A Multi-Function AAA Algorithm Applied to Frequency-Dependent Line Modeling

Preprint

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A Multi-function AAA Algorithm Applied to Frequency Dependent Line Modeling

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***Abstract*—Modeling of power distribution system components that are valid for a wide range of frequencies are crucial for highly accurate modeling of electromagnetic transient (EMT) events. This has recently become of interest due to the improvements needed for the resilient operation of distribution systems. Vector fitting (VF) is a very popular and commonly used algorithm for wide band representations of power system components in EMT simulations. In this research, we present a new multi-input rational approximation algorithm (MIAAA) and illustrate its advantages with respect to VF using examples of approximations of admittance matrices discussed in the literature. We show that MIAAA not only outperforms VF in terms of achieving better accuracy using lesser number of poles, but also has no numerical issues achieving convergence. In contrast to VF, MIAAA is not sensitive to the location of input sample points and it does not require good estimates for the location of the desired approximation poles. The novelty of this research work is the use of recent mathematical results to solve existing challenges in distribution system modeling and to develop rational approximations for power system models that intend to be optimal in terms of accuracy and performance.**

***Index Terms*—Rational approximations, function approximation, barycentric formula, vector fitting, AAA algorithm, power system modeling, computational modeling.**

I. Introduction

Electromagnetic transients programs (EMTPs) are essential tools for protection engineering and design, as well as reliability and resiliency studies on transmission and distribution systems. Examples of EMTPs include EMTP-RV [1], PSCAD [2], and ATP-EMTP [3]. Included in these EMTPs are frequency dependent (FD) line models as well as the capability to create a frequency dependent network equivalent (FDNE) from given characteristics. Essential to these FD and FDNE line models is the ability to accurately and efficiently approximate frequency dependencies via rational functions represented in partial fractions form. The partial fraction representation has an explicit, analytic Fourier transform in terms of exponential functions which is used to quickly compute time domain solutions via a recursive convolution algorithm [4].

The standard algorithm and associated software package for multi-input rational approximations used in FD and FDNE models is the vector fitting (VF) algorithm [5]. While ubiquitous, we believe that VF can be replaced by a more accurate and stable algorithm that provides shorter representations (fewer number of common poles in the rational approximations) and offers a path to achieve optimal approximation order (minimal number of poles) for any user desired target accuracy.

Adaptive Antoulas-Anderson (AAA) [6] is a well-known algorithm designed to approximate functions via rational functions through the intermediate use of a barycentric representation (a quotient of two partial fractions) which is then converted to a standard partial fractions representation. To extend AAA to problems where VF is used, two changes must be made. First, the AAA algorithm must be modified to work with multiple input functions. Second, to correctly capture asymptotic behavior in, for example, characteristic admittance matrices, AAA must be modified to be able to compute rational approximations so that their numerator and denominator may have different degrees. For the ease of understanding and effortless reproduction of this work, we present the mathematical background and formulation behind these approaches before discussing our numerical results.

The layout of the paper is as follows: in Section 2 we review the VF and the single-input AAA algorithms and in Section 3 we present the new MIAAA algorithm by explaining how to extend the single-input AAA algorithm to the multi-input setting. In Section 4 we compare the performance of MIAAA and VF on two problems from electromagnetic transients modeling. In Section 5 we conclude our work and discuss future directions of research.

We say that a rational function r has order (N, M) if its numerator is a polynomial of at most degree N, and its denominator of at most degree M, whereas r has *exact* order (N, M) if the degrees of numerator and denominator are exactly N and M.

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II. Background

We provide a brief overview of VF and detailed background on the (single input) AAA algorithm since our multi-input algorithm extends the ideas of that case. AAA is related to VF in that they are both iterative algorithms that solve a least squares problem at each step of the iteration and after convergence, both algorithms find the zeros of a rational function written in partial fractions form. Thus, both algorithms rely on iterative solutions of linear systems followed by a nonlinear algorithm to determine the desired poles of the solution. Once poles are obtained, residues of the final rational approximation can be obtained by a linear procedure.

