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Modeling and analysis of dynamic interactions between converter-dominated transmission and distribution systems

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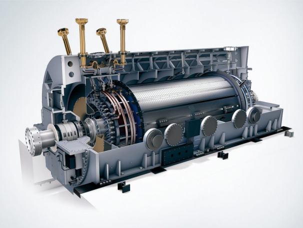
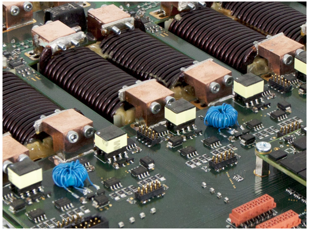
S. Nudehi, D. Groß: *Grid-forming control of three-phase and single-phase converters across unbalanced transmission and distribution systems*, IEEE TPWRS, 2022.

Challenges to standard models and analysis methods





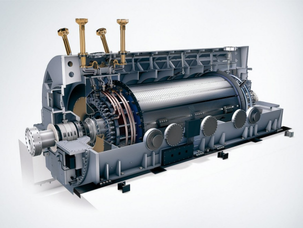
conventional multi-machine system

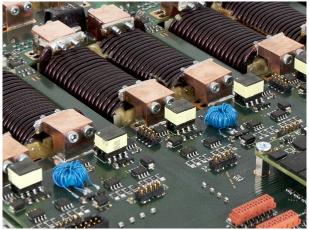
- centralized generation

emerging system

- decentralized generation

Challenges to standard models and analysis methods





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- **few** transmission-connected **SGs**

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- **grid-forming IBRs** in distribution

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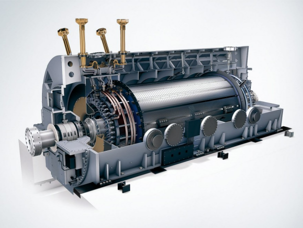

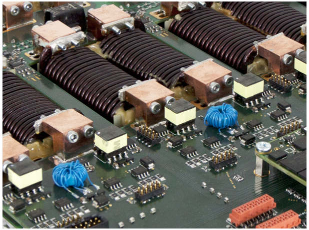

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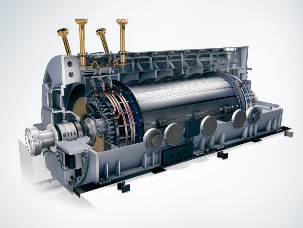

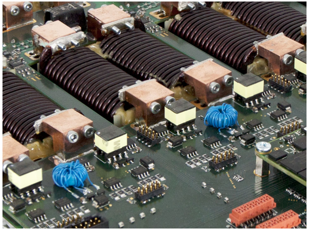

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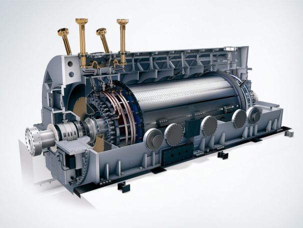

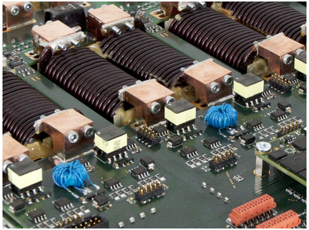

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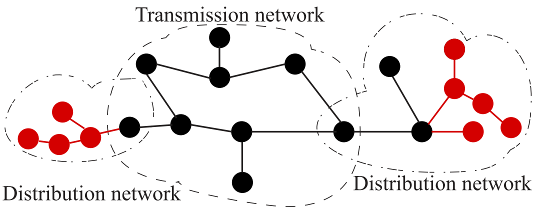
Increased complexity and dynamic interactions across distribution/transmission boundary

Today's modeling assumptions & analysis



Transmission

- power flows to uncontrollable load at grid edge

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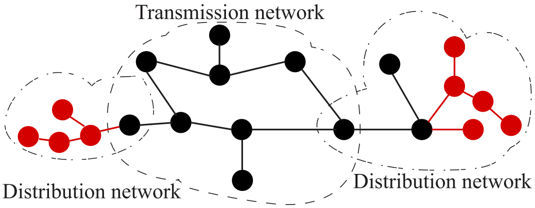

