



# Fuel Reliability, Energy Equity, and Decarbonization by Optimized Management for Grid Automation & Security

Sixth Workshop on Autonomous Energy Systems  
National Renewable Energy Laboratory, Golden, CO

**Anatoly Zlotnik**

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# Outline

- Motivation
- Pipeline network flow modeling
- Optimization formulation
- Economics
- Optimization case studies
- Transient flow
- Conclusions

# Motivation

- Natural gas is a “bridge fuel” to carbon-neutral energy systems
- Large-scale natural gas pipelines are in use for over 60 years
  - Pipelines supply >60% of U.S. heating & fuel >40% of U.S. electricity
- Hydrogen proposed as a renewable fuel blended into natural gas pipelines

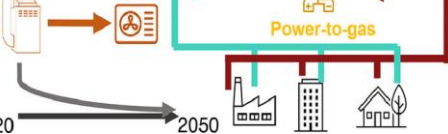
## Building resilient, integrated, decarbonized gas-electric systems

Expand energy supplies

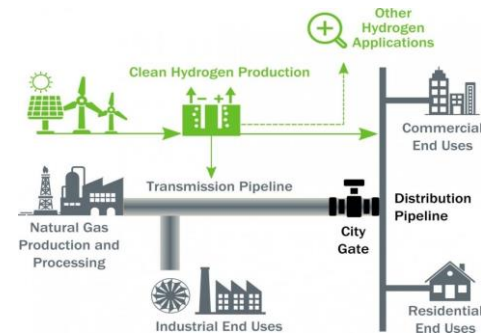


Transition final energy demands

Appliance electrification



To meet declining GHG emissions constraints



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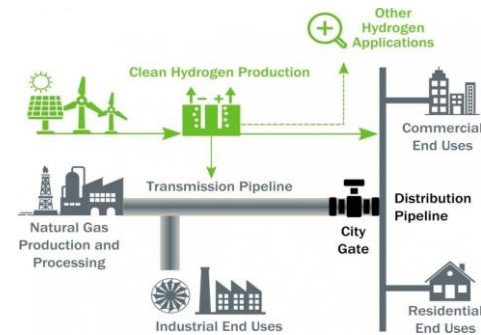
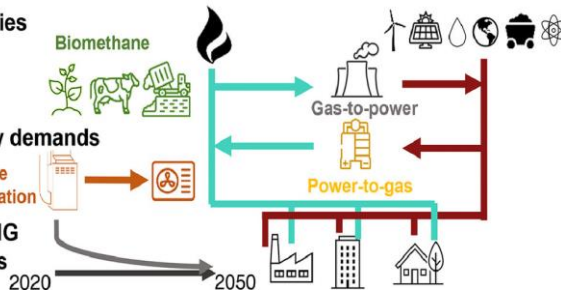
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- **By:** Optimized Management for Grid Automation & Security

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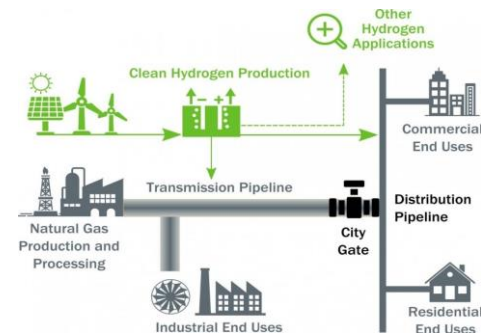
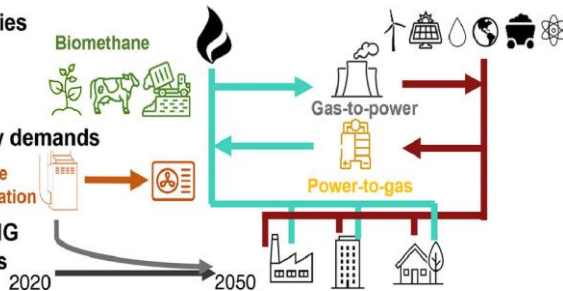
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# FREEDOM GAS

- **By:** Optimized Management for Grid Automation & Security

- U.S. Secretary of Energy Rick Perry, May 7, 2019. “Freedom Gas, the Next American Export”, <https://www.nytimes.com/2019/05/29/us/freedom-gas-energy-department.html>  
<https://www.energy.gov/articles/new-american-energy-era-secretary-perry-keynote-address-cera-week>

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- **Goal:** use capital investment in pipelines for planned lifetime and decarbonize by repurposing to transport renewable hydrogen
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- Blending hydrogen into gas pipelines raises questions of **(A)** operations:
  - (A1) what is the allowable timing, duration, and level of H<sub>2</sub> injection?
  - (A2) how to adjust compressor setpoints given variable daily loads?
  - (A3) what are new flow schedules with change in energy content by concentration?

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  - (B1) where to produce and inject hydrogen?
  - (B2) where to add compressor power?
  - (B3) how hydrogen blending changes pipeline energy transport capacity?



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- **Constraint:** Maintain predictable pipeline flows that do not affect operations
- Blending hydrogen into gas pipelines raises questions of **(A)** operations, **(B)** design, and **(C)** economics:
  - (A1) what is the allowable timing, duration, and level of H2 injection?
  - (A2) how to adjust compressor setpoints given variable daily loads?
  - (A3) what are new flow schedules with change in energy content by concentration?
  - (B1) where to produce and inject hydrogen?
  - (B2) where to add compressor power?
  - (B3) how hydrogen blending changes pipeline energy transport capacity?
  - (C1) how cost of energy transport changes with H2 blending?
  - (C2) what is the value of avoided carbon emissions?
  - (C3) what is the value of flexibility and resilience for the power grid?

# Natural gas & hydrogen basics

- Current Hydrogen pipeline systems are limited, regional, near refineries
  - H<sub>2</sub> is produced mainly (~80%) by steam-methane reforming of natural gas in U.S.
  - Can be produced using electrolysis generated by renewable electricity

Gas	Gas gravity G	MJ/kg	MJ/m <sup>3</sup>	\$/kg	\$/MJ
Natural gas	0.60-0.70	44.2	33	0.3	0.007
Hydrogen	0.0696	141.8	10	5	0.035

- Energy in hydrogen form is ~5x more expensive than NG at prevailing prices
  - The ‘three ones’ goal, \$1 for 1 kg H<sub>2</sub> within 1 decade aims to equalize this cost
- Wave speed of hydrogen is ~1090 m/s, and ~370 m/s for natural gas
  - For the same energy flow, H<sub>2</sub> transport cost is 30-50% more than NG
  - Pipeline operation & maintenance costs are 50-70% more than NG

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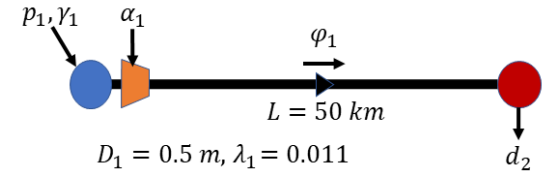
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- **Simulation** to understand effects on transients
- **Optimization** to understand effects on capacity
- **Lagrangian duality** to understand effects of economics

# Pipeline network flow modeling

- For a single pipe, boundary conditions are pressure  $p_1$  and hydrogen mass fraction  $\gamma_1$  at inlet and mass flow  $d_2$  at the outlet
- Variables: density  $\rho$ , pressure  $p$ , velocity  $u$ , & concentration  $\gamma$ ; mass flux  $\phi = \rho u$
- Boundary conditions require concentration  $\gamma_j$  specified at inflow node
- Flow on a pipe is defined by conservation of



Mass:  $\partial_t \rho + \partial_x (\rho u) = 0,$

Momentum:  $\partial_t (\rho u) + \partial_x (p + \rho u^2) = -\frac{\lambda}{2D} \rho u |u| - \rho g \frac{\partial h}{\partial x},$

Concentration:  $\partial_t \gamma + \frac{\phi}{\rho} \partial_x \gamma = 0,$

and Equation of state:  $p = \rho Z(p, \gamma) RT$

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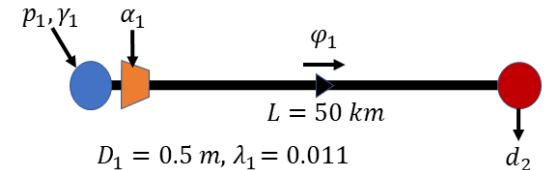
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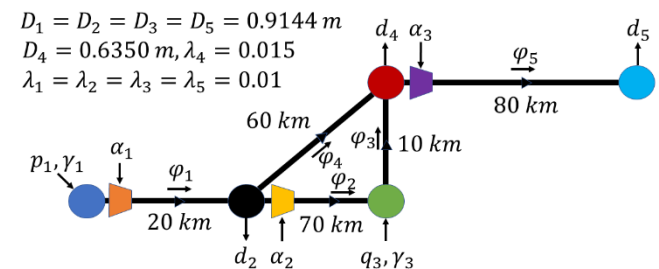
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- For a graph  $(\mathcal{E}, \mathcal{V})$ ,  $e: i \mapsto j$  denotes edges
- incoming and outgoing pipes to node  $j \in \mathcal{V}$  are  $\partial_j^+ = \{e \in \mathcal{E} \mid \exists i \in \mathcal{V} \text{ s.t. } e: i \mapsto j\}$  and  $\partial_j^- = \{e \in \mathcal{E} \mid \exists k \in \mathcal{V} \text{ s.t. } e: j \mapsto k\}$



