Computation-Efficient Optimization Algorithms for Autonomous Energy Systems

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Aug. 19, 2020
Background & Motivation
Background & Motivation

Cumulative photovoltaic capacity [MW]

Solar power

1-month voltage profile
Source: University of Hawaii
• Fast and optimal distribution systems voltage regulation
• Larger Systems: Increasing computational complexity

– Distributed and parallel computation
– Autonomous grid structure
  • No performance loss
  • Regional control and computation
  • Collaborating while preserving detailed information
Overall Goals

- **Large** distribution systems with deep renewable energy penetration
- **Fast** OPF solving
- **Optimal** solution without compromising performance (compared with centralized algorithms)
System Model & Design Intuition
Distribution System Modeling

- Radial Distribution Network
- Dist-Flow Model \([\text{Baran 1989}]
\)
\[
P_{ij} = -p_j + \sum_{k:(j,k) \in \mathcal{E}} P_{jk} + r_{ij} l_{ij},
\]
\[
Q_{ij} = -q_j + \sum_{k:(j,k) \in \mathcal{E}} Q_{jk} + x_{ij} l_{ij},
\]
\[
v_j = v_i - 2 \left( r_{ij} P_{ij} + x_{ij} Q_{ij} \right) + \frac{r_{ij}^2 + x_{ij}^2}{l_{ij}},
\]
\[\ell_{ij} v_i P_{ij}^2 + Q_{ij}^2.\]

- Lin-Dist-Flow Model \([\text{Baran 1989, Farivar 2013}]
\)
\[
v = R p + X q + \tilde{v}
\]

Resistance/Reactance of common path
\[
R_{ij} := \sum_{(\zeta, \xi) \in \mathcal{E}_i \cap \mathcal{E}_j} 2 \cdot r_{\zeta \xi}
\]
\[
X_{ij} := \sum_{(\zeta, \xi) \in \mathcal{E}_i \cap \mathcal{E}_j} 2 \cdot x_{\zeta \xi}
\]
Design Intuition

- General distribution network and sensitivity matrix
- Properties behind subtree-based network structure
- Go deeper
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Design Intuition

• General distribution network and sensitivity matrix
• Properties behind subtree-based network structure
• Go deeper
Example: Any node $i$ in subtree Area 2 and any node $j$ in subtree Area 3 share the identical common path leading back to the substation, i.e., lines 1-2-4. Therefore, $R_{ij} = R_{6,21} = r_{12} + r_{24}$, $X_{ij} = x_{12} + x_{24}$ for any $i$ within Area 2 and any $j$ within Area 3.
Solving Large OPF
Problem Formulation

• OPF Problem

$$\min_{p,q} \sum_{i \in \mathcal{N}} C_i(p_i, q_i) + C_0(P_0(p)),$$

s.t. $$\underline{v} \leq v(p, q) \leq \bar{v},$$

$$(p_i, q_i) \in \mathcal{V}_i, \forall i \in \mathcal{N}.$$

$$C_i(p_i, q_i) = (p_i - \hat{p}_i)^2 + (q_i - \hat{q}_i)^2$$

$$C_0(P_0(p)) = \alpha(P_0(p) - \hat{P}_0)^2$$

$$P_0(p) = -P_I - \sum_{i \in \mathcal{N}} p_i$$

$$v(p, q) = Rp + Xq + \tilde{v}$$

• (Regularized) Lagrangian

$$\mathcal{L}_\eta(p, q; \bar{\mu}, \mu) = \sum_{i \in \mathcal{N}} C_i(p_i, q_i) + C_0(P_0(p))$$

$$+ \mu^T(v - v(p, q)) + \bar{\mu}^T(v(p, q) - \bar{v}) - \frac{\eta}{2} \|\mu\|_2^2$$
Gradient Algorithm

