On active constraints in optimal power flow

Learning optimal solutions and identifying important constraints

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NREL Workshop, April 12 2019
Transmission System Operation

Liberalized Markets

Renewable Energy

- Distributed generation
- Large wind farms
- Cross-border trading
Transmission System Operation

Liberalized Markets

- Higher and frequently changing power flows

Renewable Energy

- Increased uncertainty
Transmission System Operation

Liberalized Markets

Renewable Energy

Higher and frequently changing power flows

Increased uncertainty

Transmission System Operator

distributed generation

large wind farms

cross-border trading
Impact of uncertainty?

How to maintain grid security?

- Chance-constrained, robust, stochastic optimization
- Adapt to uncertainty in real time!
The Optimal Power Flow Problem
Optimal Power Flow

Goal: Low cost operation, while enforcing technical limits

\[
\min \sum_{i \in G} \left( c_{2,i} p_{G,i}^2 + c_{1,i} p_{G,i} \right)
\]

s.t.

\[
f(\theta, v, p, q) = 0,
\]

\[
p_{G,g}^{\text{min}} \leq p_{G,g} \leq p_{G,g}^{\text{max}}, \ g \in G
\]

\[
q_{G,g}^{\text{min}} \leq q_{G,g} \leq q_{G,g}^{\text{max}}, \ g \in G
\]

\[
v_i^{\text{min}} \leq v_i \leq v_i^{\text{max}}, \ i \in B
\]

\[
s_{L,j}(\theta, v, p, q) \leq s_{L,j}^{\text{max}}, \ j \in L
\]

Minimize generation cost

Non-Linear AC Power Flow

Generation constraints

Voltage constraints

Transmission constraints
Optimal Power Flow

Goal: Low cost operation, while enforcing technical limits

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\min \sum_{i \in G} (c_{2,i} p_{G,i}^2 + c_{1,i} p_{G,i})
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s_{L,j}(\theta, v, p, q) \leq s_{L,j}^{\text{max}}, \ j \in L
\]

Minimize generation cost

Non-Linear AC Power Flow

Generation constraints

Voltage constraints

Transmission constraints

Observation 1:
Typically only very few transmission constraints are active at optimum!

Can be exploited algorithmically!

E.g. constraint generation

...
Optimal Power Flow

Impact of renewable energy variations/load uncertainty $\omega$?

\[ \min \sum_{i \in G} (c_{2,i} p_{G,i}(\omega)^2 + c_{1,i} p_{G,i}(\omega)) \]

s.t.
\[ f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0, \]
\[ p_{G,g}^{\text{min}} \leq p_{G,g}(\omega) \leq p_{G,g}^{\text{max}}, \ g \in G \]
\[ q_{G,g}^{\text{min}} \leq q_{G,g}(\omega) \leq q_{G,g}^{\text{max}}, \ g \in G \]
\[ v_i^{\text{min}} \leq v_i(\omega) \leq v_i^{\text{max}}, \ i \in B \]
\[ i_{L,j}(\omega) \leq i_{L,j}^{\text{max}}, \ j \in L \]

Minimize generation cost

Non-Linear AC Power Flow

Generation constraints

Voltage constraints

Transmission constraints
Optimal Power Flow

Impact of renewable energy variations/load uncertainty $\omega$?

$$\min \sum_{i \in G} (c_{2,i} p_{G,i}(\omega)^2 + c_{1,i} p_{G,i}(\omega))$$

Minimize generation cost

s.t.

$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0$,

Non-Linear AC Power Flow

$p_{G,g}^{\min} \leq p_{G,g}(\omega) \leq p_{G,g}^{\max}$, $g \in G$

Generation constraints

$q_{G,g}^{\min} \leq q_{G,g}(\omega) \leq q_{G,g}^{\max}$, $g \in G$

Voltage constraints

$v_i^{\min} \leq v_i(\omega) \leq v_i^{\max}$, $i \in B$

Transmission constraints

$i_{L,j}(\omega) \leq i_{L,j}^{\max}$, $j \in L$

Observation 2:
Typically only very few transmission constraints are ever active even for different parameters $\omega$!

