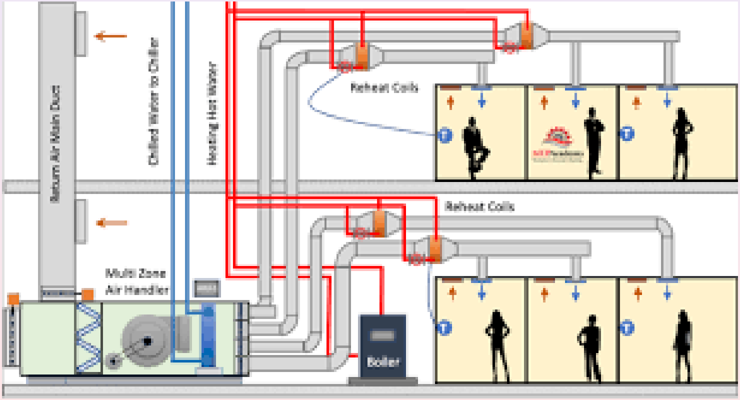
- T_o-outside and T_s-Air-Handling-Unit (AHU)
- c_o and c_s exchange rates

- $u(t)$ – control of the AHU opening
- Linearizing around "comfort" temperature/efforts
  - $0=-\underline{c}_o(\underline{T}-T_o)-c_s(\underline{T}-T_s)\underline{u}$
  - $c_o=\underline{c}_o+\sigma(t)$
  - $u(t)=\underline{u}+\phi\theta$ (+ linear feedback)
  - $\theta=T-\underline{T}$
- $\frac{d\theta}{dt}=-c(\phi)\theta+\tilde{\xi}(t)-\sigma(t)\theta$
- $c(\phi)=c_0+c_1\phi$, $c_0=\underline{c}_o+c_s\underline{u}$, $c_1=c_s(\underline{T}-T_s)$, $\tilde{\xi}(t)=\xi(t)+T_o\sigma(t)$

Dynamics of Temperature in Multi-Zone Buildings

Network (of zones) Generalization

- $\frac{d\theta_i}{dt} = -\left(c_i(\phi) + \sigma_{io}\right)\theta_i - \sum_{j\sim i}\left(\underline{c}_{ij} + \sigma_{ij}\right)(\theta_i - \theta_j) + \xi_i(t)$



White-Gaussian-Multiplicative: Fokker-Planck

- Multiplicative Noise = White Gaussian
- State Feedback Control: $\boldsymbol{u}(t) \to \boldsymbol{w}(\boldsymbol{x}(t))$ [prescribed]
- $\Rightarrow$ Fokker-Planck:
  $\left(\partial_{x_i}\left(w_i(\boldsymbol{x}) + m_{ij}x_j\right) + \kappa_{ij}\partial_{x_i}\partial_{x_j} + D_{ik;jl}\partial_{x_i}x_k\partial_{x_j}x_l\right) P(\boldsymbol{x}|\boldsymbol{w}) = 0$

Steady State Control

$$\phi^* = \arg\min_{\phi} \bar{C}(\phi), \quad \bar{C}(\phi) = \int d\boldsymbol{x} P(\boldsymbol{x}|\boldsymbol{w}_\phi) C(\boldsymbol{x}, \boldsymbol{w}_\phi)$$

$$C(\boldsymbol{x}, \boldsymbol{w}_\phi) = \underbrace{C_c(\boldsymbol{w}_\phi)}_{\text{cost of control}} + \underbrace{C_g(\boldsymbol{x})}_{\text{cost of achieving the goal, e.g.}(\boldsymbol{x}\boldsymbol{x}^T)^{q/2}}$$

Consider Examples ...

Thermal Control (single zone)

Linear feedback, M-noise short correlated

Fokker-Planck → solution → optimal

- $\left(\partial_\theta c(\phi)\theta + \kappa\partial_\theta^2 + D(\partial_\theta\theta)^2\right) P(\theta|\phi) = 0$
- $P(\theta|\phi)=\sqrt{\frac{D}{\pi\kappa}}\frac{\Gamma\left(\frac{c(\phi)}{2D}+\frac{1}{2}\right)}{\Gamma\left(\frac{c(\phi)}{2D}\right)}\left(1+\frac{D\theta^2}{\kappa}\right)^{-\frac{1}{2}-\frac{c(\phi)}{2D}}$
- MqP-stable at $\phi > \phi^{(s)} = (D(\max(q,2)-1)-c_0)/c_1$.
- optimal: $\phi^* = \frac{2D-c_0+\sqrt{(2D-c_0)^2+\beta c_1^2}}{c_1}$

General Model: Synthesis

- Linear feedback: $u_i(t) \to \sum_j \phi_{ij} x_j$
- $\boldsymbol{x}(t) = \exp(-(\boldsymbol{m}+\phi)t)\boldsymbol{W}(t)\tilde{x}$,
- $\tilde{x}$ stabilizes to a constant as t grows
- $\log P_{st}(\boldsymbol{x}\boldsymbol{f}_i^T) \propto 2\boldsymbol{f}_i\,(\boldsymbol{m}+\phi)\,\boldsymbol{f}_i^T S_i''(0) \log \frac{x_d}{\boldsymbol{x}\boldsymbol{f}_i^T}$
- Dependence on $\tilde{x}$ is "under logarithm" – thus weak and replaced by x_d
- Statistics of any norm of $\boldsymbol{x}$ is equivalent to statistics of $(\boldsymbol{x}\boldsymbol{f}_1^T)$ associated with the largest Lyapunov exponent

use white multiplicative noise

- when control is slower than $1/\bar{\lambda}_1$

Conclusions & Path Forward

- Analyzed linear dynamic system driven by additive and multiplicative noise, stabilized by feedback
- $\Rightarrow$ Algebraic = fat tail (when stabilized).
- Examples, e.g. on Thermal Control of Buildings but also in Fluid Mechanics

- Extend to complex cases, e.g. multi-zone engineered systems
- Towards data driven approaches,
  e.g. via physics-informed reinforcement learning (hierarchy of models taking advantage of the multiplicative+additive theory)

Support is Appreciated !!

- **Energy Systems**: UArizona start up + DOE/ARPA-E + NSF/RareEvents

Thanks for your attention !

- Research focused, since 1976, one of US first Ph.D. in Applied Math [dynamical, integrable systems, turbulence]
- Interdisciplinary: 100+ professors/ 26 departments/ 8 colleges across UA campus (CoS & CoE & Optics – top 3)
- Mixing traditional @ contemporary Applied Math
- 64 Ph.D students (**14**/12/13/16/10 in **2023**/22/21/20/19)
- **3 Core Courses** (1st year -- Methods, Analysis, Algorithms) https://appliedmath.arizona.edu/students/new-core-courses
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- Looking for new partnership in Applied Math for AI

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