# Pipeline network flow modeling

- **Steady Flow equations:**  $p_i^2 - p_j^2 = \frac{\lambda_e L_e}{D_e \chi_e^2} V_e(\gamma_e) \phi_e |\phi_e|$ ,  $\forall e = (i, j) \in \mathcal{E}$   
 where  $V_e = \gamma_e a_1^2 + (1 - \gamma_e) a_2^2$  is wave speed that depends on  $\gamma_e$
- **Nodal mass balance equations:**

$$\gamma_j \sum_{e \in \partial_j^-} \phi_e - \sum_{e \in \partial_j^+} \gamma_e \phi_e = \sum_{g \in \partial_j^g} s_g^{NG} - \gamma_j \sum_{g \in \partial_j^g} d_g, \quad \forall j \in \mathcal{V}, \text{ (for H2)}$$

$$(1 - \gamma_j) \sum_{e \in \partial_j^-} \phi_e - \sum_{e \in \partial_j^+} (1 - \gamma_e) \phi_e = \sum_{g \in \partial_j^g} s_g^{NG} - (1 - \gamma_j) \sum_{g \in \partial_j^g} d_g. \quad \forall j \in \mathcal{V}. \text{ (for NG)}$$
- i) **concentration continuity**, ii) **slack nodes**, iii) **compressors:**  
 i)  $\gamma_i = \gamma_e$ ,  $\forall e = (i, j) \in \mathcal{E}$ ; ii)  $p_i = \bar{p}_i$ ,  $\forall j \in \mathcal{V}_s$ ; iii)  $p_j = \alpha_c p_i$ ,  $\forall c = (i, j) \in \mathcal{C}$ .

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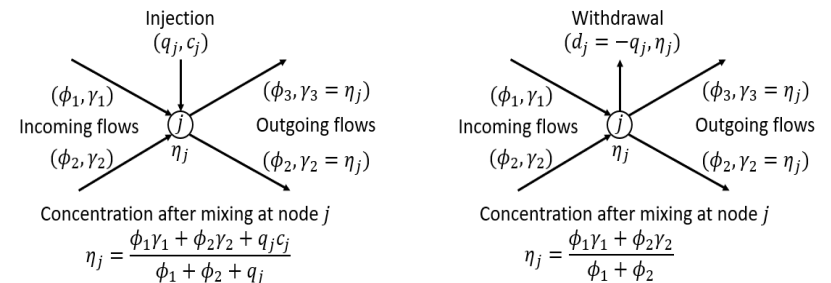
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**Challenge:** flow balance equations depend on flow direction

- Kazi, Saif R., Kaarthik Sundar, Shriram Srinivasan, and Anatoly Zlotnik. "Modeling and optimization of steady flow of natural gas and hydrogen mixtures in pipeline networks." arXiv preprint arXiv:22.12.00961 (2022).



# Pipeline network flow modeling

- **Pipeline user gNodes:** each gNode  $g \in \mathcal{G}$  is a user of the pipeline system
  - each gNode is attached to a physical location node  $j \in \mathcal{V}$
  - set of gNodes at a physical location  $j \in \mathcal{V}$  is  $\partial_j^g$
  - *suppliers* of hydrogen in set  $\mathcal{G}_s^{H2}$ , selling injection at rate  $s_g^1$  kg/s
  - *suppliers* of natural gas in set  $\mathcal{G}_s^{NG}$ , selling injection at rate  $s_g^2$  kg/s
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- **Compressor power:** approximate by adiabatic compression

$$W_c = \left( \frac{286.76 \cdot \zeta \cdot T}{G(\zeta - 1)} \right) \cdot (\alpha_c^{(\zeta - 1)/\zeta} - 1) \cdot \phi_c, \quad \forall c \in \mathcal{C},$$

- **Emissions avoided by a consumer receiving H2:**  $E_g = d_g \gamma_g \cdot \frac{R_1}{R_2} \cdot \zeta$ 
  - $d_g \gamma_g$  is the hydrogen flow in kg/s,
  - $R_{H2} = 0.06098$  &  $R_{NG} = 0.0190$  MJ/kg are calorific values for H2 & NG,
  - $\zeta = 44/18$  is approx ratio of molecular weights of CO2 and methane

- Sodwatana, Mo, Saif R. Kazi, Kaarthik Sundar, and Anatoly Zlotnik. "Optimization of Hydrogen Blending in Natural Gas Networks for Carbon Emissions Reduction." In 2023 American Control Conference (ACC), pp. 1229-1236. IEEE, 2023.

# Optimization formulation

- **Goal:** examine how flow capacity is affected by hydrogen blending – what is an appropriate economic maximum flow problem?
- **Boundary conditions:** natural gas and hydrogen enter the network in different places, and mix. How to optimize injections and withdrawals?
- **Energy capacity:** hydrogen blending affects the capacity of the pipeline to carry energy. How does it change?
- **Sensitivity analysis:** how is the energy transport capacity sensitive to parameters (energy demand, hydrogen concentration limits)?

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$R(\gamma) = (R_{H_2}\gamma + R_{NG}(1 - \gamma))$  = blend-depend. calorific value,  $\psi$  = electricity cost

- *suppliers* of hydrogen  $g \in \mathcal{G}_s^1$ , offering mass flow at rate  $s_g^{1,\max}$  kg/s at price  $c_g^1$  \$/kg
- *suppliers* of natural gas  $g \in \mathcal{G}_s^2$ , offering mass flow at rate  $s_g^{2,\max}$  kg/s at price  $c_g^2$  \$/kg
- *consumers* of mixed gas  $g \in \mathcal{G}_d$ , bidding on energy at rate  $h_g^{\max}$  MJ/s at  $c_g^d$  \$/MJ
- *global incentive* of  $c^m$  \$/kg for CO2 not emitted by consumer  $g \in \mathcal{G}_d$  at rate  $E_g$  kg/s

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  - *global incentive* of  $c^m$  \$/kg for CO2 not emitted by consumer  $g \in \mathcal{G}_d$  at rate  $E_g$  kg/s
- **Constraints:** bid and offer quantities are *upper bounds* – can be curtailed

$$\begin{aligned} 0 \leq s_g^1 \leq s_g^{1,\max}, & \quad \forall g \in \mathcal{G}_s^1, \\ 0 \leq s_g^2 \leq s_g^{2,\max}, & \quad \forall g \in \mathcal{G}_s^2, \\ 0 \leq d_g R(\gamma_g) \leq g_g^{\max}, & \quad \forall g \in \mathcal{G}_d, \end{aligned}$$

# Optimization formulation

$$\min S = \sum_{g \in \mathcal{G}} (c_g^d d_g (R_{H_2} \gamma + R_{NG} (1 - \gamma)) - c_g^{H_2} s_g^{NG} - c_g^{H_2} s_g^{NG} + c^m E_g) - \psi \sum_{c \in \mathcal{C}} W_c$$

$$\begin{aligned} \text{s.t. NG flow balance: } & (1 - \gamma_j) \sum_{k \in \delta_j^-} \phi_{jk} - \sum_{i \in \delta_j^+} (1 - \gamma_{ij}) \phi_{ij} \\ & = \sum_{m \in \delta_j^g} s_m^{NG} - (1 - \gamma_j) \sum_{m \in \delta_j^g} d_m, & \forall j \in \mathcal{V}, \end{aligned}$$

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$$\text{Pressure balance: } P_i^2 - P_j^2 = \frac{\lambda_{ij} L_{ij}}{D_{ij} A_{ij}^2} (\gamma_{ij} a_{H_2}^2 + (1 - \gamma_{ij}) a_{NG}^2) \phi_{ij} |\phi_{ij}|, \quad \forall (i, j) \in \mathcal{E},$$

$$\begin{aligned} \text{Pressure limits: } & P_j^{\min} \leq P_j, & \forall j \in \mathcal{V}, \\ & P_j = \sigma_j, & \forall j \in \mathcal{V}_s, \end{aligned}$$

$$\begin{aligned} \text{Compressor boost limits: } & P_j^2 = \alpha_{ij}^2 P_i^2, & \forall (i, j) \in \mathcal{C}, \\ & \alpha_{ij} P_i \leq P_{ij}^{\max}, & \forall (i, j) \in \mathcal{C}, \\ & 1 \leq \alpha_{ij} \leq \alpha_{ij}^{\max}, & \forall (i, j) \in \mathcal{C}, \end{aligned}$$

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$$\begin{aligned} \text{Supply limits: } & 0 \leq s_m^{H_2} \leq s_m^{\max, H_2}, & \forall m \in \mathcal{G}_s^{H_2}, \\ & 0 \leq s_m^{NG} \leq s_m^{\max, NG}, & \forall m \in \mathcal{G}_s^{NG}, \end{aligned}$$

$$\begin{aligned} \text{Demand limits: } & 0 \leq d_m, & \forall m \in \mathcal{G}_d, \\ & d_m (R_{H_2} \gamma_{j(m)} + R_{NG} (1 - \gamma_{j(m)})) \leq g_m^{\max}, & \forall m \in \mathcal{G}_d, \end{aligned}$$

- Sodwatana, Mo, Saif R. Kazi, Kaarthik Sundar, and Anatoly Zlotnik. "Optimization of Hydrogen Blending in Natural Gas Networks for Carbon Emissions Reduction." In 2023 American Control Conference (ACC), pp. 1229-1236. IEEE, 2023.