A. Vector Fitting

VF fits a set of sample values $\{f(s_n)\}_{n=1}^N$ using a rational function written in partial fraction form

$$r(s) = \sum_{m=1}^{M} \frac{r_m}{s - p_m} + d_0 + d_1 s. \tag{1}$$

The poles p_m, residues r_m, and constants d_0 and d_1 are found in the following way. A set of initial pole guesses $\{z_n\}_{1\le n\le N}$ is provided by the user and the corresponding residues and constants are computed via least squares. The procedure is iterated to obtain updated poles (pole relocation) until some convergence criteria is met and the resulting set is used as the approximation of the target poles p_m.

The algorithm is easily generalized to the multi-function case which takes as input the samples of several functions and finds partial fractions approximations of all of them using a common set of poles. For practical use, it is important to keep the number of poles (and hence, the number of parameters describing the approximations) as small as possible for a desired approximation accuracy. Because of the absence of optimization with respect to the number of poles, we show in this paper that VF generates non optimal rational approximations and that our alternative algorithm can significantly improve the total number of parameters used in the approximation.

B. Barycentric formula

Given N distinct points z_n and samples $f(z_n)$, we define the rational function B_f of order $(N-1, N-1)$

$$B_f(z) = \frac{\sum_{n=1}^{N} \frac{w_n f(z_n)}{z - z_n}}{\sum_{n=1}^{N} \frac{w_n}{z - z_n}}, \tag{2}$$

where w_n is an (arbitrary) set of non-zero *barycentric weights* and the distinct points z_n are referred to as *support points*. The function B_f, which is also a ratio of two polynomials, could have different degrees for its numerator and denominator. In particular, B_f may be the sum of a proper rational function and a polynomial. The barycentric formula (2) has two important properties.

1) Regardless of the particular choice of barycentric weights w_n, the function B_f in (2) interpolates f at the points $\{z_n\}_n$, that is
$$B_f(z_n) = f(z_n), n = 1, \ldots, N.$$
2) B_f has no poles at the points $\{z_n\}_n$.

Thus, to find the partial fraction representation of B_f we can first compute the zeros of the *barycentric denominator* $\sum_{n=1}^{N} \frac{w_n}{z - z_n}$ to obtain the poles of B_f and use these poles together with the *barycentric numerator* $\sum_{n=1}^{N} \frac{w_n f(z_n)}{z - z_n}$ to obtain the residues of B_f and, if present, polynomial coefficients.

C. AAA algorithm

Recent results (implemented in Matlab as part of open-source package Chebfun, see www.chebfun.org) show that barycentric approximations can be very accurate and lead to close to optimal solutions [6]. In contrast with VF, the support points are not successive approximations of the poles of the target function but rather carefully selected from the given set of sample points. The AAA algorithm is fast and flexible, but it does not necessarily achieve optimality in any particular norm such as L^2 or L^∞. For such problems more specialized methods are currently being developed (see [7]), where the AAA algorithm plays a role in providing an initial guess.

The main idea behind AAA is to select, from a given set of sample points $Z = \{z_n\}_{n=1}^N$ and sample values $f_n = f(z_n)$, a subset of M sample points that have the "best" properties to be used as the barycentric support points. The barycentric weights are then computed to minimize the error over the remaining $N - M$ sample points.

We review the steps of the algorithm (see [6]) assuming the selected support points are labeled $z_1, \ldots, z_M$ and $f = (f_1, \ldots, f_N)$ is the vector of sample values. To pick the first support point, we compute the largest deviation with respect to the mean of the sample values $\text{mean}(f) = \frac{1}{N}\sum_{k=1}^{N} f_k$, which is

$$\max_n |f_n - \text{mean}(f)| = |f_1 - \text{mean}(f)|. \tag{3}$$

Then, our first barycentric approximation $r_1(z)$ is

$$r_1(z) = \frac{\frac{w_1 f_1}{z - z_1}}{\frac{w_1}{z - z_1}} \equiv f_1,$$

that is, r_1 is a constant function of constant value f_1. The logic behind this step is the following. Since the sample point z_1 is responsible for the largest deviation in (3), we pick it as a support point; in this way, due to the properties of the barycentric formula, r_1 will reproduce f_1 at z_1 which was the "most problematic" sample value.

