Electric Demand Management Without Price Elasticity Models

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Acknowledgements

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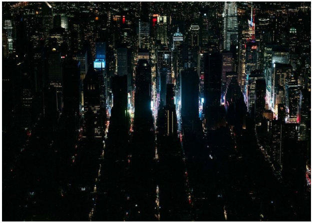
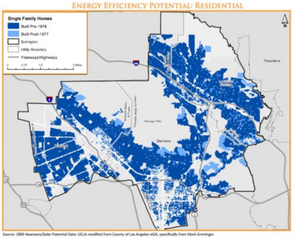
Kind support of:

Benefits of demand flexibility



Resilience

Sustainability and energy savings

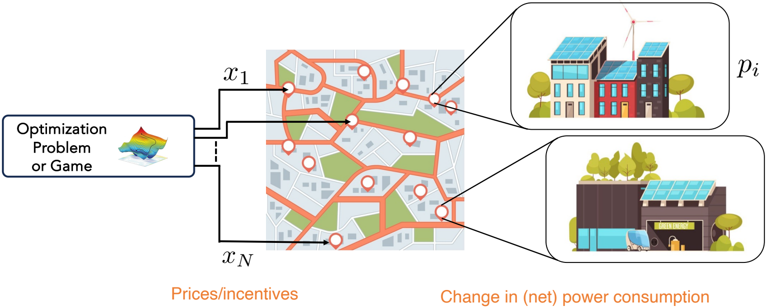


Energy equity

Infrastructure cost

Estimates in reports from RMI, NREL, U.S. EIA, and many others ...

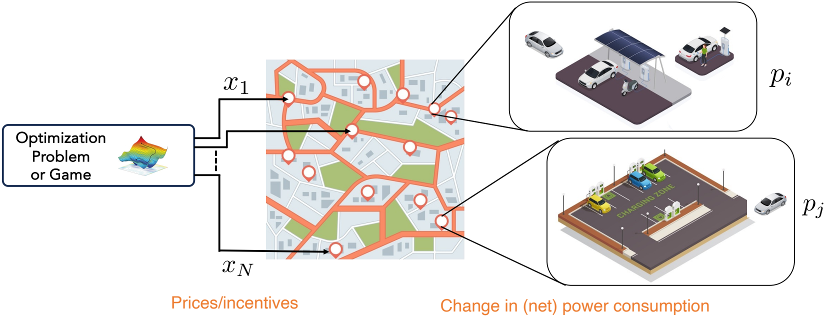
Flexibility via incentive-based control



Idea: Incentivize DER owners to adjust the power consumption or generation to provide services to the grid (at a given time-scale)

[Mohsenian-Rad et al'10], [Yang et al'15] [Li et al'16], [Zhou et al'17], [De Paola et al'17], [Gong et al'19] and many others ..

Flexibility via incentive-based control



Idea: Incentivize EV drivers change during specific times to provide services to the grid

[Sojoudi-Low'11], [Gan et al'12], [Gharesifard et al '13], [Yoon et al'15], [Paccagnan et al '18], [Perotti et al'23], and many others ..

Example of desirable outcome

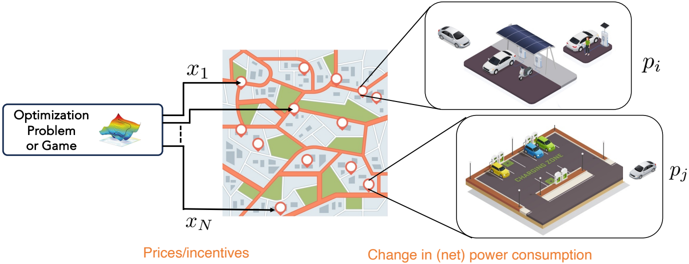
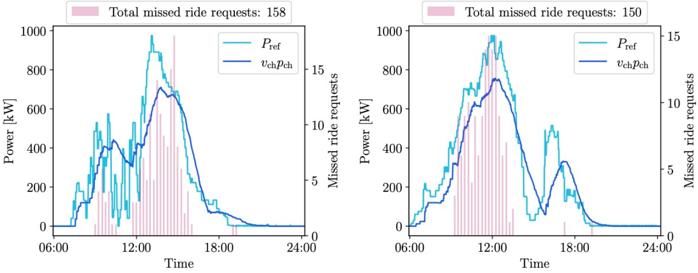