# Economics

- **Goal:** Examine price structure of optimization-based market mechanism for hydrogen and natural gas in a pipeline network
- **Hydrogen impact:** how does hydrogen blending impact the value of pipeline gas?
- **Hydrogen incentives:** how do hydrogen usage incentives to avoid carbon emissions influence the physical flow and market outcomes?
- **Decarbonization premium:** what is the additional price paid by consumers of energy in a market with hydrogen usage incentives to avoid carbon emissions?



# Economics

- **Partial Lagrangian:** include all terms that involve consumption flows  $d_g$

$$L = \sum_{g \in \mathcal{G}} \left( -c_g^d d_g R(\gamma_g) - c^m d_g \gamma_g \cdot \frac{R_1}{R_2} \cdot \zeta \right) + \sum_{j \in \mathcal{V}} \lambda_j^1 \left( \gamma_j \sum_{g \in \partial_j^g} d_g \right) \\ + \sum_{j \in \mathcal{V}} \lambda_j^2 \left( (1 - \gamma_j) \sum_{g \in \partial_j^g} d_g \right) + \sum_{g \in \mathcal{G}_d} \mu_g^l \left( -d_g R(\gamma_g) \right) + \sum_{g \in \mathcal{G}_d} \mu_g^u \left( d_g R(\gamma_g) - h_g^{\max} \right)$$

- $\lambda_j^1$  and  $\lambda_j^2$  are Lagrange multipliers on flow balance constraints for H2 and NG
- $\mu_g^l$  and  $\mu_g^u$  are Lagrange multipliers on inequality constraints for energy delivery

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- **Karush Kuhn Tucker condition for optimality:**

- derivative of  $L$  w.r.t.  $d_g$ :

$$0 = -c_g^d R(\gamma_g) - c^m \gamma_g \cdot \frac{R_1}{R_2} \cdot \zeta + \lambda_j^1 \gamma_j + \lambda_j^2 (1 - \gamma_j) - \mu_g^l R(\gamma_g) + \mu_g^u R(\gamma_g), \quad \forall g \in \mathcal{G}_d$$

- complementary slackness for energy delivery constraints:

$$\mu_g^l \cdot d_g R(\gamma_g) = 0, \quad \mu_g^u \cdot (d_g R(\gamma_g) - h_g^{\max}) = 0, \quad \forall g \in \mathcal{G}_d$$

# Economics

- **Solve derivative condition:** expression for price of withdrawn gas

$$\lambda_{j(g)} = R(\gamma_g) \cdot (c_g^d + \mu_g^l - \mu_g^u) + c^m \gamma_g \frac{R_1}{R_2} \zeta, \quad \forall g \in \mathcal{G}_d.$$

- Separate into price components:  $\lambda_{j(g)} = \tau_g^c + \tau_g^m$ ,
- *Congestion price* on gas:  $\tau_g^c = R(\gamma_g) \cdot (c_g^d + \mu_g^l - \mu_g^u)$ , or on energy:  $c_g^d + \mu_g^l - \mu_g^u$
- *Decarbonization premium* on gas:  $\tau_g^m = c^m \gamma_g \cdot \frac{R_1}{R_2} \cdot \zeta$ , or on energy:  $c^m \gamma_g \cdot \frac{R_1}{R_2} \cdot \zeta / R(\gamma_g)$

# Economics

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  - Decarbonization premium on gas:  $\tau_g^m = c^m \gamma_g \cdot \frac{R_1}{R_2} \cdot \zeta$ , or on energy:  $c^m \gamma_g \cdot \frac{R_1}{R_2} \cdot \zeta / R(\gamma_g)$
- **For a marginal consumer**,  $\mu_g^l = \mu_g^u = 0$ , so the market price of energy is the bid price  $c_g^d$  plus the decarbonization premium  $c^m \gamma_g \cdot \frac{R_1}{R_2} \cdot \zeta / R(\gamma_g)$

• Zlotnik, Anatoly, Saif R. Kazi, Kaarthik Sundar, Vitaliy Gyrya, Luke Baker, Mo Sodwatana, and Yan Brodskyi. "Effects of Hydrogen Blending on Natural Gas Pipeline Transients, Capacity, and Economics." In PSIG Annual Meeting, pp. PSIG-2312. PSIG, 2023.

# Economics

- **Take-away**: optimality conditions yield expressions for gas and energy prices, and price decomposition
- **Price components**: price is separated into prices of hydrogen and natural gas in the mixture, price of congestion, and decarbonization premium
- **Hydrogen incentives**: hydrogen incentives result in increased prices paid for energy, where incentives pass through to the consumer
- **Decarbonization premium**: depends on the mass fraction of hydrogen, so this is paid only by those consumers who get incentives

# Optimization Case Studies

- **Optimization for a single pipe**
  - Supply pressure at 6 MPa
  - Two suppliers (for H2 and NG)
  - Max 10% H2 fraction

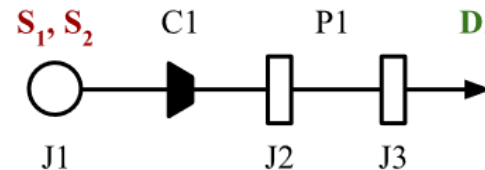
Offer price and injection limit for hydrogen and natural gas at injection node :

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$$c_{s2}^{H2s} = \$0.8/\text{kg}, s_{s2}^{\text{max}, H2} = \text{inf kg/s}$$

Bid price for blended gas at withdrawal node :

$$c_{d1}^d = \$0.019/\text{MJ}$$



- **Sensitivity analysis**

- Energy demand  $h_g^{\text{max}}$  at gNode D1 ranging from 700 to 900 MJ/s
- CO2 emissions mitigation incentive price  $c^m$  from 0 to .08 \$/kg (\$72.5/ton)
- Sensitivity w.r.t. minimum H2 fraction up to 10%

Compressor characteristics:

$$\alpha^{\text{min}} = 1.0, \alpha^{\text{max}} = 1.4$$

$$\gamma^{\text{min}} = 0, \gamma^{\text{max}} = 0.1$$

Pipe characteristics:

$$\lambda = 0.0125, l = 200 \text{ m}, d = 0.2 \text{ m}$$

$$p^{\text{min}} = 3 \text{ MPa}, p^{\text{max}} = 6 \text{ MPa}$$

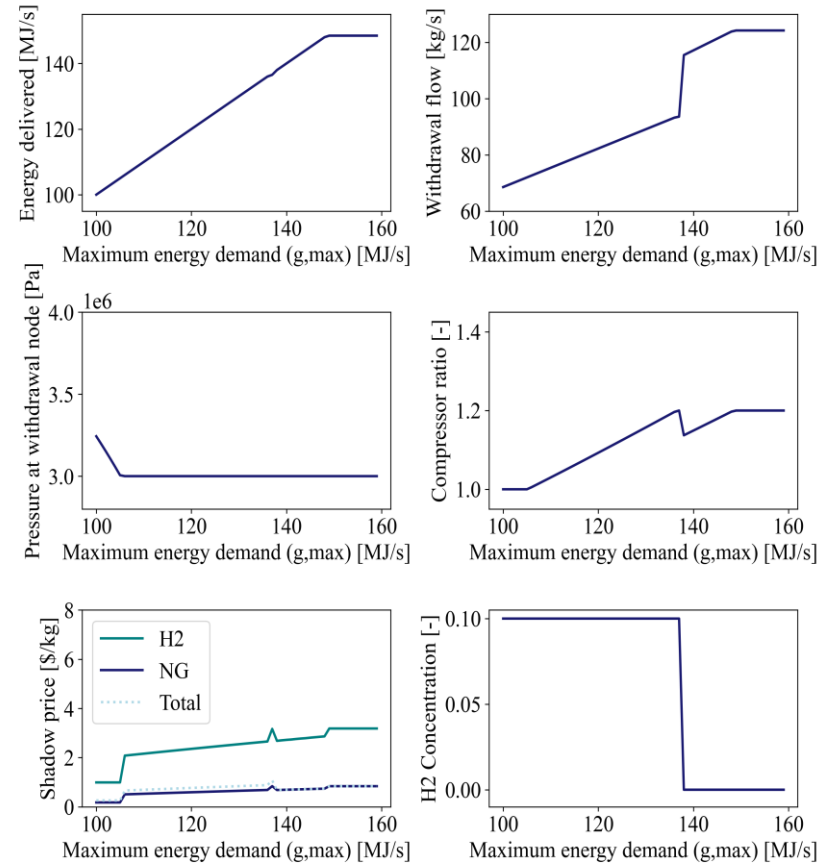
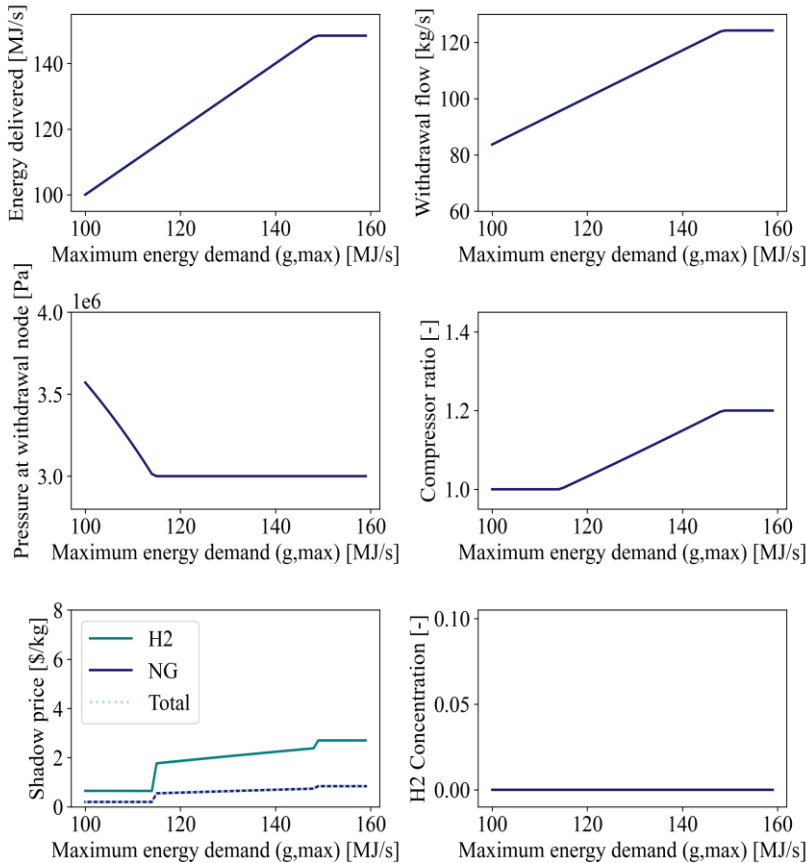
$$\gamma^{\text{min}} = 0, \gamma^{\text{max}} = 0.1$$