Transmission

- power flows to uncontrollable load at grid edge
- distribution modeled as balanced load

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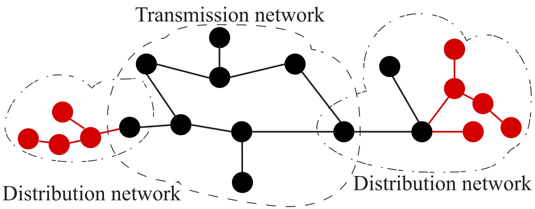

Transmission

- power flows to uncontrollable load at grid edge
- distribution modeled as balanced load
- balanced, three-phase, & constant voltage

Distribution

- transmission will absorb any imbalance
- substation modeled as infinite/slack bus
- unbalanced, single-phase, & constant freq.

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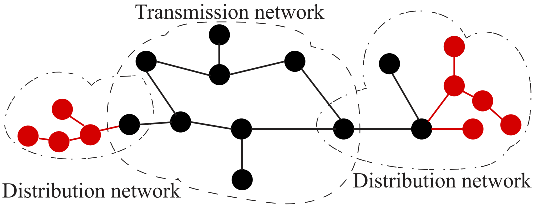

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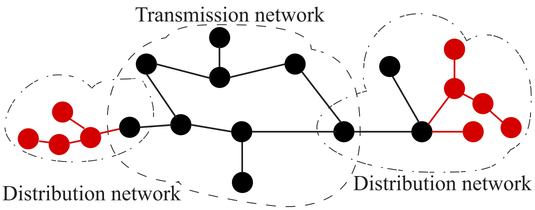

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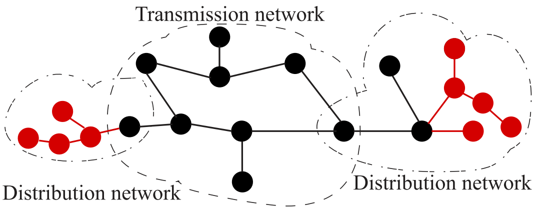
System commonly decomposed at distribution/transmission boundary

Today's modeling assumptions & analysis vs. tomorrow's system?



Non-trivial dynamics and frequency stability questions

- What if single-phase grid-forming IBRs are deployed at scale?

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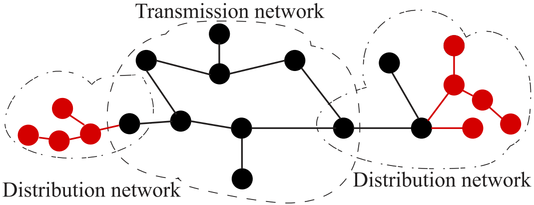

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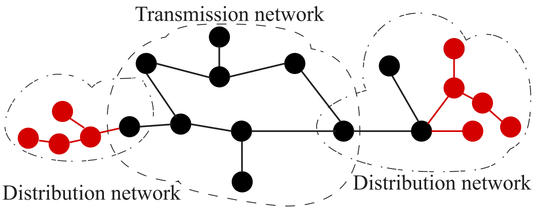

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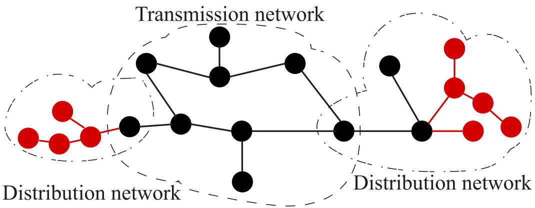
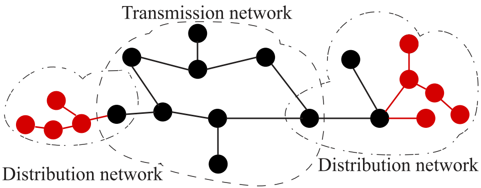


Non-trivial dynamics and frequency stability questions

- What if single-phase grid-forming IBRs are deployed at scale?
- How do single-phase and three-phase grid-forming IBRs synchronize?
- What if all generation is moved to distribution?
- How do three-phase transformer winding configurations impact synchronization?