The topic of this talk!
Is this observation true??
How can we use it??
1. Learning solutions to (power system) optimization problems through optimal active sets

2. Identifying potentially active constraints
Learning solutions to (power system) optimization problems

Yee Sian Ng, Sidhant Misra, Line Roald and Scott Backhaus, «Statistical Learning for DC Optimal Power Flow», Power System Computation Conference (PSCC), 2018

Optimal Power Flow

Impact of renewable energy variations/load uncertainty $\omega$?

$$\min \sum_{i \in G} (c_{2,i} p_{G,i}(\omega)^2 + c_{1,i} p_{G,i}(\omega))$$

s.t.

$$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0,$$

$$p_{G,g}^\text{min} \leq p_{G,g}(\omega) \leq p_{G,g}^\text{max}, \ g \in G$$

$$q_{G,g}^\text{min} \leq q_{G,g}(\omega) \leq q_{G,g}^\text{max}, \ g \in G$$

$$v_i^\text{min} \leq v_i(\omega) \leq v_i^\text{max}, \ i \in \mathcal{B}$$

$$i_{L,j}(\omega) \leq i_{L,j}^\text{max}, \ j \in \mathcal{L}$$

Minimize generation cost

Non-Linear AC Power Flow

Generation constraints

Voltage constraints

Transmission constraints
Resolve problem every 5-15 min! For each $\omega$, obtain $p^*_G(\omega)$

$$\min \sum_{i \in G} \left( c_{2,i} p_{G,i}(\omega)^2 + c_{1,i} p_{G,i}(\omega) \right)$$

s.t.

$$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0,$$

$$p_{G,g}^{\min} \leq p_{G,g}(\omega) \leq p_{G,g}^{\max}, \ g \in G$$

$$q_{G,g}^{\min} \leq q_{G,g}(\omega) \leq q_{G,g}^{\max}, \ g \in G$$

$$v_i^{\min} \leq v_i(\omega) \leq v_i^{\max}, \ i \in B$$

$$i_{L,j}(\omega) \leq i_{L,j}^{\max}, \ j \in L$$

Minimize generation cost

Non-Linear AC Power Flow

Generation constraints

Voltage constraints

Transmission constraints
Repeated solution process

**OPF at** $T_1$: $\omega_1 \rightarrow p^*_G(\omega_1)$

$$\min_{p_G(\omega)} \sum_{i \in G} (c_{2,i} p_{G,i}(\omega) + c_{1,i} p_{G,i}(\omega))$$

s.t.

$$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0,$$

$$p_{G,g}^{\min} \leq p_{G,g}(\omega) \leq p_{G,g}^{\max}, \ g \in G$$

$$q_{G,g}^{\min} \leq q_{G,g}(\omega) \leq q_{G,g}^{\max}, \ g \in G$$

$$v_i^{\min} \leq v_i(\omega) \leq v_i^{\max}, \ i \in B$$

$$i_{L,j}(\omega) \leq i_{L,j}^{\max}, \ j \in L$$

**OPF at** $T_2$: $\omega_2 \rightarrow p^*_G(\omega_2)$

$$\min_{p_G(\omega)} \sum_{i \in G} (c_{2,i} p_{G,i}(\omega) + c_{1,i} p_{G,i}(\omega))$$

s.t.

$$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0,$$

$$p_{G,g}^{\min} \leq p_{G,g}(\omega) \leq p_{G,g}^{\max}, \ g \in G$$

$$q_{G,g}^{\min} \leq q_{G,g}(\omega) \leq q_{G,g}^{\max}, \ g \in G$$

$$v_i^{\min} \leq v_i(\omega) \leq v_i^{\max}, \ i \in B$$

$$i_{L,j}(\omega) \leq i_{L,j}^{\max}, \ j \in L$$

**OPF at** $T_3$: $\omega_3 \rightarrow p^*_G(\omega_3)$

$$\min_{p_G(\omega)} \sum_{i \in G} (c_{2,i} p_{G,i}(\omega) + c_{1,i} p_{G,i}(\omega))$$

s.t.

$$f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0,$$

$$p_{G,g}^{\min} \leq p_{G,g}(\omega) \leq p_{G,g}^{\max}, \ g \in G$$

$$q_{G,g}^{\min} \leq q_{G,g}(\omega) \leq q_{G,g}^{\max}, \ g \in G$$

$$v_i^{\min} \leq v_i(\omega) \leq v_i^{\max}, \ i \in B$$

$$i_{L,j}(\omega) \leq i_{L,j}^{\max}, \ j \in L$$

...
Can we use learning to speed up the solution process by using information from past solutions \((\omega_i, p^*_G(\omega_i))\)?
First attempt:

Train a neural net
First Attempt – Train a Neural Net

Renewable energy $\omega$  

$P_G(\omega_i)$  

Dispatch
First Attempt – Train a Neural Net

- This didn’t work well...