# Optimization Case Studies

- Optimization for a single pipe - sensitivity w.r.t. demand quantity bid

No carbon credit,  $c^m = 0$

carbon credit  $c^m = \$0.055/\text{kg}$

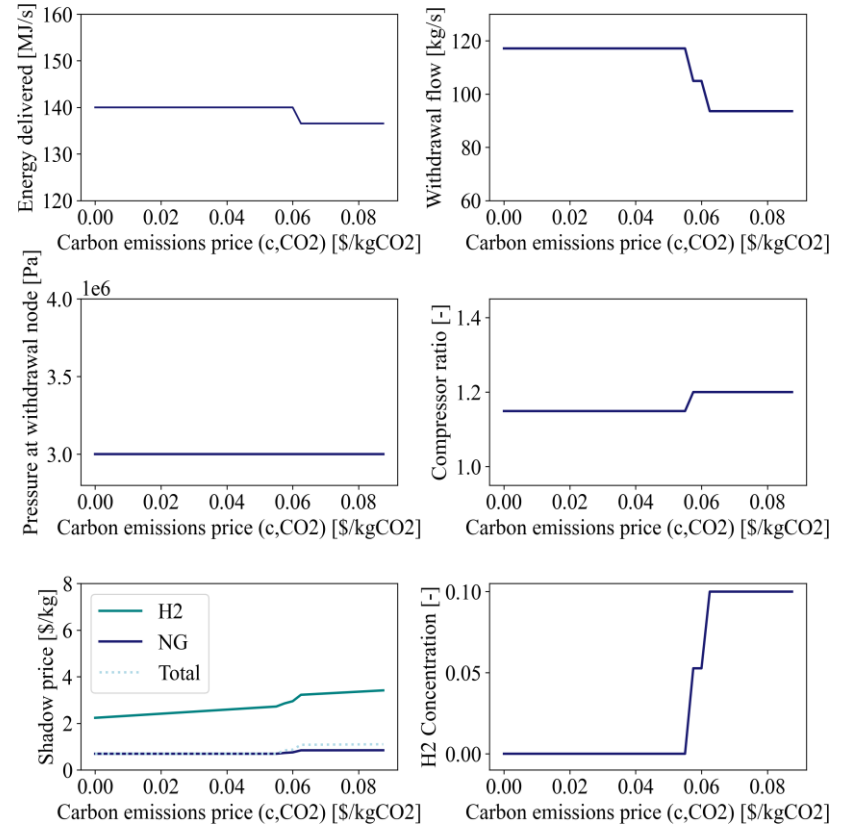
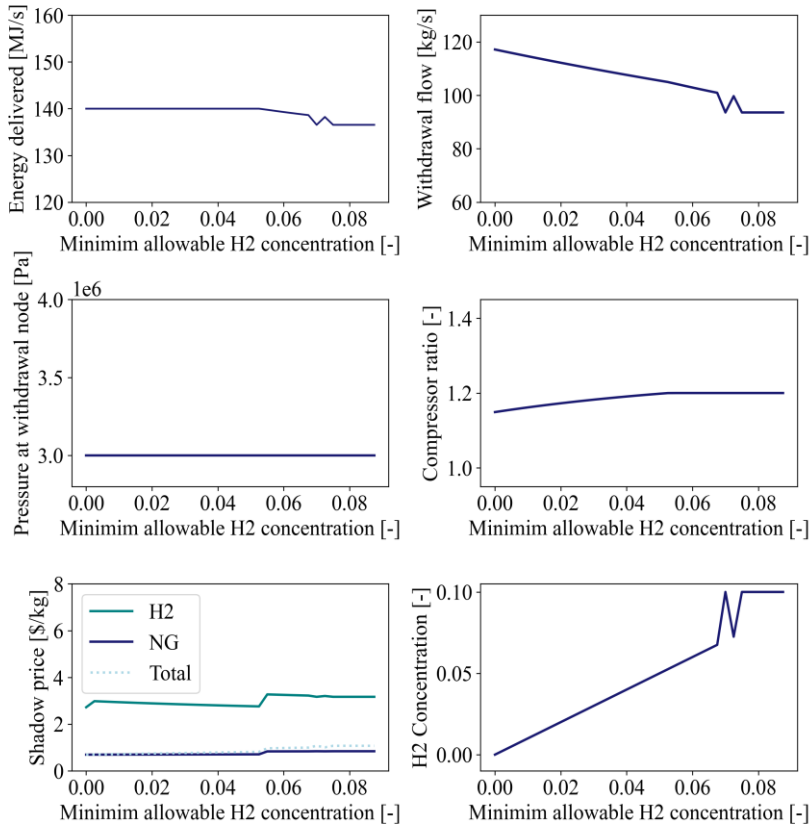


# Optimization Case Studies

- Optimization for a single pipe – sensitivity w.r.t. concentration and credit

Sensitivity w.r.t. minimum H2 fraction

Sensitivity w.r.t. CO2 offset credit





# Optimization Case Studies

- **Optimization for a network**

- 8 node, 5 pipe, 3 compressor network
- NG supply from gNode S1 at node J1
- H2 supply from gNode S2 at node J1 and from gNode S3 at node J7 at a lower price but limited quantity
- Fixed consumer D3 at J5
- Variable buyers D1 and D2 at J3 & J5

Offer price and injection limit for hydrogen and natural gas at injection nodes :

$$c_{s1}^{NGs} = \$0.2/\text{kg}, s_{s1}^{\text{max}, NG} = 155 \text{ kg/s}$$

$$c_{s2}^{H2s} = \$0.8/\text{kg}, s_{s2}^{\text{max}, H2} = \text{inf kg/s}$$

$$c_{s3}^{H2s} = \$0.7/\text{kg}, s_{s3}^{\text{max}, H2} = 10 \text{ kg/s}$$

Bid price for blended gas at withdrawal nodes :

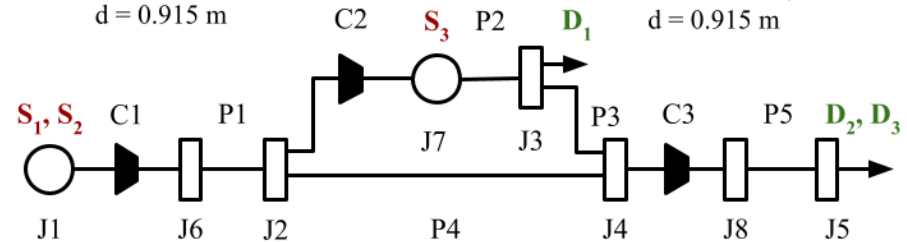
$$c_{d1}^d = \$0.019/\text{MJ}$$

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$$\begin{aligned} \text{P2: } \lambda &= 0.01, \\ l &= 70000 \text{ m}, \\ d &= 0.915 \text{ m} \end{aligned}$$

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$$\begin{aligned} \text{P1: } \lambda &= 0.01, \\ l &= 20000 \text{ m}, \\ d &= 0.915 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{P4: } \lambda &= 0.015, \\ l &= 60000 \text{ m}, \\ d &= 0.635 \text{ m} \end{aligned}$$

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  - Fixed consumer D3 at J5
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## Can H2 utilization incentives cause counter-productive results?

- Are there network topologies and market conditions for which increasing incentives to use hydrogen will lead to higher CO2 emissions?