Problem setup & review of multi-phase power flow models



- "exterior" GFM converter bus

$$\frac{\mathrm{d}}{\mathrm{d}t}\theta = \omega_0 + m_p(P^\star - P),$$
$$\frac{\mathrm{d}}{\mathrm{d}t}V = -V + V^\star + m_q(Q^\star - Q).$$

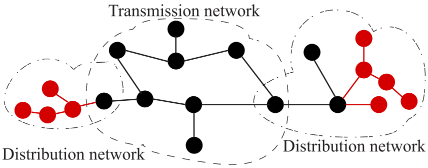
- "interior" load/GFL converter PQ bus

Overview of unbalanced multi-phase power flow models

- detailed bus injection models for OPF [1]
- branch flow models for radial networks (no ΔY) with PQ nodes [2]
- linearized models without single-phase, three-phase transformers [3]
- full generality & structural properties for existence and uniqueness of solutions [4]

[1] Girigoudar, Roald: *Linearized three-phase optimal power flow models for distribution grids with voltage unbalance*, IEEE CDC, 2021
[2] Arnold, Sankur, Dobbe, Brady, Callaway, Von Meier: *Optimal dispatch of reactive power for voltage regulation and balancing in unbalanced distribution systems*, IEEE PESGM, 2016
[3] Bernstein, Wang, Dall'Anese, Boudec, Zhao: *Load Flow in Multiphase Distribution Networks: Existence, Uniqueness, Non-Singularity and Linear Models*, IEEE TPWRS, 2018
[4] Wang, Bernstein, Le Boudec, and Paolone: *Existence and uniqueness of load-flow solutions in three-phase distribution networks*, IEEE TPWRS, 2017

A graph-theoretic approach



- connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$
  - nodes $\mathcal{N} := \mathcal{N}_{1\phi} \cup \mathcal{N}_{3\phi}$
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  - exterior nodes $\mathcal{N}_{1\phi}^{\text{ext}}, \mathcal{N}_{3\phi}^{\text{ext}}$ and interior nodes $\mathcal{N}_{1\phi}^{\text{int}}, \mathcal{N}_{3\phi}^{\text{int}}$
- three-phase branches
  - $\mathcal{E}_{3\phi} := \mathcal{E}_{\mathrm{YY}} \cup \mathcal{E}_{\mathrm{Y_g Y_g}} \cup \mathcal{E}_{\mathrm{Y_g}\Delta} \cup \mathcal{E}_{\Delta\mathrm{Y}} \cup \mathcal{E}_{\Delta\Delta} \cup \mathcal{E}_{3\pi}$
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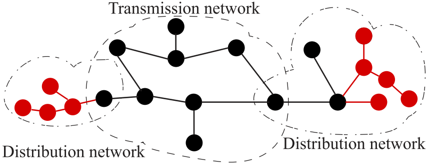

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Key steps for stability analysis

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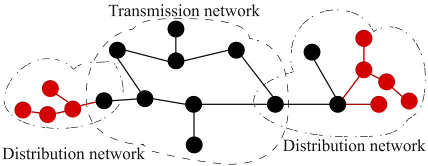

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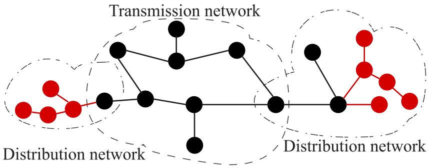

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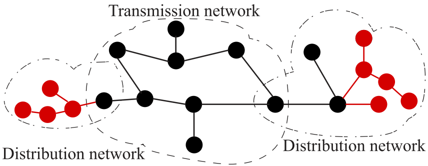

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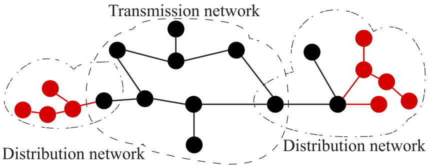