• Primal-Dual Gradient Algorithm

\[ p(t + 1) = \left[ p(t) - \epsilon \left( \nabla_p C(p(t), q(t)) - C'_0(P_0(p(t))) \cdot 1_N \\
+ R^T (\overline{\mu}(t) - \underline{\mu}(t)) \right) \right] \gamma, \]

\[ q(t + 1) = \left[ q(t) - \epsilon \left( \nabla_q C(p(t), q(t)) \\
+ X^T (\overline{\mu}(t) - \underline{\mu}(t)) \right) \right] \gamma, \]

\[ \mu(t + 1) = \left[ \mu(t) + \epsilon (v - v(t) - \eta \mu(t)) \right]_+, \]

\[ \overline{\mu}(t + 1) = \left[ \overline{\mu}(t) + \epsilon (v(t) - \overline{v} - \eta \overline{\mu}(t)) \right]_+, \]

\[ v(t + 1) = Rp(t + 1) + Xq(t + 1) + \tilde{v}. \]

• Design Motivation

  – Centrally coordinated algorithm: increasing computation in the coupling term \((O(N^2))\)
  
  – Hierarchical structure
**Primal-Dual Gradient Algorithm**

\[
p(t + 1) = \left[ p(t) - \epsilon \left( \nabla_p C(p(t), q(t)) - C'_0(P_0(p(t))) \right) \cdot 1_N + R^T (\mu(t) - \mu(t)) \right]_y, \\
qu(t + 1) = \left[ q(t) - \epsilon \left( \nabla_q C(p(t), q(t)) + X^T (\mu(t) - \mu(t)) \right) \right]_y, \\
\mu(t + 1) = \left[ \mu(t) + \epsilon (v - v(t) - \eta \mu(t)) \right]_+, \\
\bar{\mu}(t + 1) = \left[ \bar{\mu}(t) + \epsilon (v(t) - \bar{v} - \eta \bar{\mu}(t)) \right]_+, \\
v(t + 1) = Rp(t + 1) + Xq(t + 1) + \tilde{v}.
\]

**Design Motivation**

- Centrally coordinated algorithm: increasing computation in the coupling term (\(O(N^2)\))
- Hierarchical structure
Hierarchical Implementation

• Subtrees & useful properties
• Individual node update

\[
p_i(t + 1) = \left[ p_i(t) - \epsilon \left( \partial_{p_i} C_i(p_i(t), q_i(t)) - C'_0(P_0(p(t))) \right) + \sum_{j \in \mathcal{N}} R_{ij} (\bar{\mu}_j(t) - \underline{\mu}_j(t)) \right]_{\mathcal{Y}_i}
\]
Hierarchical Implementation

- **Subtrees & useful properties**
- **Individual node update**

\[
p_i(t + 1) = \left[ p_i(t) - \epsilon \left( \partial_{p_i} C_i(p_i(t), q_i(t)) - C'_0(P_0(p(t))) \right) + \sum_{j \in \mathcal{N}} R_{ij}(\overline{\mu}_j(t) - \mu_j(t)) \right] \gamma_i
\]

For clustered node \( i \in \mathcal{N}_k \)

\[
\sum_{j \in \mathcal{N}} R_{ij}(\overline{\mu}_j - \mu_j) = \sum_{j \in \mathcal{N}_k} R_{ij}(\overline{\mu}_j - \mu_j) + \sum_{j \in \mathcal{N} \setminus \mathcal{N}_k} R_{ij}(\overline{\mu}_j - \mu_j)
\]

= \sum_{j \in \mathcal{N}_k} R_{ij}(\overline{\mu}_j - \mu_j) \quad \text{within area} \ k

\sum_{h \in \mathcal{K}, h \neq k} \sum_{j \in \mathcal{N}_h} (\overline{\mu}_j - \mu_j)

+ \sum_{j \in \mathcal{N}_0} R_{n_{k,j}}(\overline{\mu}_j - \mu_j) \quad \text{unclustered nodes}

\[
= \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}_k} (\overline{\mu}_j - \mu_j) + \sum_{j \in \mathcal{N}_0} R_{ij}(\overline{\mu}_j - \mu_j)
\]