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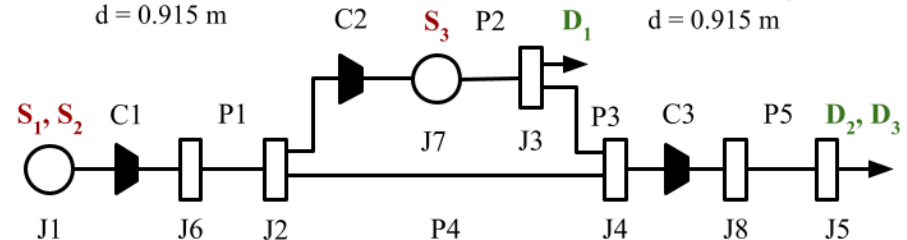
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# Optimization Case Studies

- Network incentives case study

- Baseline scenario (1)

- All parameters as given, bid price  $c_{D2}^d = 0.019$  \$/kg and CO2 price  $c^m = 0.055$  \$/kg (\$50/ton).

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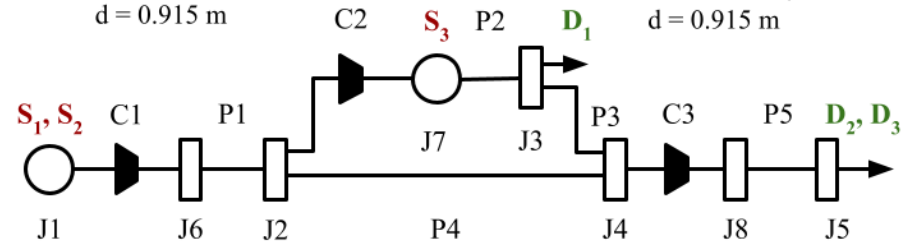
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- Bid price  $c_{D2}^d$  for energy at gNode D2 is decreased to  $c_{D2}^d = 0.0025$  \$/MJ to reflect low demand by a buyer (e.g., a gas-fired power plant) that has very elastic consumption (i.e., depends on electricity spot prices).

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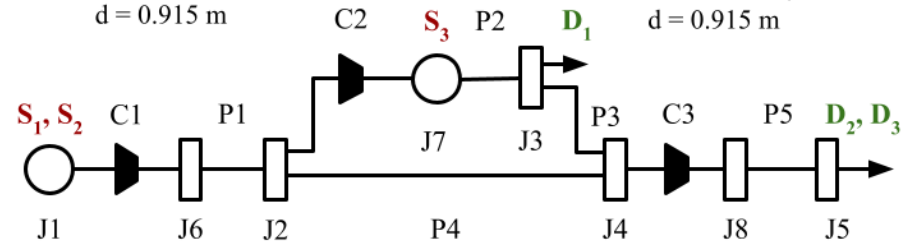
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- **High H2 incentive scenario (3)**

- CO2 price is increased to  $c^m = 0.155$  \$/kg (\$140/ton) with all else as in scenario (2)

Offer price and injection limit for hydrogen and natural gas at injection nodes :

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Bid price for blended gas at withdrawal nodes :

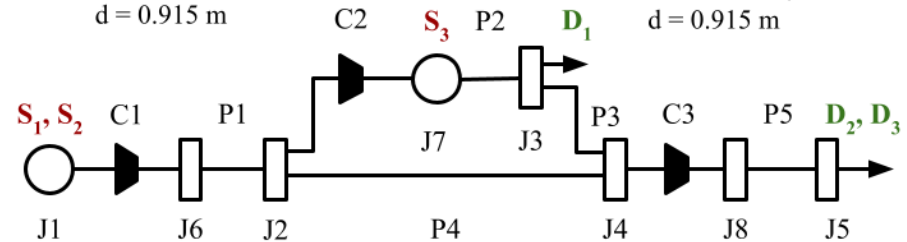
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gNodes:	S1 (NG) (J1)	S2 (H2) (J7)	D1 (J3)	D2 (J5)	D3 (Fixed) (J5)
NG Flow [kg/s] <b>101.27</b>	101	-	31.2	27.31	42.76
H2 Flow [kg/s]	-	15	7.8	2.7	4.23
Total flow [kg/s]	101	15	39	30	47
Provided energy [MJ/s]	-	-	2500	1575	2500
H2 fraction	0	0.2	0.2	0.09	0.09
Supply offer NG/H2 [\$/kg]	0.2	0.8	-	-	-
Bid price for energy [\$/MJ]	-	-	0.019	0.019	0.019
Market price NG $\lambda_j^2$ [\$/kg]	0.2	0.28	0.51	0.84	0.84
Market price H2 $\lambda_j^1$ [\$/kg]	-	0.8	1.79	3.18	3.18
Mixture price $\lambda_j$ [\$/kg]	-	-	0.77	1.05	1.05
Incentive [\$/kg] premium $\tau_g^m$	-	-	0.0863	0.0388	0.0388

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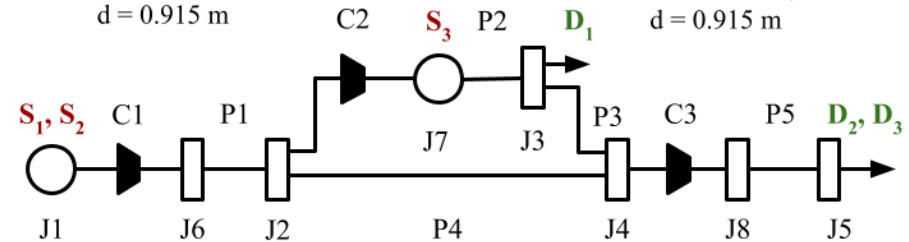
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## • Low price at D2 scenario (2)

- Bid price  $c_{D2}^d$  for energy at gNode D2 is decreased to  $c_{D2}^d = 0.0025$  \$/MJ to reflect low demand by a buyer

gNodes:	S1 (NG) (J1)	S2 (H2) (J7)	D1 (J3)	D2 (J5)	D3 (Fixed) (J5)
NG Flow [kg/s] <b>76.7</b>	77	-	31.2	0	45.5
H2 Flow [kg/s]	-	11	7.8	0	3.42
Total flow [kg/s]	77	11	39	0	49
Provided energy [MJ/s]	-	-	2500	0	2500
H2 fraction	0	0.2	0.2	0.07	0.07
Supply offer NG/H2 [\$/kg]	0.2	0.8	-	-	-
Bid price for energy [\$/MJ]	-	-	0.019	0.0025	0.019
Market price NG $\lambda_j^2$ [\$/kg]	0.2	0.28	0.19	0.17	0.17
Market price H2 $\lambda_j^1$ [\$/kg]	-	0.8	0.93	1.02	1.02
Mixture price $\lambda_j$ [\$/kg]	-	-	0.338	0.23	0.23
Incentive [\$/kg] premium $\tau_g^m$	-	-	0.0863	0.0302	.0302

Offer price and injection limit for hydrogen and natural gas at injection nodes :

$$c_{s1}^{NGs} = \$0.2/\text{kg}, s_{s1}^{\text{max}, NG} = 155 \text{ kg/s}$$

$$c_{s2}^{H2s} = \$0.8/\text{kg}, s_{s2}^{\text{max}, H2} = \text{inf kg/s}$$

$$c_{s3}^{H2s} = \$0.7/\text{kg}, s_{s3}^{\text{max}, H2} = 10 \text{ kg/s}$$

Bid price for blended gas at withdrawal nodes :

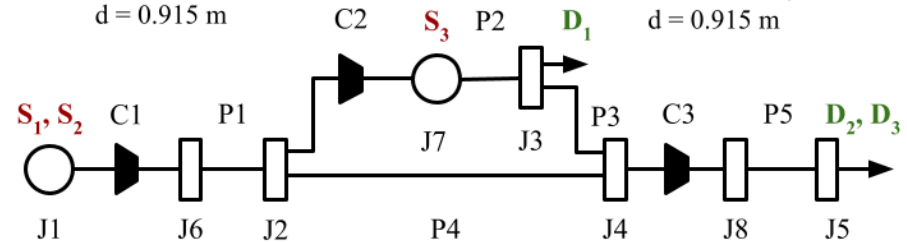
$$c_{d1}^d = \$0.019/\text{MJ}$$

$$c_{d2}^d = \$0.019/\text{MJ}$$

$$c_{d3}^d = \$0.0/\text{MJ}$$

$$P2: \lambda = 0.01, \\ l = 70000 \text{ m}, \\ d = 0.915 \text{ m}$$

$$P3: \lambda = 0.01, \\ l = 10000 \text{ m}, \\ d = 0.915 \text{ m}$$



$$P1: \lambda = 0.01, \\ l = 20000 \text{ m}, \\ d = 0.915 \text{ m}$$

$$P4: \lambda = 0.015, \\ l = 60000 \text{ m}, \\ d = 0.635 \text{ m}$$

$$P5: \lambda = 0.01, \\ l = 80000 \text{ m}, \\ d = 0.915 \text{ m}$$

Compressor characteristics:

$$\alpha^{\min} = 1.0, \alpha^{\max} = 1.4 \\ \gamma^{\min} = 0, \gamma^{\max} = 0.1$$

Pipe characteristics:

$$P^{\min} = 3 \text{ MPa}, P^{\max} = 6 \text{ MPa} \\ \gamma^{\min} = 0, \gamma^{\max} = 0.1$$