Figure from [Perotti et al'23]

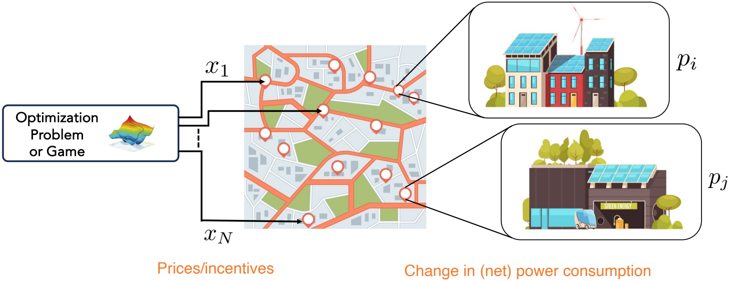


Price-based or incentive-based control

A stylized formulation:

$$\min_x f(x) := \left(\sum_{i=1}^{N} \mathcal{E}_i(x_i) - p^* \right)^2$$

where $p_i = \mathcal{E}_i(x_i)$ modeling the elasticity to prices or the response of DER owners



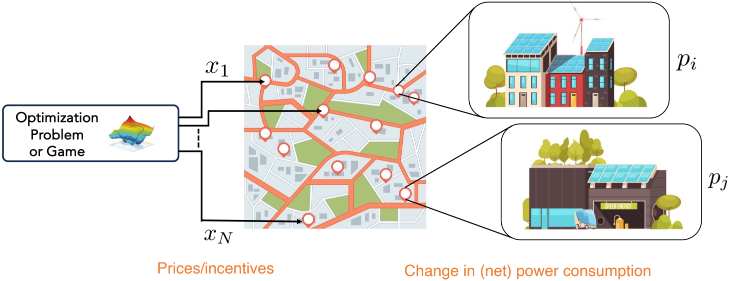
Note: need to know (perfectly) the functions $\mathcal{E}_i(x_i)$ for all $i = 1, \ldots, N$

Price-based or incentive-based control

A stylized formulation:

$$\min_x f(x) := \left(\sum_{i=1}^{N} \mathcal{E}_i(x_i) - p^* \right)^2 + \sum_{i=1}^{N} \mathcal{E}_i(x_i) x_i$$

where $p_i = \mathcal{E}_i(x_i)$ modeling the elasticity to prices or the response of DER owners



Note: need to know (perfectly) the functions $\mathcal{E}_i(x_i)$ for all $i = 1, \ldots, N$

Price-based or incentive-based control

A stylized bi-level formulation:

$$\min_{x,p} f(x,p) := \sum_{i=1}^{N} (p_i - p_i^*)^2 + \sum_{i=1}^{N} p_i x_i$$

$$\text{s.t.} \quad p_i \in \operatorname*{argmin}_{r \in \mathcal{P}} C_i(r) - x_i r \;\; i = 1, \ldots, N$$

where

$$\operatorname*{argmin}_{r \in \mathcal{P}} C_i(r) - x_i r$$

is the best response to prices of DER owner or aggregator i

What is this? Re: Classical Stackelberg game between a leader (utility company) and followers (DER owners)

Note: need to know (perfectly) the functions $C_i(r) - x_i r$ for all $i = 1, \ldots, N$

Elasticity and response are uncertain

From the U.S. Energy Information Administration: "price elasticity of demand, or the percentage change in energy consumption relative to the percentage change in prices, all other factors being equal."

What about "other factors"?

- Anxiety for low state of charge
- Different driving patterns
- Traffic congestion
- Tourists driving through the area
- ...

Bottom line: we cannot easily model $\mathcal{E}_i(x_i)$ or $C_i(x_i)$

Elasticity and response are uncertain

From the U.S. Energy Information Administration: "price elasticity of demand, or the percentage change in energy consumption relative to the percentage change in prices, all other factors being equal."