Reduce/Recycle computation!
Hierarchical Implementation

- Subtrees & useful properties
- Individual node update

\[
p_i(t + 1) = \left[ p_i(t) - \epsilon \left( \partial_{p_i} C_i(p_i(t), q_i(t)) - C'_0(P_0(p(t))) \right) + \sum_{j \in \mathcal{N}} R_{ij} (\bar{\mu}_j(t) - \mu_j(t)) \right] y_i
\]

For unclustered node \( i \in \mathcal{N}_0 \)

\[
\sum_{j \in \mathcal{N}} R_{ij} (\bar{\mu}_j - \mu_j)
\]

\[
= \sum_{j \in \mathcal{N}_k} R_{ij} (\bar{\mu}_j - \mu_j) + \sum_{j \in \mathcal{N} \setminus \mathcal{N}_k} R_{ij} (\bar{\mu}_j - \mu_j)
\]

\[
= \sum_{j \in \mathcal{N}_k} R_{ij} (\bar{\mu}_j - \mu_j) + \sum_{h \in \mathcal{K}, h \neq k} R_{n_h^n_k} \sum_{j \in \mathcal{N}_h} (\bar{\mu}_j - \mu_j)
\]

\[
+ \sum_{j \in \mathcal{N}_0} R_{n_k^0ij} (\bar{\mu}_j - \mu_j)
\]

within all areas

unclustered nodes
Hierarchical Implementation

- Mathematically equivalent to the classic (distributed) gradient algorithm
- New implementation adapted for networked AGs structure
  - Design by exploring network and linearization structure
  - Parallel computation of coupling terms (computation bottleneck!)
    - Reducing and recycling computational load
    - Node-wise information preserved within AGs
Hierarchical Implementation

• Mathematically equivalent to the classic (distributed) gradient algorithm
• New implementation adapted for networked AGs structure
  – Design by exploring network and linearization structure
  – Parallel computation of coupling terms (computation bottleneck!)
    – Reducing and recycling computational load
    – Node-wise information preserved within AGs

AG-based hierarchical distributed implementation
Hierarchical Implementation

• Equivalent to centrally coordinated implementation
• Computational complexity reduction
  – Ideal (probably unrealistic) situation: $O(N^2) \rightarrow O(N^{4/3})$
  – Optimal/More clustering?
• Privacy preservation
  – Node-wise information preserved
  – Topology information within areas preserved
Multi-Phase System Extension

- **Linearized multi-phase dist-flow** [Gan 2016]

\[
\mathbf{v}_\Xi = R_\Xi \mathbf{p}_\Xi + X_\Xi \mathbf{q}_\Xi + \tilde{\mathbf{v}}_\Xi
\]

\[
\mathbf{v}_\Xi = \begin{bmatrix} [v_1^\phi ]_{\phi \in \Phi_1}^T, \ldots, [v_N^\phi ]_{\phi \in \Phi_N}^T \end{bmatrix}^T \in \mathbb{R}^{N_\Xi}
\]

\[
\mathbf{p}_\Xi = \begin{bmatrix} [p_1^\phi ]_{\phi \in \Phi_1}^T, \ldots, [p_N^\phi ]_{\phi \in \Phi_N}^T \end{bmatrix}^T \in \mathbb{R}^{N_\Xi}
\]

\[
\mathbf{q}_\Xi = \begin{bmatrix} [q_1^\phi ]_{\phi \in \Phi_1}^T, \ldots, [q_N^\phi ]_{\phi \in \Phi_N}^T \end{bmatrix}^T \in \mathbb{R}^{N_\Xi}
\]

\[
\frac{\partial}{\partial p_i^\phi} v_j^\phi = 2 \Re \{ Z_{ji}^\phi \cdot e^{-j2\pi(\varphi-\phi)/3} \}
\]

\[
\frac{\partial}{\partial q_i^\phi} v_j^\phi = -2 \Im \{ Z_{ji}^\phi \cdot e^{-j2\pi(\varphi-\phi)/3} \}
\]

\[
Z_{ji}^\phi = \sum_{(\zeta,\xi) \in \mathcal{E}_j \cap \mathcal{E}_i} z_{\zeta,\xi}^\phi : \text{impedance of common path!}
\]