# Optimization Case Studies

- **High H2 incentive scenario (3)**

- CO2 price is increased to  $c^m = 0.155$  \$/kg (\$140/ton) with all else as in scenario (2)

gNodes:	S1 (NG) (J1)	S2 (H2) (J7)	D1 (J3)	D2 (J5)	D3 (Fixed) (J5)
NG Flow [kg/s] <b>101.27</b>	101	-	31.2	27.31	42.76
H2 Flow [kg/s]	-	15	7.8	2.7	4.23
Total flow [kg/s]	101	15	39	30	47
Provided energy [MJ/s]	-	-	2500	1575	2500
H2 fraction	0	0.2	0.2	0.09	0.09
Supply offer NG/H2 [\$/kg]	0.2	0.8	-	-	-
Bid price for energy [\$/MJ]	-	-	0.019	0.0025	0.019
Market price NG $\lambda_j^2$ [\$/kg]	0.2	0.203	0.18	0.11	0.11
Market price H2 $\lambda_j^1$ [\$/kg]	-	0.8	1.12	1.72	1.72
Mixture price $\lambda_j$ [\$/kg]	-	-	0.368	0.255	0.255
Incentive [\$/kg] premium $\tau_g^m$	-	-	0.2431	0.1094	0.1094

Offer price and injection limit for hydrogen and natural gas at injection nodes :

$$c_{s1}^{NGs} = \$0.2/\text{kg}, s_{s1}^{\text{max}, NG} = 155 \text{ kg/s}$$

$$c_{s2}^{H2s} = \$0.8/\text{kg}, s_{s2}^{\text{max}, H2} = \text{inf kg/s}$$

$$c_{s3}^{H2s} = \$0.7/\text{kg}, s_{s3}^{\text{max}, H2} = 10 \text{ kg/s}$$

Bid price for blended gas at withdrawal nodes :

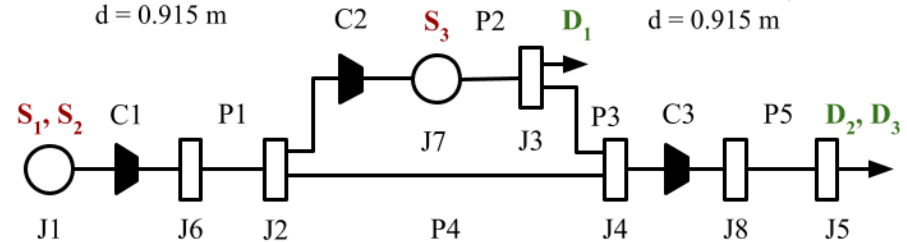
$$c_{d1}^d = \$0.019/\text{MJ}$$

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• Zlotnik, Anatoly, Saif R. Kazi, Kaarthik Sundar, Vitaliy Gyrya, Luke Baker, Mo Sodwatana, and Yan Brodskyi. "Effects of Hydrogen Blending on Natural Gas Pipeline Transients, Capacity, and Economics." In PSIG Annual Meeting, pp. PSIG-2312. PSIG, 2023.



# Optimization Case Studies

- **Take-away:** solution behaves intuitively for a single pipe, but may be counter-intuitive for a network
- **Price components:** prices change non-monotonically with parameters of the optimization problem
- **Hydrogen and capacity:** adding hydrogen decreases energy capacity of a constrained system, but may increase transported energy if the system is not constrained to begin with
- **Influence of incentives:** incentives to consumers who receive hydrogen pass through, but may increase total NG consumption

# Transient Flow

- **Goal:** examine how flow transients are affected by hydrogen blending
- **Boundary conditions:** natural gas and hydrogen enter the network in different places, possibly with time-dependent profiles, and mix
- **Monotonicity properties:** for homogeneous gas, pressures and flows are ordered if boundary conditions are ordered. What happens in inhomogeneous mixing?
- **Stability properties:** how fast can hydrogen injections change without causing unstable pressure waves?

# Transient Flow

- **Dynamics** in each pipe:

Mass:	$\partial_t \rho + \partial_x(\rho v) = 0,$
Momentum:	$\partial_t(\rho u) + \partial_x(p + \rho v^2) = -\frac{\lambda}{2D} \rho v  v ,$
Concentration:	$\partial_t \eta^{(m)} + v \partial_x \eta^{(m)} = 0,$
and Equation of state:	$p = \sigma^2 \rho$

- $\lambda$  and  $D$  are friction factor and pipe diameter
- Velocity  $v$
- Mass fraction  $\gamma^{(m)}$  ( $m = 1$  for NG or  $m = 2$  for H<sub>2</sub>)
- Density  $\rho$  and pressure  $p = \sigma^2 \rho$
- $\sigma$  is the wave speed of the mixture,  $\sigma^2 = \sigma_1^2 \gamma^{(1)} + \sigma_2^2 \gamma^{(2)}$

# Transient Flow

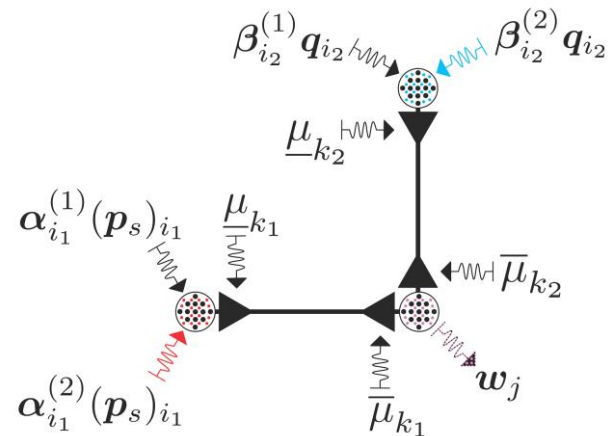
- Nodal mass balance equations:**

$$\sum_{k \in \rightarrow j} \eta_{ij}(t, L_{ij}) \phi_{ij}(t, L_{ij}) - \sum_{k \in j \rightarrow} \eta_{jk}(t, 0) \phi_{jk}(t, 0) = \gamma_j^{(m)} \sum_{g \in \partial_j^g} w_g(t), \quad \forall j \in \mathcal{V}, \quad m = 1, 2$$

- i) slack mass fraction, ii) slack node and compressor:**

i)  $\eta_{k_1}(t, 0) = \alpha_{i_1}^{(m)}(t), \quad \forall e = (i, j) \in \mathcal{E};$  ii)  $p_{k_1}(t, 0) = \underline{\mu}_{k_1}(p_s)_i(t) \quad , \quad \forall j \in \mathcal{V}_s;$

- Slack node pressure:  $p_s(t)$
- Slack node mass fraction:  $\alpha_s^{(m)}(t)$
- Non-slack injection:  $q(t)$
- Non-slack mass fraction:  $\beta^{(m)}(t)$
- Withdrawal:  $w(t)$
- Compression:  $\underline{\mu}_k(t)$
- Regulation:  $\bar{\mu}_k(t)$



# Transient Flow

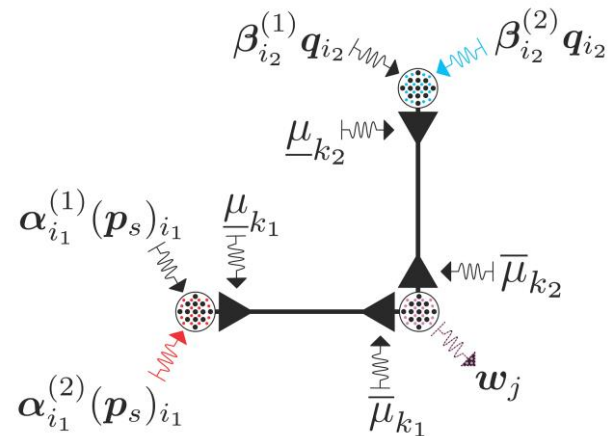
- **Discretization** in space results in a DAE control system:

$$R\dot{p} = Q_d^T \varphi - w$$

$$R\dot{p} = Q_d^T \left( \left( |Q_s| \sigma_s^2 + |Q_d| \frac{p}{\rho} \right) \odot \varphi \right) - \left( I_q \sigma_d^2 + I_d \frac{p}{\rho} \right) \odot w$$

$$0 = M_s p_s + M_d p + LK \frac{\varphi \odot \varphi}{I\rho}$$

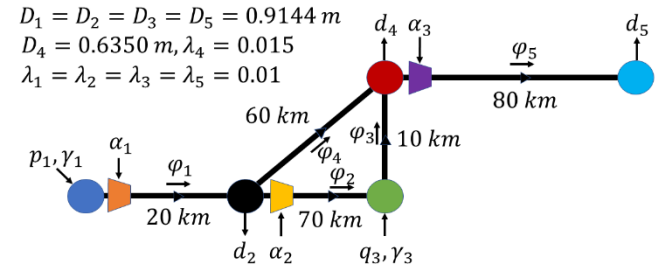
- **Equivalent representations**
- **Monotone ordering theorems**



- Baker, Luke S., Saif R. Kazi, and Anatoly Zlotnik. "Transitions from Monotonicity to Chaos in Gas Mixture Dynamics in Pipeline Networks." PRX Energy 2, 033008 (2023).