For the next step, we sample r_1 at the remaining sample points $\{z_2, \ldots, z_N\}$ and compute the largest deviation

$$\max_{2\le n\le N} |f_n - r_1(z_n)| = |f_2 - r_1(z_2)| = |f_2 - f_1|. \tag{4}$$

Defining our next barycentric approximation $r_2(z)$ as

$$r_2(z) = \frac{\frac{w_1 f_1}{z - z_1} + \frac{w_2 f_2}{z - z_2}}{\frac{w_1}{z - z_1} + \frac{w_2}{z - z_2}}, \tag{5}$$

we note that now, by construction, $r_2(z_n) = f_n$ for $n = 1, 2$ and that the numerator and denominator of r_2 have at most degree 1. We substitute z by $z_3, z_4, \ldots z_N$ in (5) to determine the weights w_1, w_2, then approximate $r_2(z_n)$ by f_n, and finally multiply by the denominator in (5) to form the (linear in the weights!) system

$$f_n\left(\frac{w_1}{z_n - z_1} + \frac{w_2}{z_n - z_2}\right) = \frac{w_1 f_1}{z_n - z_1} + \frac{w_2 f_2}{z_n - z_2}, n = 3, \ldots, N. \quad (6)$$

We write (6) as $Lw = 0$, where L is the $(N-2) \times 2$ Loewner matrix of entries $L(n,l) = \frac{f_n - f_l}{z_n - z_l}$. The previous system is solved in least squares sense with the additional constraint

$$\|w\| = \sqrt{|w_1|^2 + |w_2|^2} = 1$$

which prevents the (unwanted) solution $w = 0$.

Once the weights are obtained, we can evaluate r_2 at the sample points and compute

$$\max_{3 \le n \le N} |f_n - r_2(z_n)| = |f_3 - r_2(z_3)|. \quad (7)$$

We repeat the previous additional steps to compute the weights and build the next barycentric approximation. If all goes well, at the final step M, we found an approximation r_M such that

$$r_M(z_n) = f_n, 1 \le n \le M, \; r_M(z_n) \approx f_n, M+1 \le n \le N.$$

D. Secular equation

The AAA algorithm finds the rational approximation in barycentric form (2). To obtain the partial fractions approximation, we need to find the zeros of the barycentric denominator. This type of problem can be formulated as a so-called secular equation for which several algorithms have been developed [8, Chap. 9] but the standard results only apply to partial fraction representations of orders (n,n) or $(n-1,n)$ (see [9, p. 231] and [10]). However, in many cases, the barycentric denominators are not of those orders. This causes the standard single-input AAA algorithm's partial fraction computation to fail. We have developed an approach to secular equations of arbitrary order and in the examples below our preliminary algorithm has been used. We will report the details of this new algorithm in a forthcoming paper.

III. Proposed Methodology

Starting from a set of K functions $\{\varphi_k\}_{k=1}^{K}$ with sample values from a common set $Z = \{z_n\}_{n=1}^{N} \subseteq \mathbb{C}$, we iteratively build barycentric approximations following the ideas in Section II. Similar to vector fitting, we find a common set of M support points $\{z_n\}_{n=1}^{M}$ and weights w_n so that the barycentric functions

$$B_k(z) = \frac{\sum_{n=1}^{M} \frac{w_n \varphi_k(z_n)}{z - z_n}}{\sum_{n=1}^{M} \frac{w_n}{z - z_n}}, \; k = 1, \ldots, K \quad (8)$$

are used to approximate the functions φ_k, $k = 1, \ldots, K$. Since all B_k have a common barycentric denominator, the final rational approximations, written in partial fractions form, have a common set of poles (and, of course, different residues and, if present, polynomial coefficients).

At step m of the iteration, a greedy choice described below is used to select an additional support point z_m from the initial sample set Z. We then use least squares to compute a set of common weights $w_1, ..., w_m$ for the intermediate barycentric approximations

$$B_k^{(m)}(z) = \frac{N_k^{(m)}(z)}{D^{(m)}(z)} = \frac{\sum_{n=1}^{m} \frac{w_n \varphi_k(z_n)}{z - z_n}}{\sum_{n=1}^{m} \frac{w_n}{z - z_n}} \quad (9)$$

and compute the difference between $B_k^{(m)}$ and φ_k on $Z^m = Z \diagdown \{z_1, ..., z_m\}$ to pick the next support point.