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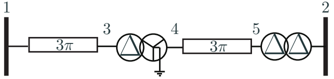
- formalize overall network model via $\mathcal{G}$ & generalized incidence matrix B
- link graph topology to properties of linear model (e.g., symmetry, nullspace, ...)
- conditions for generalized Kron reduction to remove PQ nodes
- reveal conditions on topology for dynamic stability

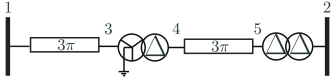
Basic assumptions on the network topology

Definition 1 (interior-exterior node connected network)

The network $\mathcal{G}$ is interior-exterior node connected if, for any interior node $k \in \mathcal{N}^{\text{int}}$, the subgraph $\mathcal{G}_{\text{sync}} := (\mathcal{N}, \mathcal{E}_{\text{sync}}) \subseteq \mathcal{G}$ contains a path to an exterior node $l \in \mathcal{N}_{3\phi}^{\text{ext}}$ that, starting from $k \in \mathcal{N}^{\text{int}}$, traverses all edges from their primary terminal to their secondary terminal.



(a)



(b)

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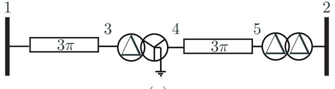

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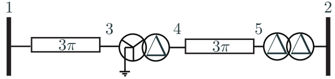
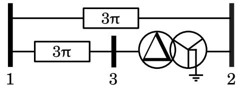


(b)

Assumption 1 (Well-posed network)

1. $\mathcal{G}$ is simple, connected, and interior-exterior node connected,
2. the number of transformer branches of a specific type, voltage ratio, and orientation traversed by any path between any two nodes is identical,
3. the graph $\mathcal{G}_{1\phi} := (\mathcal{N}, \mathcal{E}_{1\phi})$ contains no path connecting any three-phase nodes.



- linearized at trivial solution (i.e., nominal voltage, zero power)
- real and reactive powers $S_{\delta,i,k} = (P_{\delta,i,k}, Q_{\delta,i,k}) \in \mathbb{R}^{2n_i}$
- voltage phase angles and magnitudes $V_{\delta,i} = (\theta_{\delta,i}, v_{\delta,i}) \in \mathbb{R}^{2n_i}$
- $J_{i,k} \succcurlyeq 0$, $\mathsf{J}_{ii} \succcurlyeq 0$, and $J_{i,k}(\nu_{n_i}, \nu_{n_k}) = \mathbb{0}_{2(n_i+n_k)}$ with $\nu_n = (\mathbb{1}_n, \mathbb{0}_n)$

$$\begin{bmatrix} S_{\delta,i,k} \\ S_{\delta,k,i} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathsf{J}_{ii} \\ \mathsf{J}_{ik} \end{bmatrix} b_{i,k} \begin{bmatrix} \mathsf{J}_{ii} \\ \mathsf{J}_{ik} \end{bmatrix}^{\mathsf{T}}}_{=: J_{i,k}} \begin{bmatrix} V_{\delta,i} \\ V_{\delta,k} \end{bmatrix}$$

Type	P_{ii}	R_{ii}	P_{ik}	R_{ik}
Y⏚Y⏚ transformer	I_3	$\mathbb{0}_{3\times3}$	I_3	$\mathbb{0}_{3\times3}$
Y⏚Y transformer	P_1	P_2	P_1	P_2
Y⏚Δ transformer	I_3	$\mathbb{0}_{3\times3}$	P_3	P_4
YY transformer	P_1	P_2	P_1	P_2
YΔ transformer	P_1	P_2	P_3	P_4
ΔΔ transformer	P_1	P_2	P_1	P_2
Three-phase line	I_3	$\mathbb{0}_{3\times3}$	I_3	$\mathbb{0}_{3\times3}$
Single-phase branch	1	0	$\mathcal{I}_{i,k}$	$\mathbb{0}_{n_k\times1}$

$$\mathsf{J}_{ii} = \begin{bmatrix} \mathsf{P}_{ii} & -\mathsf{R}_{ii}^{\mathsf{T}} \\ -\mathsf{R}_{ii} & \mathsf{P}_{ii} \end{bmatrix}, \quad \mathsf{J}_{ik} = \begin{bmatrix} -\mathsf{P}_{ik} & -\mathsf{R}_{ik} \\ \mathsf{R}_{ik} & -\mathsf{P}_{ik} \end{bmatrix}$$

$$\mathsf{P}_1 := \frac{1}{12}\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}, \quad \mathsf{P}_2 := \frac{\sqrt{3}}{12}\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\mathsf{P}_3 := \frac{1}{4}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad \mathsf{P}_4 := \frac{\sqrt{3}}{12}\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Network model & generalized Kron reduction