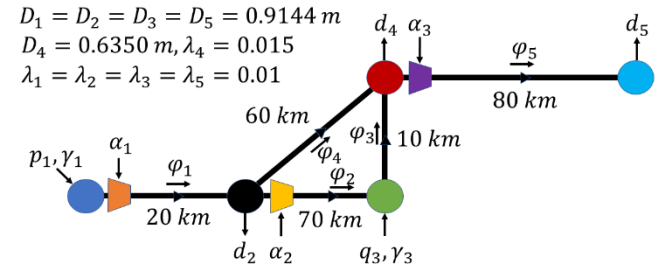
# Transient Flow

- Test network with 5 nodes, 5 pipes, and 3 compressors
- Natural gas enters at slack node 1 (blue) with pressure at 10 Mpa (1450 psi)
- Baseline withdrawal of 100 and 300 kg/s (220.46 and 661.38 lb/s) at nodes 2 (black) and 5 (cyan)
- Hydrogen is injected at node 3 (green)
- Suppose wave speed for hydrogen is  $a_1 = 2.8a_2$  where the wave speed for natural gas is  $a_2 = 377$  m/s (843.3 miles/hour)
- Energy content of mixture is computed with heating values of  $R_1 = 141.8$  MJ/kg for hydrogen and  $R_2 = 44.2$  MJ/kg for natural gas ( $R_1 = 0.06098$  and  $R_2 = 0.0190$  mmbtu/lb)



# Transient Flow

- Consider simulations comparing a baseline with no hydrogen injection with a case of hydrogen injection at node 3 (green)
- Examine simulations for 3 scenarios
- **Scenario (a):** constant withdrawals and compression ratios
- **Scenario (b):** constant withdrawals at nodes 2 (black) and 5 (cyan) with proportional\* feedback control of compression ratios
- **Scenario (c):** transient withdrawals at nodes 2 (black) and 5 (cyan) with proportional\* feedback control of compression ratios
- \*gains of 0.5 for comps. 1 (orange) & 3 (purple); 0.1 for comp. 2 (yellow)
- Hydrogen injections ramp from 0 at  $t = 5.5$  hours to 3 kg/s (6.61 lb/s) at  $t = 6.5$  hours



# Transient Flow

- **Scenario (a):** constant withdrawals and compression ratios
- Dots: baseline; lines: with hydrogen

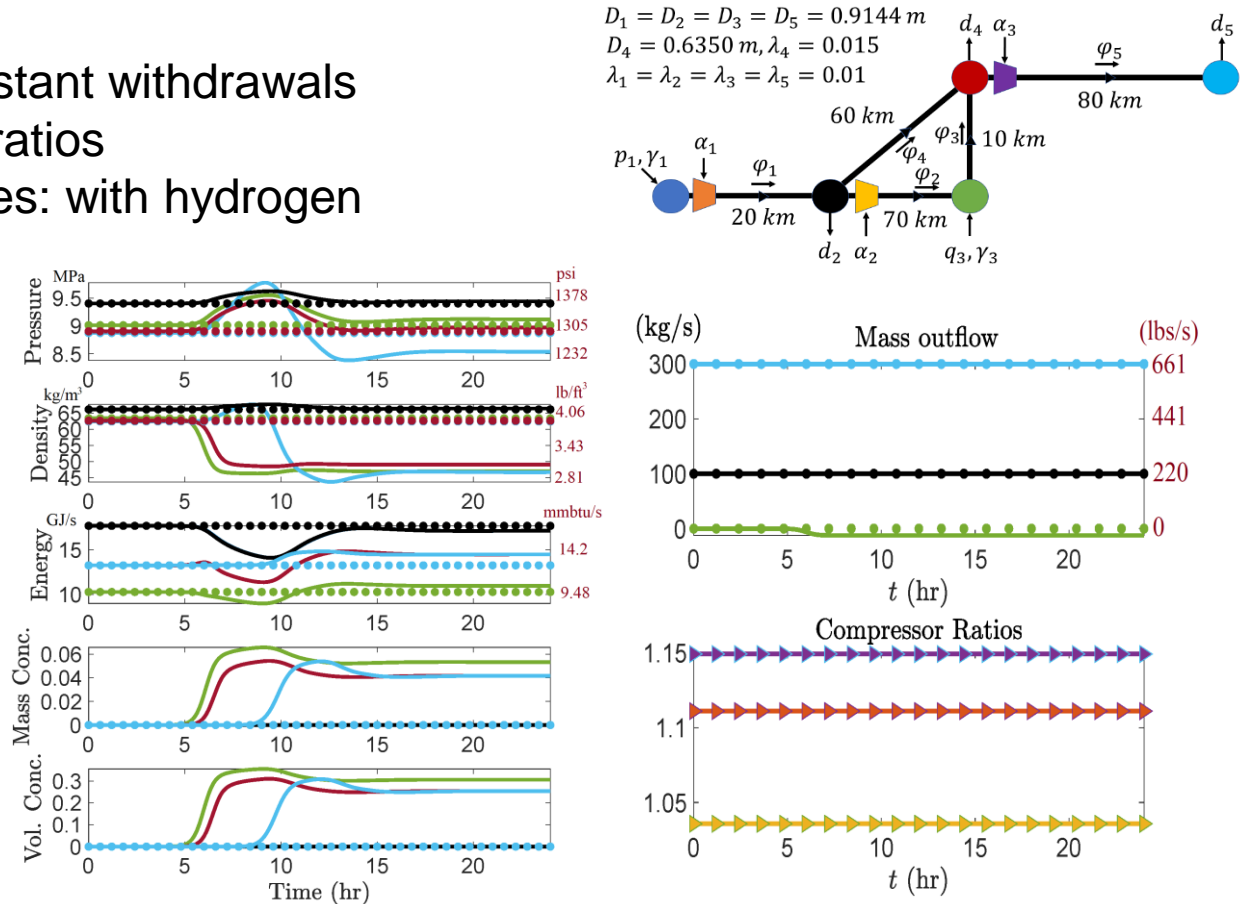
- Pressures (MPa)

- Densities (kg/m<sup>3</sup>)

- Energy (GJ/s)

- Mass frac. H2 (%)

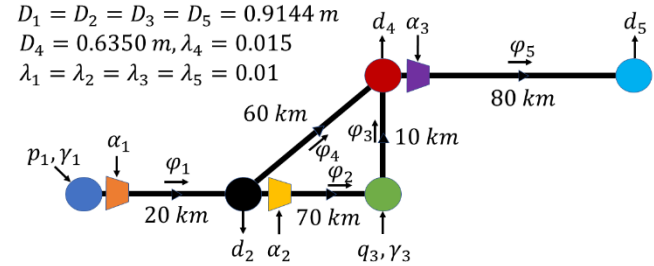
- Volume frac. H2



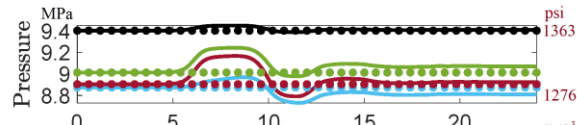


# Transient Flow

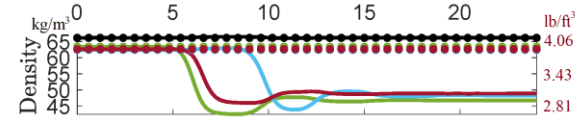
- **Scenario (b)**: constant withdrawals at nodes 2 (black) and 5 (cyan) with proportional feedback control of compression ratios



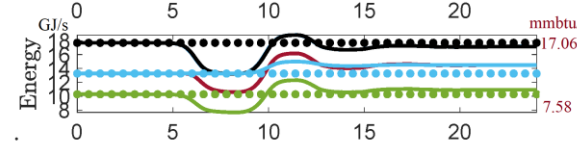
- Pressures (MPa)



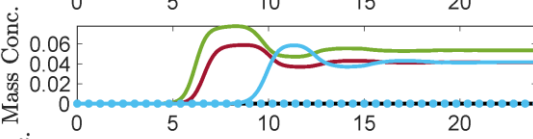
- Densities (kg/m<sup>3</sup>)



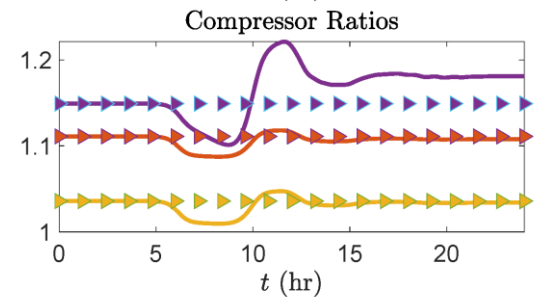
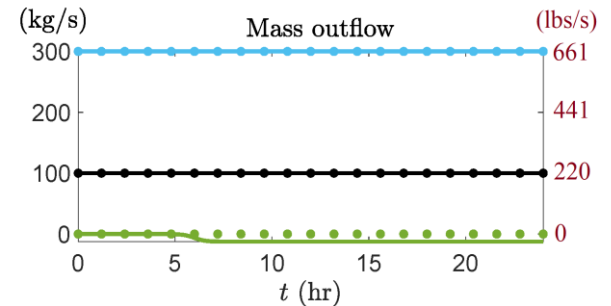
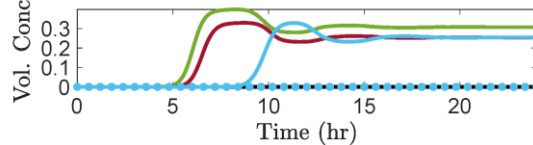
- Energy (GJ/s)