(DISCLAIMER: I will admit that we gave up quite fast!)
First Attempt – Train a Neural Net

Renewable energy $\omega$ → $P_G(\omega_i)$ Dispatch

• This didn’t work well...
  – Hard to satisfy safety constraints!

First Attempt – Train a Neural Net

- This didn’t work well...
  - Hard to satisfy safety constraints!
  - Projection back onto feasible space cause suboptimality...

First Attempt – Train a Neural Net

- This didn’t work well...
  - Hard to satisfy safety constraints!
  - Projection back onto feasible space cause suboptimality…
  - Challenging: High-dimensional input → High dimensional output

First Attempt – Train a Neural Net

- **This didn’t work well...**
  - Hard to satisfy safety constraints!
  - Projection back onto feasible space cause suboptimality…
  - Challenging: High-dimensional input $\rightarrow$ *High dimensional output*

- **This can work well under some circumstances**
  Wide enough and deep enough, and with enough data! [Karg and Lucia, 2018]
We have a mathematical optimization problem

– can we use more information about the problem structure?
Think again:

How can we leverage pre-existing knowledge about the solution?
Think again:
How can we leverage pre-existing knowledge about the solution?

New idea:
Learn the optimal set of active constraints!
Optimal set of active constraints

Set of constraints that are active at optimum!

- Equality (power flow) constraints are always active

- Only very few of the inequality constraints are active
  - Generation constraints
  - Voltage constraints
  - Transmission constraints
Learn optimal set of active constraints

- Why?
  - Optimal active set is the “minimal” information we need to recover optimal solution
  - Inherently encodes information about physical constraints and technical limits
  - Finite, low dimensional object
  - Nice physical interpretation (power system operational pattern)
Learn optimal set of active constraints

\[ \text{Renewable energy} \rightarrow \text{Predict optimal active set} \rightarrow \text{Optimal active set} \rightarrow \text{Predict/recover optimal solution} \rightarrow \text{Optimal dispatch} \]

- \( A^*(\omega_i) \)

• Related to explicit MPC
  - Explicit MPC – each optimal active set corresponds to an optimal affine control policy
    [Bemporad et al, 2002], [Pannochia, Rawlings, Wright, 2007], [Zeilinger et al, 2011], [Karg and Lucia, 2018], …

Look only at specific classes of problems
Not very scalable
Do not consider input distribution over \( \omega \)
Ensemble Policy

Realization $\omega$

Candidates for optimal active set

Solve problem given the active set

$\rightarrow$ Solve a reduced problem with fewer constraints!

$\rightarrow$ Solve a set of linear equations (linear problem)!

Easier than solving the full optimal power flow problem
Ensemble Policy

Realization $\omega$

$A_1 \quad A_2 \quad \ldots \quad A_n$

$p_1(\omega) \quad p_2(\omega) \quad \ldots \quad p_n(\omega)$

Infeasible Feasible Low cost Feasible High cost

Candidates for optimal active set

Solve problem given the optimal active set

Evaluate cost and feasibility
Ensemble Policy

Realization $\omega$

$A_1 \quad A_2 \quad \ldots \quad A_n$

$p_1(\omega) \quad p_2(\omega) \quad \ldots \quad p_n(\omega)$

Infeasible Feasible Low cost Feasible High cost

Candidates for optimal active set

Solve problem given the optimal active set

Evaluate cost and feasibility

Pick best (optimal?) solution
Limits of the Approach

Works well if the number of active sets $\mathcal{A}_n$ is small!

Total number of possible active sets is exponential 😞

Maybe only a few are practically relevant? 😊
Limits of the Approach

How to identify the collection of relevant active sets?