More specifically, the first support point z_1 is chosen so that

$$\max_{1 \le n \le N} \sum_{k=1}^{K} |\varphi_k(z_n) - \|\Phi_k\|| = \sum_{k=1}^{K} |\varphi_k(z_1) - \|\Phi_k\||$$

where Φ_k is the vector of entries $\{\varphi_k(z_n)\}_{1 \le n \le N}$ and $\|\cdot\|$ denotes the discrete 2-norm. At iteration $m > 1$, the next support point z_m is chosen from $Z^{m-1} = Z \diagdown \{z_1, ..., z_{m-1}\}$ such that

$$\max_{n \in Z^{m-1}} \sum_{k=1}^{K} \left|\varphi_k(z_n) - B_k^{(m-1)}(z_n)\right| = \sum_{k=1}^{K} \left|\varphi_k(z_m) - B_k^{(m-1)}(z_m)\right|. \quad (10)$$

As for the single input AAA algorithm, at each iteration step we check which of the remaining sample points is responsible for the largest deviation and pick it as an additional support point. Due to property 2 in Section II-B, the updated barycentric approximations will reproduce φ_k at z_m and eliminate the large error caused by z_m.

To determine the set of common weights at step m, we seek approximations

$$B_k^{(m)}(z) = \frac{N_k^{(m)}(z)}{D^{(m)}(z)} \approx \varphi_k(z) \quad (11)$$

but instead, minimize the linearized equation with respect to w

$$\min_{1 \le k \le K} \left\|N_k^{(m)}(z) - \varphi_k(z) D^{(m)}(z)\right\|_{z \in Z^m}, \|w\| = 1. \quad (12)$$

We terminate the iteration when $\max_k \left|B_k^{(m)}(z) - \varphi_k(z)\right|_{z \in Z}$ is less than the desired tolerance (all the rational approximations are accurate with respect to the maximum norm).

To solve (12), we rewrite $N_k^{(m)}(z) - \varphi_k(z) D^{(m)}(z)$ as

$$\sum_{l=1}^{m} \frac{\varphi_k(z_l) w_l}{z_n - z_l} - \sum_{l=1}^{m} \frac{\varphi_k(z_n) w_l}{z_n - z_l} = (L_k w)_n, n = M+1, \ldots, N,$$

where L_k is the $N - M \times M$ Loewner matrix of entries $L_k(n,l) = \frac{\varphi_k(z_n) - \varphi_k(z_l)}{z_n - z_l}$. We then find the least squares solution w to the problem

$$Lw = 0, \|w\| = 1, \text{ where } L = \begin{bmatrix} L_1 \\ \vdots \\ L_K \end{bmatrix}.$$

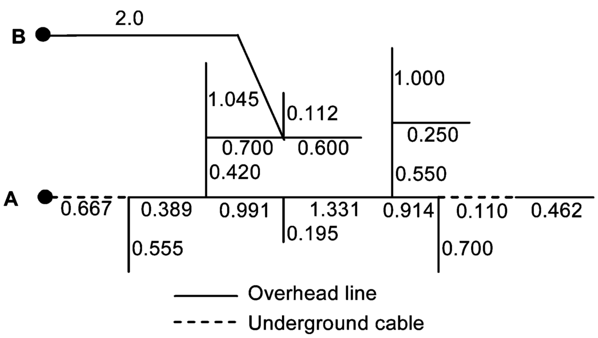

Fig. 1. Power system diagram for the admittance matrix example in [11], [12]. Numbers represent lengths of lines in kilometers.