- Mass frac. H<sub>2</sub> (%)



- Volume frac. H<sub>2</sub>



# Transient Flow

- **Scenario (c)**: transient withdrawals at nodes 2 (black) and 5 (cyan) with proportional feedback control of compression ratios

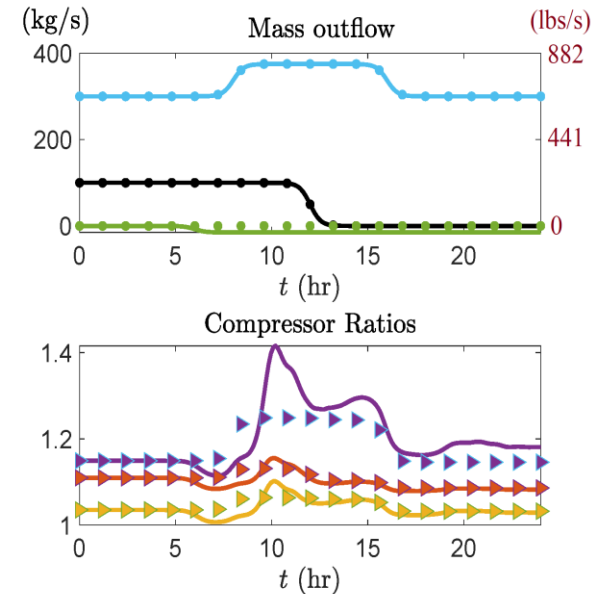
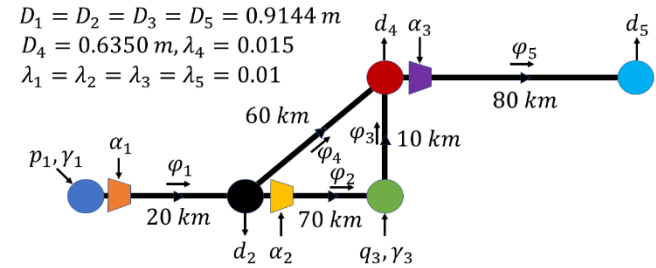
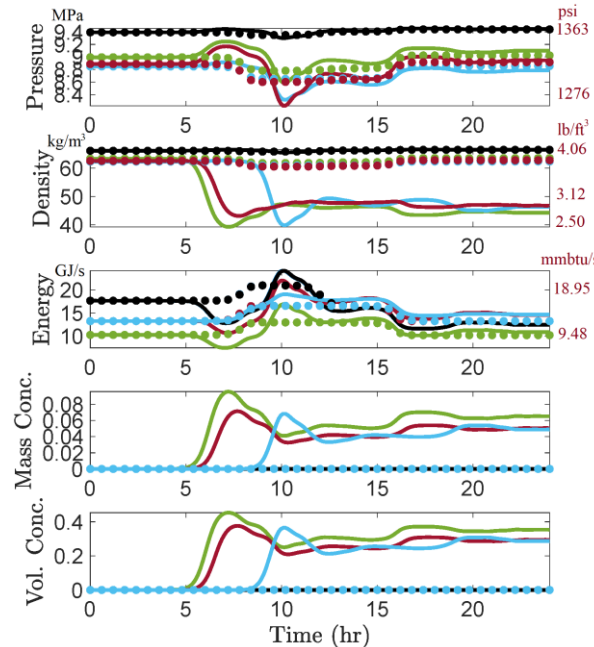
- Pressures (MPa)

- Densities (kg/m<sup>3</sup>)

- Energy (GJ/s)

- Mass frac. H<sub>2</sub> (%)

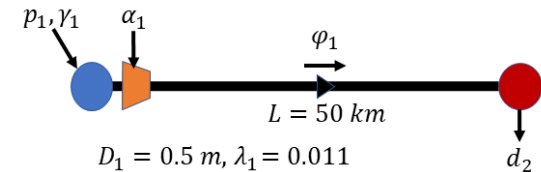
- Volume frac. H<sub>2</sub>



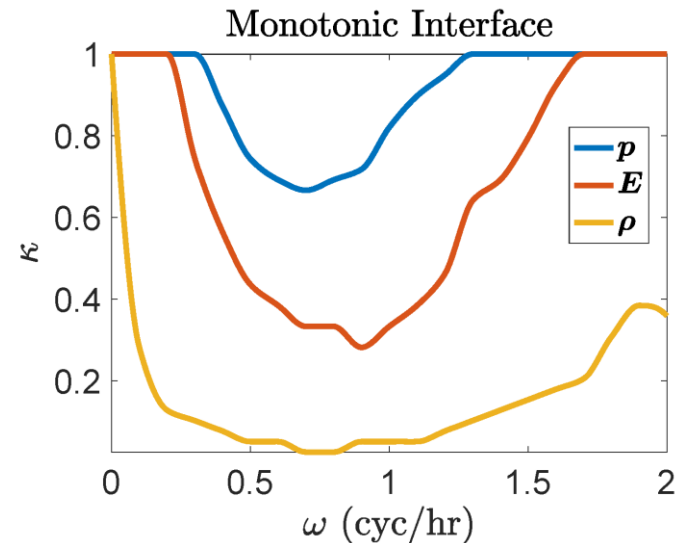
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# Transient Flow

- **Monotone ordering:** if  $d_2^{(a)}(t) < d_2^{(b)}(t)$ , then  $p_2^{(a)}(t) > p_2^{(b)}(t)$ .
- Monotone ordering does not in general hold if  $\gamma_1(t) > 0$  is transient
- Under what variation of  $\gamma_1(t)$  does monotonicity hold?



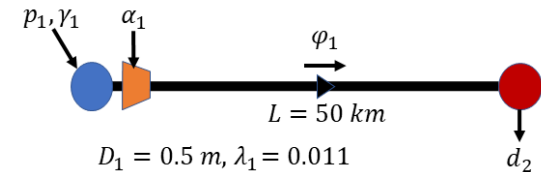
- Suppose  $p_1 = 7$  MPa and  $\gamma_1(t) = \bar{\gamma}_1 \cdot (1 + \kappa \sin(2\pi\omega t))$ , with  $\bar{\gamma}_1 = 0.04$
- Vary amplitude  $\kappa$  and frequency  $\omega$  of hydrogen mass fraction fluctuation and test for crossovers of pressures for  $d_2(t) = 120, 140, 160$  kg/s (264.5544, 308.6468, and 352.7392 lb/s)



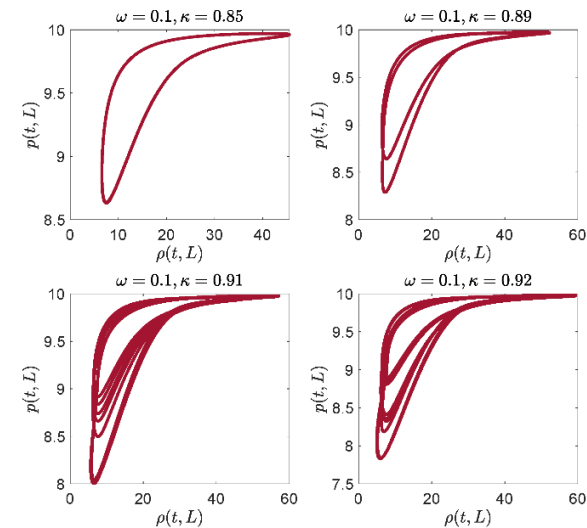
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# Transient Flow

- Are there solutions where the boundary conditions  $p_1 = 7$  Mpa and  $\gamma_1(t)$  are time-periodic, but the pressure  $p_2(t)$  at node 2 is irregular (chaotic)?



- Suppose  $a_1 = 4a_2$  where the wave speed for natural gas is  $a_2 = 338.38$  m/s (756 mph)
- Let  $\gamma_1(t) = \bar{\gamma}_1 \cdot (1 + \kappa \sin(2\pi\omega t))$ , with  $\bar{\gamma}_1 = 0.2$
- After 400 hours of simulation, trajectories do not form a closed periodic orbit in pressure-density phase space



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# Transient Flow

- **Take-away:** blending hydrogen creates counter-intuitive transient behavior – sometimes injecting more gas (or withdrawing less) can actually *lower* pressure
- **State dynamics:** adding hydrogen can increase energy content, while lowering density and pressure
- **Pressure dynamics:** monotonicity is largely preserved for pressure dynamics, which is good from point of view of gas controllers
- **Chaotic response:** too much variability in hydrogen injection leads to pressure waves that do not dissipate

# Conclusions

- **Take-away:** hydrogen blending impacts transient, capacity, and economics of pipeline networks designed for natural gas
- **Moving forward:** pipeline simulation and optimization is needed to evaluate the consequences of proposed projects
- **Pipeline design:** evaluation of hydrogen siting proposals
- **Regulatory policy:** evaluation of economic and CO2 emissions impacts

# Conclusions

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Hydrogen blending enables:

Fuel Reliability  
Energy Equity  
Decarbonization

## FREEDOM GAS

By: Optimized Management for Grid Automation & Security



# Thank You!

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[azlotnik@lanl.gov](mailto:azlotnik@lanl.gov)