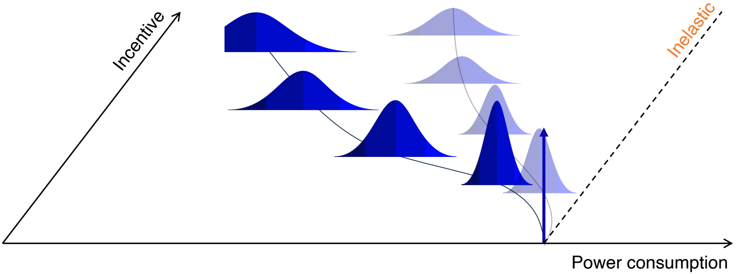
What about "other factors"?

- Different control systems
- Different preferences
- Different weather
- Traveling
- ...

Bottom line: we cannot easily model $\mathcal{E}_i(x_i)$ or $C_i(x_i)$

A simple approach based on a simple observation

The likelihood of changing power consumption depends on the price *and* other factors that are difficult to model



Modeling uncertain elasticity with decision-dependent problems:

$$x^* \in \underset{x \in \mathcal{X}}{\operatorname{argmin}} \left\{ F(x) := \underset{p \sim D(x)}{\mathbb{E}} f(x, p) \right\}$$

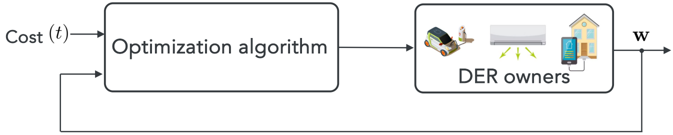
where $D : \mathbb{R}^d \to \mathcal{P}(\mathbb{R}^k)$ is a distributional map

Main goal

Decision-dependent problem (t time index):

$$x_t^* \in \operatorname*{argmin}_{x \in \mathcal{X}_t} \left\{ F_t(x) := \mathop{\mathbb{E}}_{p \sim D_t(x)} f_t(x, p) \right\}$$

Objective (informal): Design an algorithm to dispatch prices x_t based on i) the cost $f_t(x, p)$ and ii) demand measurements p_t, that are "as close as possible" to the "optimal prices" $\{x_t^*\}$



Key operating assumptions: the algorithm has no access to elasticity models or the map $D_t(x)$

Challenges

Challenge #1: Cost may be non-convex in many settings

Challenge #2: We cannot compute the gradient

Why? Gradient requires distribution information:

$$\nabla F(x) = \mathop{\mathbb{E}}_{p \sim D(x)} [\nabla_x f(x, p) + f(x, p) \nabla_x \log \rho(p|x)]$$

An answer to #1: notion of equilibrium point

Definition. A point $\bar{x} \in \mathcal{X}_i$ is an equilibrium point if:

$$\bar{x} \in \operatorname*{argmin}_{x \in \mathcal{X}_t} \mathop{\mathbb{E}}_{p \sim D_t(\bar{x})} f_t(x, p).$$

An equilibrium point is is optimal for the distribution that it induces on p.

Equilibrium Points

Two key results from [Perdomo et al'20]

Existence of equilibria). Suppose that
(i) $x \mapsto f(x,p)$ is continuous and convex,
(ii) $D : \mathbb{R}^d \to (\mathcal{P}(\mathbb{R}^k), W_1)$ is continuous,
(iii) $\mathcal{X}$ is convex and compact.
Then there exists $\bar{x} \in \mathcal{X}$ such that

$$\bar{x} \in \operatorname*{argmin}_{x \in \mathcal{X}} \mathop{\mathbb{E}}_{p \sim D(\bar{x})} f(x,p).$$

Theorem (Unique Equilibrium Point). If $x \mapsto f(x,p)$ is γ-strongly convex, $\nabla_x f$ is L-Lipschitz, D is ε-Lipchitz, and $\frac{\varepsilon L}{\gamma} < 1$, then $\bar{x}$ is unique.

ε-Lipschitz distributional map: $W_1(D(x), D(y)) \leq \varepsilon \|x - y\|$ for all $x, y \in \mathbb{R}^d$.