IV. Results

In order to compare MIAAA with VF, we consider two examples discussed in the literature. The first one, a problem with multiple resonance peaks, is presented in [13, Example 4a], [11] and in [12, Section III.D]. A FDNE approximation is performed on a single column of a 6 by 6 terminal admittance matrix of a power distribution system. The system has two 3-phase buses as terminals, denoted as A and B in Figure 1 and the admittance matrix has been sampled in the frequency range 10Hz-100kHz. To illustrate the results, the first two entries, $f(1)$ and $f(2)$, and their approximations using MIAAA and VF are displayed in Figure 2 and the fitting errors of both methods are displayed in Figure 3. The MIAAA approximation is always below the target accuracy 10^{-6} and fits the linear section of the function with error close to double precision, while VF performs significantly worse. Importantly, MIAAA also provides a better approximation order since it only requires 36 common poles to reach the target accuracy in contrast with VF which requires 50 poles. Although MIAAA was also faster than VF, we do not provide speed comparisons since additional work is necessary for a more thorough comparison and, for the intended applications, the final number of poles used in the approximation is the dominant factor in running time and not the running time of VF or MIAAA.

In Table I, we report the misfit error on the 6 functions being approximated, computed using the metrics

$$\varepsilon_{RMS} = \sqrt{\sum_{n=1}^{N}\sum_{k=1}^{K} |\varphi_k(z_n) - \phi_k(z_n)|^2/(N \cdot K)}, \quad (13)$$

$$\varepsilon_{relative} = \sum_{n=1}^{N}\sum_{k=1}^{K} \left| \frac{\varphi_k(z_n) - \phi_k(z_n)}{\varphi_k(z_n)} \times 100 \right| /(N \cdot K), \quad (14)$$

where ϕ_k is the MIAAA or VF approximation of the k^{th} function φ_k.

The second example as well as the VF results reported in Table II are taken from [12, Section III.C]. We approximate all entries of the 2×2 admittance matrix Y of a simple pi-circuit with the goal of illustrating the impact on accuracy as

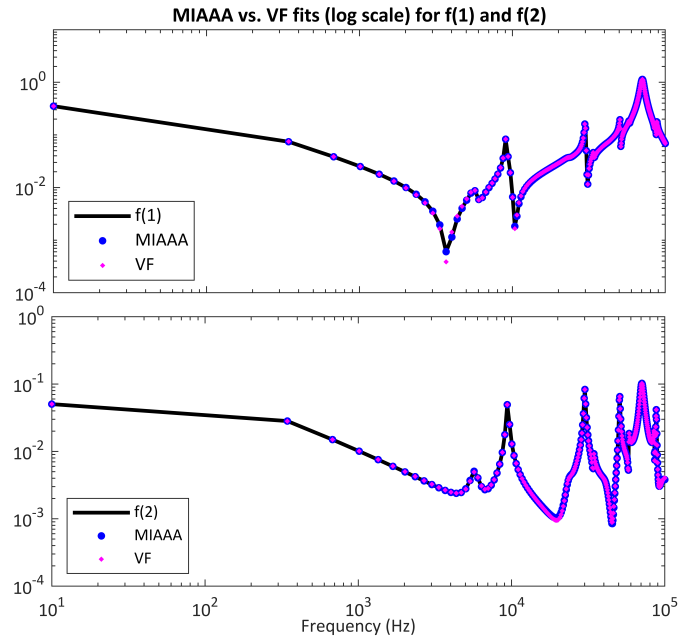

Fig. 2. Comparing vector fitting and MIAAA approximations of the admittance matrix example in [13], [11], [12]. We display results for two entries, $f(1)$, top, and $f(2)$, bottom, as functions of frequency.