- **Multi-Phase OPF Problem**

\[
\min_{\mathbf{p}_\Xi, \mathbf{q}_\Xi} \sum_{i \in \mathcal{N}} \sum_{\phi \in \Phi_i} C_i^\phi (p_i^\phi, q_i^\phi) + C_0(P_0(\mathbf{p}_\Xi)),
\]

s.t. \[
v_i^\phi \leq v_i^\phi(\mathbf{p}_\Xi, \mathbf{q}_\Xi) \leq \bar{v}_i^\phi, \phi \in \Phi_i, \forall i \in \mathcal{N},
\]

\[
(p_i^\phi, q_i^\phi) \in \mathcal{Y}_i^\phi, \phi \in \Phi_i, \forall i \in \mathcal{N}.
\]

\[
P_0 = - \sum_{\phi \in \Phi_0} \sum_{i \in \mathcal{N}} \sum_{\phi \in \Phi_i} p_i^\phi
\]
Multi-Phase System Extension

- (Regularized) Lagrangian

\[
\mathcal{L}^\phi_\eta(p_\Xi, q_\Xi; \mu_\Xi, \overline{\mu}_\Xi) = \sum_{i \in \mathcal{N}} \sum_{\phi \in \Phi_i} C_i^\phi(p_i^\phi, q_i^\phi) + C_0(P_0(p_\Xi)) + \mu_\Xi^\top (v_\Xi - v_\Xi(p_\Xi, q_\Xi)) + \overline{\mu}_\Xi^\top (v_\Xi(p_\Xi, q_\Xi) - \overline{v}_\Xi) - \frac{\eta}{2} \| \mu_\Xi \|^2.
\]

- Primal-dual gradient algorithm

\[
p_i^\phi(t + 1) = \left[ p_i^\phi(t) - \epsilon \left( \partial_{p_i^\phi} C_i^\phi(p_i^\phi(t), q_i^\phi(t)) - C'_0(P_0(p_\Xi(t))) + \sum_{j \in \mathcal{N}} \sum_{\phi \in \Phi_j} \partial_{p_i^\phi} v_j^\phi(\mu_j^\phi(t) - \mu_j^\phi(t)) \right) \right] \gamma_i^\phi,
\]

\[
q_i^\phi(t + 1) = \left[ q_i^\phi(t) - \epsilon \left( \partial_{q_i^\phi} C_i^\phi(p_i^\phi(t), q_i^\phi(t)) + \sum_{j \in \mathcal{N}} \sum_{\phi \in \Phi_j} \partial_{q_i^\phi} v_j^\phi(\mu_j^\phi(t) - \mu_j^\phi(t)) \right) \right] \gamma_i^\phi,
\]

\[
\mu_i^\phi(t + 1) = \left[ \mu_i^\phi(t) + \epsilon (v_i^\phi - v_i^\phi(t) - \eta \mu_i^\phi(t)) \right]_+,
\]

\[
\overline{\mu}_i^\phi(t + 1) = \left[ \overline{\mu}_i^\phi(t) + \epsilon (v_i^\phi(t) - \overline{v}_i^\phi - \eta \overline{\mu}_i^\phi(t)) \right]_+,
\]

\[
v_\Xi(t + 1) = R_\Xi p_\Xi(t + 1) + X_\Xi q_\Xi(t + 1) + \tilde{v}_\Xi.
\]
Multi-Phase System Extension