- generalized oriented incidence matrix encodes topology

$$B := \begin{bmatrix} B_1 & \dots & B_{|\mathcal{E}|} \end{bmatrix} = \begin{bmatrix} B_{1,1} & \dots & B_{1,|\mathcal{E}|} \\ \vdots & \ddots & \vdots \\ B_{|\mathcal{N}|,1} & \dots & B_{|\mathcal{N}|,|\mathcal{E}|} \end{bmatrix} \in \mathbb{R}^{n_{\mathcal{N}} \times n_{\mathcal{E}}}$$

- $B_{l_i,l} = \mathsf{J}_{\mathsf{l_i l_i}}$ and $B_{l_i,l} = \mathsf{J}_{\mathsf{l_i l_k}}$ and $B_{j,l} = \mathbb{0}_{2n_i \times 2n_l}$ otherwise for all edges $l \in \mathbb{N}_{[1,|\mathcal{E}|]}$
- overall network model $S_\delta = \underbrace{BWB^\mathsf{T}}_{=:J} V_\delta$ partitioned into exterior and interior nodes

$$\underbrace{\begin{bmatrix} S_\delta^{\text{ext}} \\ S_\delta^{\text{int}} \end{bmatrix}}_{=S_\delta} = \underbrace{\begin{bmatrix} J_{\text{ext}} & J_{\text{c}} \\ J_{\text{c}}^\mathsf{T} & J_{\text{int}} \end{bmatrix}}_{=J} \underbrace{\begin{bmatrix} V_\delta^{\text{ext}} \\ V_\delta^{\text{int}} \end{bmatrix}}_{=V_\delta},$$

Lemma 1 (Generalized Kron reduction)

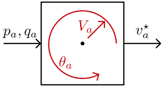
If the network is connected and interior-exterior node connected, then J_{int} has full rank and

$$S_{\delta,\text{ext}} = (J_{\text{ext}} - J_{\text{c}} J_{\text{int}}^{-1} J_{\text{c}}^\mathsf{T})\, V_\delta^{\text{ext}} + J_{\text{c}} J_{\text{int}}^{-1} S_{\delta,\text{int}}.$$

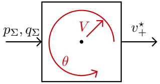
Grid-forming droop control of single-phase and three-phase VSCs

- droop coefficient $m_d = m_p = m_q/\tau$ to simplify notation
- single-phase GFM droop

$$\begin{bmatrix} \dot{\theta}_{\delta,i} \\ \dot{v}_{\delta,i} \end{bmatrix} = -\frac{1}{\tau} \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{=:\mathsf{F}_i\mathsf{F}_i^\mathsf{T}} \begin{bmatrix} \theta_{\delta,i} \\ v_{\delta,i} \end{bmatrix} - m_d \begin{bmatrix} P_{\delta,i} \\ Q_{\delta,i} \end{bmatrix}$$

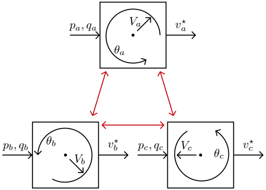


- standard three-phase droop



$$\begin{bmatrix} \dot{\gamma}_i \\ \dot{\vartheta}_i \end{bmatrix} = -\begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{\tau} \end{bmatrix} \begin{bmatrix} \gamma_i \\ \vartheta_i \end{bmatrix} - m_d \underbrace{(I_2 \otimes \mathbb{1}_3^\mathsf{T})}_{=:\mathsf{E}^\mathsf{T}} \begin{bmatrix} P_{\delta,i} \\ Q_{\delta,i} \end{bmatrix}, \quad \begin{bmatrix} \theta_{\delta,i} \\ v_{\delta,i} \end{bmatrix} = \mathsf{E} \begin{bmatrix} \gamma_i \\ \vartheta_i \end{bmatrix}$$