High probability that one of the active sets is optimal for a new realization $\omega$

Do NOT search entire parameter space!
Using Sampling to Learn Important Active Sets
Learning Collection of Optimal Active Sets

- samples of input parameters

all possible active sets

color: discovered active sets
grey: undiscovered active sets
Learning Collection of Optimal Active Sets

- samples of input parameters

all possible active sets

$\mathcal{A}^1 \rightarrow \mathcal{A}^N \rightarrow \mathcal{A}^{N+1} \rightarrow \cdots \rightarrow \mathcal{A}^{N+\exp(m)}$

color: discovered active sets
grey: undiscovered active sets
Learning Collection of Optimal Active Sets

- samples of input parameters

all possible active sets

$\mathcal{A}^1 \quad \mathcal{A}^2 \quad \ldots \quad \mathcal{A}^N \quad \mathcal{A}^N+1 \quad \mathcal{A}^N+2 \quad \ldots \quad \mathcal{A}^N+\exp(m)$

- $\omega_1$ axis
- $\omega_2$ axis

color: discovered active sets
grey: undiscovered active sets
Learning Collection of Optimal Active Sets

- samples of input parameters

$\mathcal{A}^1$ to $\mathcal{A}^N$, $\mathcal{A}^{N+1}$, ..., $\mathcal{A}^{N+\exp(m)}$

Collection of observed active sets

$\mathcal{O}_M = \{\mathcal{A}^1\}$
Learning Collection of Optimal Active Sets

\[ \mathcal{O}_M = \{ \mathcal{A}^1 \} \]

- samples of input parameters
- all possible active sets
  - color: discovered active sets
  - grey: undiscovered active sets

Collection of observed active sets
Learning Collection of Optimal Active Sets

Collection of observed active sets

$\mathcal{O}_M = \{\mathcal{A}_1, \mathcal{A}_2\}$

- samples of input parameters
- all possible active sets
- color: discovered active sets
- grey: undiscovered active sets
Learning Collection of Optimal Active Sets

- samples of input parameters

\[ \omega_1 \]

\[ \omega_2 \]

\[ \mathcal{A}^2 \rightarrow \mathcal{A}^N \rightarrow \mathcal{A}^{N+1} \rightarrow \cdots \rightarrow \mathcal{A}^{N+\text{exp}(m)} \]

Collection of observed active sets

\[ \mathcal{O}_M = \{ \mathcal{A}^1, \mathcal{A}^2 \} \]

- all possible active sets
  - color: discovered active sets
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Learning Collection of Optimal Active Sets

\[ \mathcal{O}_M = \{ \mathcal{A}^1, \mathcal{A}^2 \} \]

- samples of input parameters
- all possible active sets
- color: discovered active sets
- grey: undiscovered active sets
Learning Collection of Optimal Active Sets

Collection of observed active sets

$O_M = \{A^1, A^2, A^N\}$

- samples of input parameters
- all possible active sets
  - color: discovered active sets
  - grey: undiscovered active sets
Learning Collection of Optimal Active Sets

\[ \mathcal{O}_M = \{ A^1, A^2, \ldots, A^N \} \]

• samples of input parameters

all possible active sets
color: discovered active sets
grey: undiscovered active sets

Collection of observed active sets
Learning Collection of Optimal Active Sets

When do I stop???
Streaming Algorithm to Learn Collection of Optimal Active Sets
Learning Collection of Optimal Active Sets

Training data:
Renewable energy realizations and corresponding active set $(\omega_i, A_i^+)$

Goal: Find a active sets that together have a high probability of being optimal!
1. Observe optimal active sets for M samples

$\mathcal{A}^*_1, \mathcal{A}^*_2, \ldots, \mathcal{A}^*_M$

Collection of observed active sets
Learning Collection of Optimal Active Sets

1. Observe optimal active sets for $M$ samples

$\mathcal{A}_1^\star \quad \mathcal{A}_2^\star \quad \ldots \quad \mathcal{A}_M^\star$

Collection of observed active sets

2. Check “rate of discovery” for $W$ samples

$\mathcal{A}_1^\star \quad \mathcal{A}_2^\star \quad \ldots \quad \mathcal{A}_W^\star$

How frequently do we observe sets we have not seen before?

Rate of discovery: $R_{M,W} = \frac{N_{\text{unobserved}}}{N}$

where $W = \frac{2Y}{\epsilon^2} \max\{\log(M), \log(\underline{M})\}$

$\underline{M} = 1 + \left(\frac{Y}{\delta(y-1)}\right)^{\frac{1}{y-1}}$
Learning Collection of Optimal Active Sets

1. Observe optimal active sets for $M$ samples

\[ \mathcal{A}_1^*, \mathcal{A}_2^*, \ldots, \mathcal{A}_M^* \]

Collection of observed active sets

- If the rate of discovery is below the threshold $R_{M,W} \leq \alpha - \epsilon$, stop.