Example: if $D(x) = \mathcal{N}(Mx, \Sigma)$, then $\varepsilon = \|M\|_2$.

Time-varying Optimal Pricing Problem Revisited

Time-varying optimal pricing problem (t time index):

$$x_t^* \in \underset{x \in \mathcal{X}_t}{\operatorname{argmin}} \left\{ F_t(x) := \underset{p \sim D_t(x)}{\mathbb{E}} [f_t(x,p)] \right\}$$

Assumptions: $x \mapsto f_t(x,p)$ is γ-strongly convex, $\nabla_x f_t$ is L-Lipschitz, D_t is ε-Lipchitz, and $\frac{\varepsilon L}{\gamma} < 1$ for all t.

Goal: track the trajectory of equilibia $\{\bar{x}_t\}_{t \in \mathbb{N}}$; i.e. bound $\limsup_{t \to \infty} \|x_t - \bar{x}_t\|$

Connecting to the previous examples:

$$\sum_{i=1}^{N} \mathcal{E}_i(x_i) - p^* \Bigg)^2 \quad \text{is replaced by:} \quad \underset{p \sim D_t(x)}{\mathbb{E}} \left[\Bigg(\sum_{i=1}^{N} p_i - p^* \Bigg)^2 \right]$$

Online Equilibrium Gradient Descent

A "conceptual" online equilibrium seeking:

$$x_{t+1} = \mathrm{proj}_{\mathcal{X}_t}\left(x_t - \eta_t \, \mathbb{E}_{p\sim D_t(x_t)} \nabla_x f_t(x_t, p)\right)$$

Theorem (Error Bound) [Wood-Bianchin-Dall'Anese '22] Assume that:
(i) $x \mapsto f_t(x,p)$ is γ-strongly convex and $\nabla_x f_t$ is L-Lipschitz, (ii) D_t is ε-Lipchitz, (iii) $\frac{\varepsilon L}{\gamma} < 1$. Then,

$$\|x_t - \bar{x}_t\| \leq (\rho + \eta\varepsilon L)^t \|x_0 - \bar{x}_0\| + \Delta(1 - (\rho + \eta\varepsilon L))^{-1}, \quad \forall t \in \mathbb{N}$$

where $\rho = \max\{|1-\eta\gamma|, |1-\eta L|\}$ and $\Delta := \sup_{i\in\mathbb{N}}\{\|\bar{x}_{i+1} - \bar{x}_i\|\}$.

Corollary (Linear Tracking). If $\eta \in \left(0, \frac{2}{(1+\varepsilon)L}\right)$, then

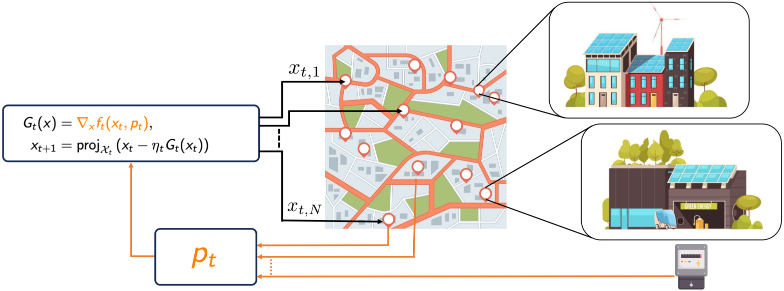
$$\limsup_{t\to\infty} \|x_t - \bar{x}_t\| \leq \Delta(1 - \rho - \eta\varepsilon L)^{-1}.$$

Closed-loop Equilibium Seeking

Idea: replace $\mathbb{E}_{p\sim D_t(x_t)} \nabla_x f_t(x_t, p)$ with a single-sample gradient estimate

$$G_t(x) = \nabla_x f_t(x_t, p_t), \quad p_t \sim D_t(x_t)$$
$$x_{t+1} = \text{proj}_{\mathcal{X}_t}(x_t - \eta_t G_t(x_t))$$

This leads to a feedback system: deploy prices and then measure power



Analysis

Perturbation of the conceptual equilibirum seeking method with error:

$$e_t := \nabla_x f_t(x_t, p_t) - \mathbb{E}_{p \sim D_t(x_t)} \nabla_x f_t(x_t, p)$$