- single-phase droop & phase-balancing feedback $\mathsf{S}_i = I_2 \otimes \sqrt{12}\mathsf{P}_4$ [5]



$$\begin{bmatrix} \dot{\theta}_{\delta,i} \\ \dot{v}_{\delta,i} \end{bmatrix} = -\left(\frac{1}{\tau}\mathsf{F}_i\mathsf{F}_i^\mathsf{T} + k_{\mathrm{bal},i}\mathsf{S}_i\mathsf{S}_i^\mathsf{T}\right) \begin{bmatrix} \theta_{\delta,i} \\ v_{\delta,i}, \end{bmatrix} - m_d \begin{bmatrix} P_{\delta,i} \\ Q_{\delta,i} \end{bmatrix}$$

[5] Bhagwat, Groß: *Three-phase grid-forming droop control for unbalanced systems and fault ride through* , IEEE PES GM, 2023

Properties of the closed loop

- closed-loop system matrix

$$J_{\mathsf{cl}} := E^\top \left(m_d J + \sum\nolimits_{i \in \mathcal{N}^{\mathsf{ext}}} \frac{1}{\tau} F_i F_i^\top + k_{\mathsf{bal},i} S_i S_i^\top \right) E,$$

- closed-loop dynamics after eliminating interior nodes

$$\tfrac{\mathrm{d}}{\mathrm{d}t} x = -(J_{\mathsf{cl,ext}} - J_{\mathsf{cl,c}} J_{\mathsf{cl,int}}^{-1} J_{\mathsf{cl,c}}^\top) x = J_{\mathsf{cl,red}}$$

Theorem 1 (Nullspace of J_{cl})

Consider a connected network $\mathcal{G} = (\mathcal{E}, \mathcal{N})$ that satisfies Assumption 1. Assume that one of the following holds:

1. there exists $i \in \mathcal{N}_{3\phi}^{\mathsf{ext}}$ such that $k_{\mathsf{bal},i} \in \mathbb{R}_{>0}$,
2. there exists at least one path between two exterior nodes that contains at least one $\mathrm{Y_g}\Delta$ branch and all $\mathrm{Y_g}\Delta$ branches are traversed in the same orientation.

Then $J_{\mathsf{cl}} \xi = \mathbb{0}_{6|\mathcal{N}_{3\phi}|+2|\mathcal{N}_{1\phi}|}$ if and only if $\xi_i \in \mathrm{span}(\nu_{n_i})$.

leverages synchronization through subgraph $\mathcal{G}_{\mathsf{sync}} := (\mathcal{N}, \mathcal{E}_{\mathsf{sync}})$ with $\mathcal{E}_{\mathsf{sync}} := \mathcal{E}_{\mathrm{Y_g}\mathrm{Y_g}} \cup \mathcal{E}_{3\pi} \cup \mathcal{E}_{\mathrm{Y_g}\Delta} \cup \mathcal{E}_{1\phi}$

Stability conditions

Theorem 2 (Asymptotic stability)

Consider a interior-exterior node connected network that satisfies Assumption 1. Moreover, one of the following holds:

1. there exists a three-phase VSC $i \in \mathcal{N}_{3\phi}^{\text{ext}}$ using standard three-phase droop control,
2. there exists a three-phase VSC $i \in \mathcal{N}_{3\phi}^{\text{ext}}$ using generalized three-phase droop control with $k_{\text{bal},i} \in \mathbb{R}_{>0}$,
3. there exists at least one path between the two exterior nodes that contains at least one $Y_g\Delta$ branch and all $Y_g\Delta$ branches are traversed in the same orientations.

Then, the system is asymptotically stable with respect to the subspace $x_i = \nu_{n_i}$ for all $i \in \mathcal{N}^{\text{ext}}$.