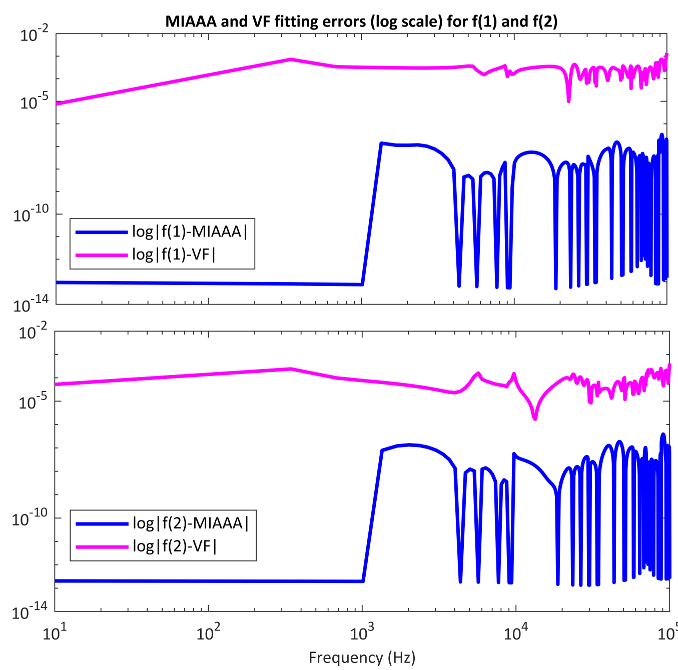

Fig. 3. Comparing vector fitting and MIAAA fitting errors for the two functions shown in Figure 2. The MIAAA errors correspond to the final partial fraction approximations, not the intermediate barycentric approximations.

TABLE I
Relative and RMS errors for vector fitting and MIAAA approximations of functions $f(1), \dots, f(6)$.

Technique	Order	ε_{RMS}	$\varepsilon_{relative}$
VF	50	$1.563 \cdot 10^{-4}$	2.556
MIAAA	36	$6.159 \cdot 10^{-8}$	$2.702 \cdot 10^{-3}$

we increase the order of the approximation. The entries of the symmetric matrix Y (admittances of the circuit elements of the pi-model) are given by

$$Y_{11} = Y_a + Y_b, \quad Y_{21} = Y_{12} = -Y_a, \quad Y_{22} = Y_a + Y_c \quad (15)$$

where

$$Y_a(s) = \frac{2}{s+5} + \frac{20 \pm 50j}{s + 30 \mp 1000j} + 0.4 \quad (16)$$

$$Y_b(s) = \frac{6}{s+12} + \frac{17 \pm 30j}{s + 35 \mp 3000j} + 0.2 \quad (17)$$

$$Y_c(s) = \frac{4}{s+10} + \frac{12 \pm 24j}{s + 15 \mp 5500j} + 0.3. \quad (18)$$

TABLE II
COMPARISON OF VF AND MIAAA TECHNIQUES FOR FITTING THE 2×2 ADMITTANCE MATRIX GIVEN BY EQUATIONS (15) – (18)

Technique	Order	ε_{RMS}	$\varepsilon_{relative}$
VF	7	2.20×10^{-3}	8.33×10^{-2}
MIAAA		2.18×10^{-4}	3.24×10^{-2}
VF	8	2.13×10^{-4}	1.00×10^{-2}
MIAAA		1.63×10^{-5}	1.97×10^{-3}
VF	9	5.10×10^{-15}	2.69×10^{-13}
MIAAA		3.75×10^{-16}	4.15×10^{-14}

We fit all entries of the matrix Y using VF and MIAAA with a fix order, that is, by rational functions with a fix number of common poles. We compute the approximation errors for 7, 8, and 9 poles and report the results in Table II. The RMS and relative errors are defined as before, using (13) and (14), respectively.

As expected, the sharp drop in errors of both VF and MIAAA approximations at order 9 are due to the fact that there are only 9 distinct poles in the entries of the admittance matrix. We observe that MIAAA outperforms VF for all orders considered and, in all but one case, MIAAA outperforms VF by an order of magnitude.

V. CONCLUSIONS

We presented the MIAAA algorithm, a novel, very general approach to obtain rational approximations and showed that outperforms the commonly used VF algorithm on two examples from the literature of power distribution system models. However, given the generality of MIAAA, we intend to use it for many other applications.

We plan to use MIAAA to model underground cables, transmission lines, transformers, and other power system components. as well as to generate rational functions representations for complex components and utilize these representations for digital real time simulations. We expect that employing a near-optimal number of poles will have a strong impact on performance and will help to create complex models that can be run in real time. We also plan to work on additional MIAAA features like enforcing passivity as part of the algorithmic steps and explore how to extend to the multi-input setting the optimization (minimal order) approach developed in [14], [15], [16]. For this optimization approach, MIAAA will play the critical role of providing an initial, sub-optimal but accurate, rational approximation.

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