2. Check “rate of discovery” for $W$ samples

\[ \mathcal{A}_1^*, \mathcal{A}_2^*, \ldots, \mathcal{A}_W^* \]

How frequently do we observe sets we have not seen before?

Rate of discovery: $R_{M,W} = \frac{N_{\text{unobserved}}}{N}$

where $W = \frac{2^\gamma}{\epsilon^2} \max\{\log(M), \log(M)\}$

\[ M = 1 + \left( \frac{\gamma}{\delta(y-1)} \right)^{1/y-1} \]
Learning Collection of Optimal Active Sets

1. Observe optimal active sets for M samples

Collection of observed active sets

2. Check “rate of discovery” for W samples

How frequently do we observe sets we have not seen before?

- If the rate of discovery is below the threshold $R_{M,W} \leq \alpha - \epsilon$, stop.

- If the rate of discovery is too high, add more samples.

Guarantees performance at termination

[Misra, Roald, Ng, 2018]
1. Observe optimal active sets for $M$ samples

2. Check “rate of discovery” for $W$ samples

Collection of observed active sets

How frequently do we observe sets we have not seen before?

- If the rate of discovery is below the threshold $R_{M,W} \leq \alpha - \epsilon$, stop.

- If the rate of discovery is too high, add more samples.

Guaranteed to converge!

Guarantees performance at termination

[Misra, Roald, Ng, 2018]
Learning Collection of Optimal Active Sets

1. Observe optimal active sets for $M$ samples

Collection of observed active sets

$\mathcal{A}_1^* \quad \mathcal{A}_2^* \quad \ldots \quad \mathcal{A}_M^* \quad \mathcal{A}_{M+1}^*$

2. Check “rate of discovery” for $W$ samples

How frequently do we observe sets we have not seen before?

$\mathcal{A}_1^* \quad \mathcal{A}_1^* \quad \ldots \quad \mathcal{A}_W^* \quad \mathcal{A}_{W+1}^*$

• If the rate of discovery is below the threshold $R_{M,W} \leq \alpha - \epsilon$, stop.

• If the rate of discovery is too high, add more samples.

Guaranteed to converge fast for low number of optimal active sets! [Misra, Roald, Ng, 2018]
Practicability of the approach

Realization $\omega$

- Collection of optimal active sets
- Practicability of the approach

No assumptions on distribution
No assumptions on problem structure

Guaranteed to converge fast for low number of optimal active sets!
Results for the (linear) DC Optimal Power Flow Problem
Streaming Algorithm Results – PGLib-OPF v 17.08

Probabilistic guarantee: \( \mathbb{P}_\omega(\pi(U_M)) < \alpha = 0.05, \)

Max. difference: \( \epsilon = 0.04 \)

Confidence level: \( \delta = 0.01 \)

Hyperparameter: \( \gamma = 2 \)

\( W = \frac{2\gamma}{\epsilon^2} \max\{\log M , \log M\} \) with \( M = 1 + \left( \frac{\gamma}{\delta(\gamma-1)} \right)^{\frac{1}{\gamma-1}} \)

Termination: \( R_{M,W} \leq 0.01 \)

Initial \( W \): \( W = 13'259 \)

(constant until \( M = 201 \))
Streaming Algorithm Results – PGLib-OPF v 17.08

Probabilistic guarantee: $\mathbb{P}_\omega(\pi(\mathcal{U}_M)) < \alpha = 0.05,$

Max. difference: $\epsilon = 0.04$

Confidence level: $\delta = 0.01$

Hyperparameter: $\gamma = 2$

Termination: $R_{M,W} \leq 0.01$

\[ W = \frac{2\gamma}{\epsilon^2} \max\{\log M, \log M\} \quad \text{with} \quad \underline{M} = 1 + \left(\frac{\gamma}{\delta(\gamma-1)}\right)^{\frac{1}{\gamma-1}} \]

Initial $W$: $W = 13'259$

(constant until $M = 201$)

Uncertain loads:

Normal distribution $\omega \sim \mathcal{N}(0, \sigma = 0.03d)$

Uniform distribution with support $\omega \in [-0.09d, 0.09d]$
Example – RTE 1951 bus test case

When there are few relevant active sets, the algorithm terminates fast!
Example – PSERC 200 bus test case

Number of active sets
5

Rate of discovery
0.0069

When there are few relevant active sets, the algorithm terminates fast!
Example – PSERC 200 bus test case