- Decompose of the coupling terms

For clustered node $i \in N_k$

$$2 \sum_{j \in N} \sum_{\varphi \in \Phi_j} \Re \{ \overline{Z}_{ji}^{\varphi} e^{-j2\pi(\varphi-\frac{\varphi}{3})} \} \cdot (\overline{\mu}_i^\varphi (t) - \mu_i^\varphi (t))$$

$$= 2 \Re \left\{ \sum_{\varphi \in \Phi_0} e^{-j2\pi(\varphi-\frac{\varphi}{3})} \left( \sum_{j \in \varphi \cap N_k} \overline{Z}_{ji}^{\varphi} (\overline{\mu}_i^\varphi (t) - \mu_i^\varphi (t)) \right) \right\}$$

$$+ \sum_{n_h \in N^\varphi} \sum_{j \in \varphi \cap N_h} (\overline{\mu}_i^\varphi (t) - \mu_i^\varphi (t))$$

$$+ \sum_{j \in \varphi \cap N_0} (\overline{\mu}_i^\varphi (t) - \mu_i^\varphi (t))$$

$$= 2 \Re \left\{ \sum_{\varphi \in \Phi_0} e^{-j2\pi(\varphi-\frac{\varphi}{3})} \left( \sum_{n_h \in N^\varphi} \sum_{j \in \varphi \cap N_h} \overline{Z}_{ji}^{\varphi} (\overline{\mu}_i^\varphi (t) - \mu_i^\varphi (t)) \right) \right\}$$

within area $k$

within other areas

unclustered nodes
Multi-Phase System Extension

• Decompose of the coupling terms

For unclustered node $i \in \mathcal{N}_0$

\[
2 \sum_{j \in \mathcal{N}} \sum_{\varphi \in \Phi_j} \Re \{ Z_{ji}^{\varphi} e^{-j2\pi(\varphi-\phi)/3} \} \cdot (\bar{\mu}_j^\varphi(t) - \mu_j^\varphi(t)) = 2\Re \left\{ \sum_{\varphi \in \Phi_0} e^{-j2\pi(\varphi-\phi)/3} \left( \sum_{j \in \mathcal{N}^\varphi \cap \mathcal{N}_k} Z_{ji}^{\varphi} (\bar{\mu}_j^\varphi(t) - \mu_j^\varphi(t)) + \sum_{n_k^0 \in \mathcal{N}^\varphi, j \in \mathcal{N}^\varphi \cap \mathcal{N}_{h, h \neq k}} Z_{n_k^0 n_k}^{\varphi} \sum_{j \in \mathcal{N}^\varphi \cap \mathcal{N}_h} (\bar{\mu}_j^\varphi(t) - \mu_j^\varphi(t)) \right) \right. \\
\left. + \sum_{j \in \mathcal{N}^\varphi \cap \mathcal{N}_0} Z_{ji}^{\varphi} (\bar{\mu}_j^\varphi(t) - \mu_j^\varphi(t)) \right\}
\]

within all areas

unclustered nodes
Numerical Setup

- Synthetic 11,000-node test feeder
  - IEEE 8,500
  - EPRI CKT7
- Primary side voltage control
  - 4,521-node network
  - 1,043 controllable loads
- Clustered into 4 areas consisting of 154—357 controllable loads
- Three-phase hierarchical distributed algorithm
OPF Numerical Results

- **2-level**: 4-fold speed improvement
  - “Free” speed improvement
  - Parallel implementation: 2.5 times more
- **3-level**: 31.3% further improvement
OPF Numerical Results

- Voltage regulation

Controlled Voltage

Voltage magnitude, p.u.