Theorem (Mean Error Bound). Suppose that
(i) f_t is γ_t-strongly convex and L_t-smooth,
(ii) D_t is ε_t-Lipschitz continuous.
Then, if $\frac{\varepsilon_t L_t}{\gamma_t} < 1$,

$$\mathbb{E}[\|x_t - \bar{x}_t\|] \leq a_t \|x_0 - \bar{x}_0\| + \sum_{i=1}^{t} \Big(b_i \left(\Delta_i + \eta_i \mathbb{E}[\|e_t\|]\right)$$

with $a_t := \Pi_{i=1}^{t}(\rho_i + \eta_i \varepsilon L_i)$ and

$$b_i = \left\{ \Bigg(\begin{array}{ll} 1 & i = t \\ \Pi_{k=i+1}^{t}(\rho_k + \eta_k \varepsilon_k L_k) & i < t \end{array} \right.$$

Analysis

Perturbation of the conceptual equilibirum seeking method with error:

$$e_t := \nabla_x f_t(x_t, p_t) - \mathbb{E}_{p\sim D_t(x_t)} \nabla_x f_t(x_t, p)$$

Model: Each entry of e_t is a Sub-Weibull random variable

Definition (Sub-Weibull rv). A random variable $X \in \mathbb{R}$ is sub-Weibull if $\exists\, \theta > 0$ such that (s.t.) one of the following conditions is satisfied:

(i) $\exists\ \nu_1 > 0$ s.t. $\mathbb{P}[|X| \geq \epsilon] \leq 2e^{-(\epsilon/\nu_1)^{1/\theta}}$, $\forall\, \epsilon > 0$.

(ii) $\exists\ \nu_2 > 0$ s.t. $\|X\|_k \leq \nu_2 k^{\theta}$, $\forall\, k \geq 1$

where $\|X\|_k := (\mathbb{E}[|X|^k])^{1/k}$.

Short-hand notation: $X \sim \mathrm{subW}(\theta, \nu)$ means sub-Weibull rv according to (ii).

Analysis

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Model: Each entry of e_t is a Sub-Weibull random variable

What is it?

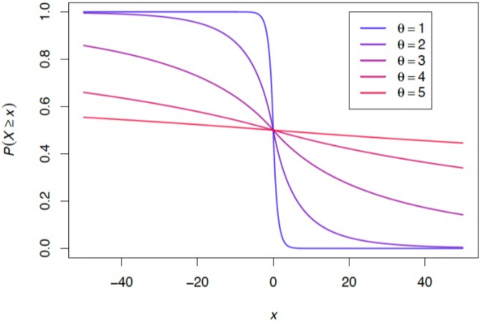


Image from [Vladimirova et al'20]

A Step-Wise Error Bound

Theorem (Stochastic Error Bound) Under the same assumptions of the previosu theorem, let $\|e_t\| \sim \mathrm{subW}(\theta_t, \nu_t)$ Then, if $\frac{\varepsilon_t L_t}{\gamma_t} < 1$, for any $\delta \in (0,1)$, the following bound holds with probability $\geq 1 - \delta$.

$$\|x_t - \bar{x}_t\| \leq c(\theta) \log^{\theta}\left(\frac{2}{\delta}\right)\left(a_t \|x_0 - \bar{x}_0\| + \sum_{i=1}^{t} b_i \left(\Delta_i + \eta_i \nu_i\right)\right)$$

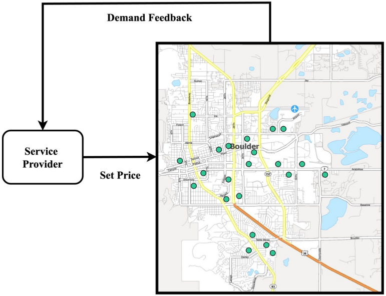
Corollary. Let $\lambda \in (0,1)$ and

$$\eta_t \in \left[\frac{1-\lambda}{\gamma_t + \varepsilon_t L_t}, \frac{1+\lambda}{L_t(1+\varepsilon_t)}\right]$$