Number of active sets 163
Rate of discovery 0.0099

When there are many relevant active sets, the algorithm terminates slowly!
Streaming Algorithm Results – PGLib-OPF v 17.08

Max. undiscovered: \( \alpha = 0.05 \), Max. difference: \( \epsilon = 0.04 \)  
Termination: \( R_{M,W} \leq 0.01 \)

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System sizes ranging from 3 to 1981 nodes
Streaming Algorithm Results – PGLib-OPF v 17.08

Max. undiscovered: $\alpha = 0.05$, Max. difference: $\epsilon = 0.04$  Termination: $R_{M,W} \leq 0.01$

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Normal distribution $\omega^2 \sim N(0, \sigma = 0.03d)$
Uniform distribution with support $\omega \in [-0.09d, 0.09d]$
Streaming Algorithm Results – PGLib-OPF v 17.08

Max. undiscovered: $\alpha = 0.05$, Max. difference: $\epsilon = 0.04$  
Termination: $R_{M,W} \leq 0.01$

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Streaming Algorithm Results – PGLib-OPF v 17.08

Max. undiscovered: $\alpha = 0.05$, Max. difference: $\epsilon = 0.04$  
Termination: $R_{M,W} \leq 0.01$

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Few active sets!
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Few active sets!

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Few active sets!  
Terminates fast.  
High probability of optimal solutions!
Max. undiscovered: $\alpha = 0.05$, Max. difference: $\epsilon = 0.04$  

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| High-Complexity | | | | | | | | | |
| case73.ieee.rts | 19 | 1258 | 17'844 | 0.0087 | 0.9931 | 130 | 22'000 | 24'977 | 0.0136 | - |
| case300.ieee | 24 | 1257 | 17'842 | 0.0073 | 0.9919 | 293 | 9095 | 22'789 | 0.0099 | 0.9897 |
| case200.psrc | 174 | 4649 | 21'112 | 0.0099 | 0.9909 | 236 | 6741 | 22'040 | 0.0099 | 0.9901 |
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**few active sets!**

**terminates fast.**

**high probability of optimal solutions!**

**not always:**

**large number of active sets**
Max. undiscovered: \( \alpha = 0.05 \), Max. difference: \( \epsilon = 0.04 \)  
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**Few active sets!**  
**Terminates fast.**  
**High probability of optimal solutions!**  
**Not always:**  
**Large number of active sets**  
**Slow to terminate.**
Max. undiscovered: $\alpha = 0.05$, Max. difference: $\epsilon = 0.04$  
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**Few active sets!**
**Terminates fast.**
**High probability of optimal solutions!**

**Not always:**
**Large number of active sets**
**Slow to terminate.**
**Lower probability of optimal solutions!**
Practical Implications for Power Systems Operation
IEEE 300 bus system with normally distributed load

Increasing parameter uncertainty = Increasing number of optimal active sets
IEEE 300 bus system with normally distributed load

Increasing parameter uncertainty = Increasing number of optimal active sets

«Power systems operation becomes more unpredictable and complex with increasing uncertainty»

General perception among system operators
IEEE 300 bus system with normally distributed load

Number of unique active sets

Increasing parameter uncertainty = Increasing number of optimal active sets

«Power systems operation becomes more unpredictable and complex with increasing uncertainty»

General perception among system operators

What does this imply for system risk? Price stability?
# 1 – Leverage pre-existing knowledge (mathematical model) improves learning outcomes

# 2 – Using active sets as an intermediate step is useful
- encodes all information about optimal solution
- finite (and typically low?) number of active sets

# 3 – Streaming algorithm establishes practicability of the task
- Probabilistic performance guarantees
- Guaranteed to terminate
- Guaranteed to terminate fast for nice problems

• Quite general strategy
  – Streaming algorithm can work for very general problems: Non-convex AC OPF, mixed integer problems …
  – Disclaimer: Application must be such that the number of active sets is small.
  – Alternative strategy: Learn possible active constraints instead of active sets
Outlook

Realization $\omega$

$\mathcal{A}_1 \quad \mathcal{A}_2 \quad \ldots \quad \mathcal{A}_n$

Classification!

[Deka and Misra, 2019]
Realization $\omega$

Classification!