Node index on primary side

Voltage w/o control
Voltage w/ default control
Voltage w/ OPF control
Distributed Version of the Algorithm
Objective

- AES cellular control structure
  - Multi-layer hierarchy
  - Distributed coordination between cells in the same layer
    - Reduced reliance on central controller (flexibility, robustness, privacy)
- By merging
  - Complexity reduction method
  - Distributed feedback-based algorithm (review later)

Low complexity feedback-based algorithm that is flexible to various communication scenarios
Review of Distributed Feedback-Based Algorithm

• Recall the OPF Problem that we attempt to solve

\[
\min_{p,q} \sum_{i \in \mathcal{N}} C_i(p_i, q_i),
\]

s.t. \( v = Rp + Xq + \tilde{v} \),

\( v \leq v \leq \bar{v} \),

\( (p_i, q_i) \in \mathcal{Y}_i, \quad \forall i \in \mathcal{N} \)

• Primal-dual method \( x_i = [p_i, q_i]^T \)

\[
\dot{x}_i = \Pi_{\mathcal{Y}_i} \left( - \left[ \nabla_{p_i} C_i + \nabla_{q_i} C_i + R_i^T (\bar{\mu} - \mu) \right] \right), \forall i \in \mathcal{N}
\]

\[
\dot{\mu} = \Pi_\mathbb{R}_+^{\mathbb{R}_+} \left( v(x) - \bar{v} \right)
\]

\[
\dot{\underline{\mu}} = \Pi_\mathbb{R}_+^{\mathbb{R}_+} \left( \bar{v} - v(x) \right)
\]

Need all-to-all communication because \( R \) and \( X \) are non-sparse

A distributed algorithmic framework applicable to systems with underlying non-sparse network graph
Abstraction of Power Networks

- For online OPF
  - **Sensor**: voltage magnitude
  - **Actuator**: active/reactive power

- Each sensor communicates with at least one actuator

- Actuators’ network is connected (no need to match physical network)
• Main idea
  ➢ Each actuator has an estimate of all the dual variables ($\bar{\mu}$ and $\bar{\mu}$)
    ▪ Focus on $\bar{\mu}$ as similar logics apply to $\bar{\mu}$

$\bar{\mu} = \begin{bmatrix} \bar{\mu}_1 & \cdots & \bar{\mu}_M \end{bmatrix} \in \mathbb{R}^{N \times M}$

Copy of $\bar{\mu}$ at node 1 (or $\bar{\lambda}^{(1)}$)

All the copies of $\bar{\mu}_2$ (or $\bar{\lambda}_2$)

➢ The nodes use distributed communication reaches consensus for $\bar{\lambda}_i$ for all $i$ (at an optimum of $\bar{\mu}_i$, $\bar{\lambda}_i^*$)
Review of Distributed Feedback-Based Algorithm

• Proposed updating rule:

\[ \dot{x}_i = \mathcal{M}_i(x_i, \bar{\lambda}^{(i)}) \]

\[ \dot{\bar{\lambda}}_k = b_k \prod_{\mathcal{T} \in \mathcal{N}} \left( v_k - \bar{v}_k \right) - L \bar{\lambda}_k \quad \forall k = 1, \cdots, M. \]

Primal update only needs local variables

Distributed communication for consensus

Using voltage measurement to find a dual optimum \( \bar{\mu}_i^* \)

• Distributed communication is only used for the consensus
• Provably convergence
• Event-triggered version available
• Plug-and-play capability
• **Drawback:** requires \( NM \) number of variables (originally \( M \))
• Recall the complexity reduction trick
Complexity Reduction on the Distributed Algorithm

• All the actuators in each area reach consensus of
  – Dual variables in the same area
  – Sum of the dual variables in each of other areas