Stability conditions

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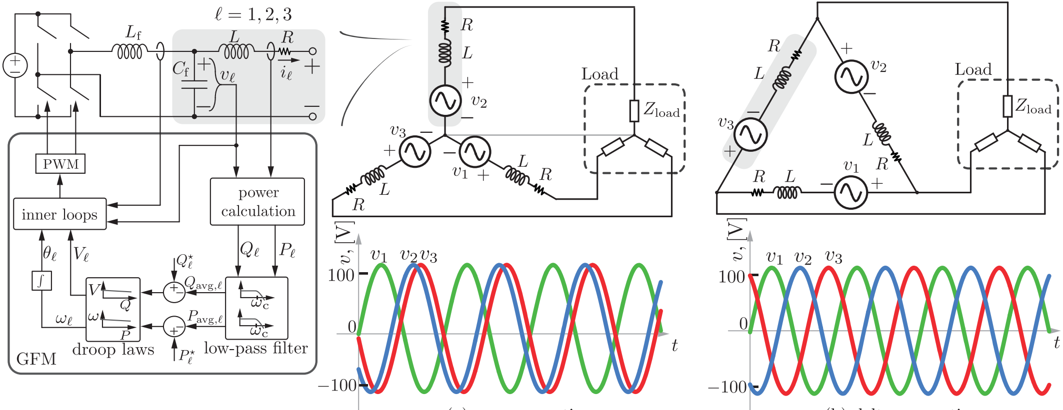
3. there exists at least one path between the two exterior nodes that contains at least one Y⏚Δ branch and all Y⏚Δ branches are traversed in the same orientations.

Then, the system is asymptotically stable with respect to the subspace $x_i = \nu_{n_i}$ for all $i \in \mathcal{N}^{\text{ext}}$.

Results in the literature are special cases of Theorem 2:

- conditions for balanced three-phase network with standard three-phase droop control
- conditions for single-phase network with single-phase droop
- spontaneous phase-balancing of ΔY-connected single-phase GFM converters

Spontaneous phase-balancing of single-phase GFM converters [6]

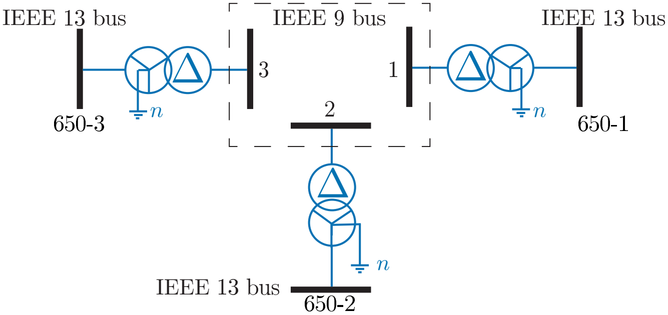


(a) wye connection

(b) delta connection

[6] Lu, Dhople, Zimmanck, Johnson: *Spontaneous Phase Balancing in Delta-Connected Single-Phase Droop-Controlled Inverters*, IEEE TPEL, 2022

Case study


IEEE 13 bus
650-3
n
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3
1
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IEEE 13 bus
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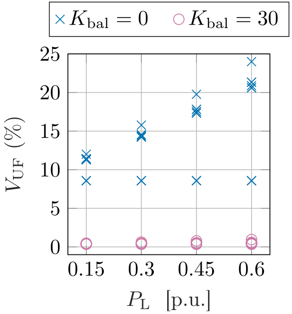
IEEE 13 bus
650-2


$K_{\mathrm{bal}} = 0$
$K_{\mathrm{bal}} = 30$
V_{UF} (%)
25
20
15
10
5
0
0.15
0.3
0.45
0.6
P_{L} [p.u.]

Conclusions & Future Work

Take home messages

- linearized quasi-steady-state model of unbalanced multi-phase systems
- tractable for dynamic stability analysis
- highlights conditions on network topology for synchronization

Ongoing work

- steady-state analysis
  - sharing of load unbalance and voltage unbalance
  - bounds on voltage unbalance as function of load, network, and control gains
- sensitivity analysis of linearization errors for typical branches
- distributed control for mitigating phase imbalances in distribution systems