Efficient solution?
Active set solver, local approximation, …
Preliminary results for the (non-linear, non-convex) AC Optimal Power Flow Problem
AC Optimal Power Flow

Max. undiscovered: $\alpha = 0.1$, Max. difference: $\epsilon = 0.05$

Termination: $R_{M,W} \leq 0.05$

RTE 1951 bus test case

Rate of discovery of new optimal active sets

(did not terminate)

PSERC 200 bus test case

Rate of discovery of new optimal active sets

0.039
AC Optimal Power Flow

Max. undiscovered: $\alpha = 0.1$, Max. difference: $\epsilon = 0.05$

Termination: $R_{M,W} \leq 0.05$

**RTE 1951 bus test case**

Rate of discovery of new optimal active sets

Rate of discovery of new active constraints

**PSERC 200 bus test case**

Rate of discovery of new optimal active sets

Rate of discovery of new active constraints
AC Optimal Power Flow

Max. undiscovered: $\alpha = 0.1$, Max. difference: $\epsilon = 0.05$

Termination: $R_{M,W} \leq 0.05$

RTE 1951 bus test case

Only 164 of 5192 transmission line constraints ever active

PSERC 200 bus test case

Only 28 of 490 transmission line constraints ever active
1. Learning solutions to (power system) optimization problems through optimal active sets

2. Identifying potentially active constraints
Identifying potentially active constraints

Daniel K. Molzahn
Georgia Tech


Optimal Power Flow

\[
\min_{P_G(\omega)} \sum_{i \in G} (c_{2,i} p_{G,i}(\omega)^2 + c_{1,i} p_{G,i}(\omega))
\]

s.t.

\[
f(\theta(\omega), v(\omega), p(\omega), q(\omega)) = 0,
\]

\[
p_{G,g}^{\min} \leq p_{G,g}(\omega) \leq p_{G,g}^{\max}, \quad g \in G
\]
\[
q_{G,g}^{\min} \leq q_{G,g}(\omega) \leq q_{G,g}^{\max}, \quad g \in G
\]
\[
v_i^{\min} \leq v_i(\omega) \leq v_i^{\max}, \quad i \in B
\]
\[
i_{L,j}(\omega) \leq i_{L,j}^{\max}, \quad j \in L
\]

Before, we learned constraints that are \textit{likely} to be active.

Now we want to understand which constraints can \textit{possibly} be active!

\textbf{Feasible set} in the direction of the cost function.

\textbf{The full feasible set}
Optimization-based constraint screening

Main idea:
Minimize/maximize the value of the constraints!

\[
\min v_i \quad \text{or} \quad \max v_i
\]

s.t.
\[
f(\theta, v, p, q) = 0,
\]

\[
p_{g,g}^{\min} \leq p_{g,g} \leq p_{g,g}^{\max}, \quad g \in G
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\[
q_{g,g}^{\min} \leq q_{g,g} \leq q_{g,g}^{\max}, \quad g \in G
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\[
v_i^{\min} \leq v_i \leq v_i^{\max}, \quad i \in B
\]

\[
s_{L,j}(\theta, v, p, q) \leq s_{L,j}^{\max}, \quad j \in L
\]

Minimize/maximize voltage, currents ...

Non-Linear AC Power Flow

Generation constraints

Voltage constraints

Transmission constraints

If \( \max v_i \leq v_i^{\max} \)
and \( \min v_i \geq v_i^{\min} \)

Voltage will never go out of bounds!
Optimization-based constraint screening

**Main idea:**
Minimize/maximize the value of the constraints!

\[
\begin{align*}
\min & \quad v_i / \max & \max & \quad v_i \\
\text{s.t.} & \quad f(\theta, v, p, q) = 0, \\
& \quad p_{G,g}^{\min} \leq p_{G,g} \leq p_{G,g}^{\max}, \ g \in G \\
& \quad q_{G,g}^{\min} \leq q_{G,g} \leq q_{G,g}^{\max}, \ g \in G \\
& \quad v_i^{\min} \leq v_i \leq v_i^{\max}, \ i \in B \\
& \quad s_{L,j}(\theta, v, p, q) \leq s_{L,j}^{\max}, \ j \in L
\end{align*}
\]

Minimize/maximize voltage, currents …

Non-Linear AC Power Flow

- Connections to optimization-based bound tightening
  [C. Coffrin, Hijazi, and Van Hentenryck, 2015]

- Results for DC OPF
  [Ardakani and F. Bouffard, 2013, 2015]
  [Madani, Lavaei, and Baldick, 2017]

- Our interest:
  - Large ranges of load
  - AC OPF (distribution grids)
Distribution grids – AC Optimal Power Flow