Then $\sup_{t \geq 0}\{\rho_t + \varepsilon_t L_t \eta_t\} \leq \lambda < 1$, and

$$\Pr\left(\limsup_{t \to \infty} \|x_t - \bar{x}_t\| \leq \frac{\Delta + \eta \nu}{1 - \lambda}\right) = 1.$$

Example: EV Market Problem



Example of cost function:

$$f_t(x) = \sum_{i=1}^{N} \frac{\gamma_{t,i}}{2} x_i^2 - p_i x_i + u_{t,i} x_i$$

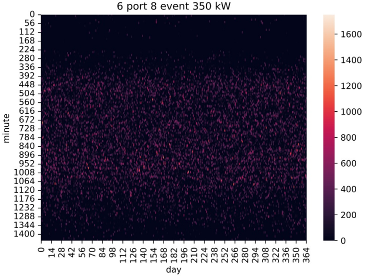
- Profit: $p_i x_i$
- Utility cost: $u_{t,i} x_i$
- Quality-of-service or equity: $\frac{\gamma_{t,i}}{2} x_i^2$

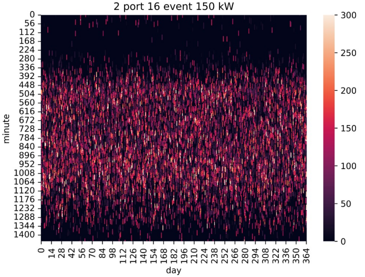
Example of distributional map:

$$p \overset{d}{=} p_{t,0} + E_t x$$

- Price elasticity of demand: E_t
- Stationary demand: $p_{t,0}$

Data

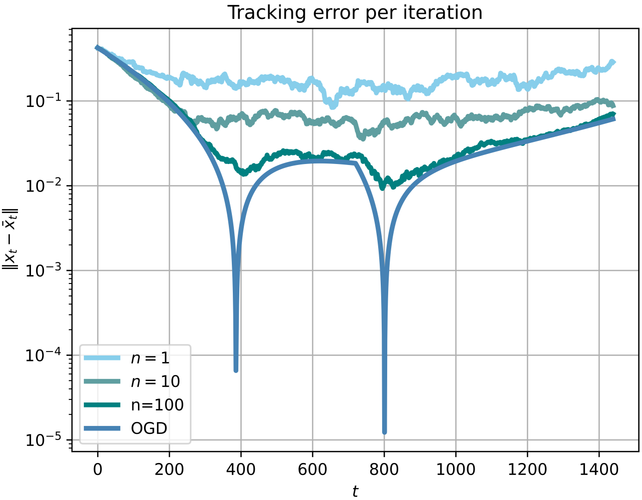




Demand Data

- 18 total stations
- Vary in port power, number of ports, and demand rate
- Data from NREL [Gilleran et al'21]

Results



Tracking error per iteration
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\|x_t - \bar{x}_t\|$
0
200
400
600
800
1000
1200
1400
t
$n = 1$
$n = 10$
n=100
OGD

Extensions

- Saddle-point problem [Wood-Dall'Anese '23]

$$\min_{x \in \mathcal{X}} \max_{y \in \mathcal{Y}} \left\{ F(x,y) := \mathop{\mathbb{E}}_{p \sim D(x,y)} [f(x,y,p)] \right\} \Bigg($$

$x \in \mathbb{R}^d$, $y \in \mathbb{R}^n$, $w \in \mathbb{R}^k$.

Motivation: Competitive energy markets between two EV charging providers.

- Multi-player monotone games [soon]

$$\min_{x_i \in \mathcal{X}_i} \left\{ F(x_i, x_{-i}) := \mathop{\mathbb{E}}_{p \sim D(x_i, x_{-i})} [f(x_i, x_{-i}, p)] \right\} \Bigg(\quad i = 1, \ldots, P$$

Motivation: Multi-operator or multi-utility competitive energy markets

Conclusions

- Elasticity difficult to model in modern energy systems
- Decision-dependent formulations to model uncertain elasticity
- Feedback-based gradient method to design prices
- Performance assessment relative to equilibria

Thank you!

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