• New partition of $\bar{\mu}$:

$$\bar{\mu} = \begin{bmatrix} \bar{\mu}_{1,1} & \cdots & \bar{\mu}_{1,M_1} \\ \bar{\mu}_{2,1} & \cdots & \bar{\mu}_{2,M_2} \\ \vdots & \ddots & \vdots \\ \bar{\mu}_{K,1} & \cdots & \bar{\mu}_{K,M_K} \end{bmatrix}$$

\begin{align*}
\text{Area 1} & \quad \text{Area 2} & \quad \cdots & \quad \text{Area } K \\
\begin{bmatrix}
\lambda_{a,1}^{(1)} & \cdots & \lambda_{a,1}^{(M_a)} \\
\lambda_{a,2}^{(1)} & \cdots & \lambda_{a,2}^{(M_a+1)} \\
\vdots & \ddots & \vdots \\
\lambda_{a,N_a}^{(1)} & \cdots & \lambda_{a,N_a}^{(M_a+1)} \\
\end{bmatrix} & \quad \begin{bmatrix}
\lambda_{a,1}^{(1)} & \cdots & \lambda_{a,1}^{(M_a+1)} \\
\lambda_{a,2}^{(1)} & \cdots & \lambda_{a,2}^{(M_a+K-1)} \\
\vdots & \ddots & \vdots \\
\lambda_{a,N_a}^{(1)} & \cdots & \lambda_{a,N_a}^{(M_a+K-1)} \\
\end{bmatrix} & \quad \cdots \quad \begin{bmatrix}
\lambda_{a,1}^{(1)} & \cdots & \lambda_{a,1}^{(M_a+1)} \\
\lambda_{a,2}^{(1)} & \cdots & \lambda_{a,2}^{(M_a+K-1)} \\
\vdots & \ddots & \vdots \\
\lambda_{a,N_a}^{(1)} & \cdots & \lambda_{a,N_a}^{(M_a+K-1)} \\
\end{bmatrix}
\end{align*}

$\in \mathbb{R}^{N_a \times (M_a+K-1)}$

Each of them surrogates the sum of the dual variables in another area
Complexity Reduction on the Distributed Algorithm

- **Similar updating rule:**

\[
\dot{x}_{a,i} = M_i(x_{a,i}, \bar{\lambda}_a^{(i)}), \\
\dot{\bar{\lambda}}_{a,k} = b_{a,k} \Pi_{\mathbb{R}^N_a} (v_{a,k} - \bar{v}_{a,k}) - L_a \bar{\lambda}_{a,k}, \quad \forall k = 1, \ldots, M_a \\
\dot{\bar{\lambda}}_{a,k} = \sum_{j \in \mathcal{N}_h} b_{h,j}^{(k)} \Pi_{\mathbb{R}^N_a} (v_{h,j} - \bar{v}_{h,j}) - L_a \bar{\lambda}_{a,k}, \quad \forall k = M_a + 1, \ldots, M_a + K - 1.
\]

- **Soft constraints on the distributed communication**
  - Each sensor communicates with at least one actuator in every other area
  - Every row (and column) of the off-diagonal block \( L_{ik} \) has at least one non-zero entry
  - Off-diagonal blocks \( L_{ik} \) defines \( b_{h,j}^{(k)} \)
Complexity Reduction on the Distributed Algorithm

- Equivalent to the original algorithm
- Number of copies is greatly reduced:
  - From $NM$ to $\sum_{a \in \mathcal{K}} N_a (M_a + K - 1)$

- The reduced amount of the multiplications are translated to less amount of variables in the distributed approach

- Proof of convergence
  - Mild assumptions
    - The objective function is continuously differentiable and strongly convex
    - The constraint set is nonempty, compact, and convex
    - The problem is feasible and the Slater condition is satisfied
  - LaSalle’s invariance principle to show convergence to invariant set where the copies of the dual variables are at the optimal point
Hierarchical Distributed Control Algorithm

• Assume each area has a regional coordinator (RC)
  – Distributed communication between RC
  – Number of variables further reduced
    from \( \sum_{a \in K} N_a (M_a + K - 1) \) to \( K^2 + M \)
  – Similar proof of convergence

• Flexibility to adjust the algorithm to various operation scenarios

• Future works:
  – Check how the feedback-based algorithm complement with other controls (e.g. black start, LTC)
Q&A