- Consider ranges of **load variations** (not controllable by the system operator)
- Voltage constraints only on buses we **monitor/control**

\[
\begin{align*}
\min & \quad v_i / \max \quad v_i \\
\text{s.t.} & \quad f(\theta, v, p, q) = 0, \\
& \quad p_{D,i}^{\min} \leq p_{D,i} \leq p_{D,i}^{\max}, \quad i \in N \\
& \quad q_{D,i}^{\min} \leq q_{D,i} \leq q_{D,i}^{\max}, \quad i \in N \\
& \quad v_i^{\min} \leq v_i \leq v_i^{\max}, \quad i \in C
\end{align*}
\]

Minimize/maximize voltage

Non-Linear AC Power Flow

Load variations

Voltage constraints on nodes with measurements/control

Valid bounds:
Use convex relaxation.

**QC relaxation with bound tightening**

[Coffrin, Hijazi and Van Hentenryck, 2016 & 2017]

Challenging:
- non-standard objective (relaxation is weak)
- low-voltage solutions
- …
Can we certify safe operations?

IEEE 123 bus system – single-phase equivalent

[Bolognani and Zampieri, 2016]

Start out assuming only one node with controlled voltage
Can we certify safe operations?

Increasing load variability, increasing voltage range!

violations??

Add more controllable nodes, and tighten the voltage limits!
Can we certify safe operations?

Added controllability and tighter voltage range on Bus 32
Can we certify safe operations?

Added controllability and tighter voltage range on Bus 11
Redundant constraints in DC optimal power flow

How many constraints can ever be active in DC optimal power flow?

\[
\begin{align*}
\min_{P_G} & \quad C_G^T P_G \\
\text{s.t.} & \quad \sum_{i=1}^{N_B} (P_G(i) - P_{D(i)}) = 0 \\
& \quad 0 \leq P_G \leq P_G^{\max}, \\
& \quad -P_L^{\max} \leq M (P_G - P_D) \leq P_L^{\max},
\end{align*}
\]

Minimize/maximize constraints

Power balance \quad \text{Non-redundant}

Generation constraints \quad \text{Non-redundant}

Transmission constraints \quad \text{Often redundant}

Allow power demand \( P_{D(i)} \) to vary \( \pm X \% \) where \( 0 \leq X \leq 100 \)

Relax generator lower bounds to \( 0 \)
Results on PGLib-OPF test cases

Percentage of line flow constraints

Remaining non-redundant constraints

± % Load variation

case1951_rte

100%

75%

50%

25%

0%

0% 25% 50% 75% 100%

1.0% 3.2% 6.4% 8.9% 13.3%
Results on standard test cases
Optimization-based constraint screening

• **Main idea:**
  – Solve optimization problems that minimize/maximize the value of the constraints
    (If the problems are hard to solve, use relaxations to obtain valid lower/upper bounds!)
  – Identify constraints that cannot be violated -> redundant constraints
  – Identify constraints that can be violated -> potentially important constraints

• **Works really well for power flow optimization!**

• **We can use this to**
  (1) identify constraints that need to be monitored/controlled
  (2) reduce the number of considered constraints
  (3) …
THANK YOU!

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Summary of streaming algorithm results

1. Guaranteed to terminate, no need to decide on the number of M samples apriori
   Definition of the window size W and termination criterion

Theorem 1 and 2 [Misra, Roald, Ng, 2018]:

If the window size $W(M)$ is defined as

$$W = \frac{2\gamma}{\epsilon^2} \max\{\log M, \log \underline{M}\} \quad \text{with} \quad \underline{M} = 1 + \left(\frac{\gamma}{\delta^{(\gamma-1)}}\right)^{\frac{1}{\gamma-1}}$$

Then $\mathbb{P}(\pi(U_M) - R_{M,W} \leq \epsilon \quad \forall \ M > 1) \leq 1 - \delta$

$\epsilon$ difference between true and empirical probability of unobserved active sets

$\delta$ confidence level

$\gamma$ hyperparameter $> 1$

2. Guaranteed to terminate fast for benign systems

Theorem 3 [Misra, Roald and Ng] If a (small) number of relevant active sets $K_0$ that contains more than $1 - \alpha_0$ probability mass, then with probability at least $1 - \delta - \delta_0$ the algorithm terminates in less iterations than

$$M = \frac{1}{\alpha - \alpha_0} \left(K_0 \log 2 + \log \frac{1}{\delta_0}\right)$$