Circle Diagram Approach for Self Excited Induction Generators

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Abstract-When an induction generator is connected to a utility line supply, the voltage and frequency at the terminal output are the same as the voltage and frequency of the utility line supply to which the generator is connected. The reactive power needed by the induction generator is supplied by the utility and the real power is returned to the utility. With a fixed frequency dictated by the utility, the induction machine starts generating above the synchronous speed. The range of speed is also limited by the slip. At a very high slip, the copper losses increases as the current increases.

On the other hand, in an isolated operation, the induction generator operates in self-excitation mode. It determines its own voltage and frequency. These two quantities depend on the size of the AC capacitor, the induction machine parameters, the electrical load, and the speed of the generator. The operating speed of the induction generator is extended without generating excessive loss. This paper presents an analytical study by utilizing a circle diagram to illustrate the operation of the induction generator in isolated operation. The steady-state calculations are presented to support the analysis. Possible applications for the system in variable-speed generation are currently under investigation. The output can be directly connected to equipment that is non-sensitive to the frequency (a heater, battery charger, etc.) or can be connected to a converter to get a fixed-frequency AC output.

I. INTRODUCTION

An induction machine can be operated as an isolated generator with no connection to the utility supply. When connected to the utility, the reactive power needed by the induction generator is supplied by the utility. For an isolated operation, with the reactive power needed by the induction generator must be compensated by a local source such as a three-phase AC capacitor or solid-state excitation [1-3]. While the solid-state excitation provides a variable size of reactive power [2], the application of solid-state excitation is generally accompanied by harmonics generated by the converter and additional switching losses. The physical diagram of induction generator in an isolated operation is presented in Fig. 1. There are two possible connections to couple the load to the terminal of the induction generator: the series compensation [4] and the parallel compensation. Delta connection can also be implemented; however, for the simplicity of the analysis; a wye connected system is considered. The equivalent circuit in Fig. 2a can be generalized to account for any frequency operation, as shown in Fig. 2b.

In an isolated operation, the conservation of real and reactive power must be preserved [5,6]. The equation governing the system can be simplified by looking at the impedance or admittance of the induction machine. To operate in an isolated operation, the total admittance of the induction machine must be zero. The voltage of the system is determined by the flux and frequency of system. Thus it is easier to start the analysis from a node at one end of the magnetizing branch. Note that the term IMPEDANCE in this paper is the conventional impedance divided by the frequency. The term ADMITTANCE in this paper corresponds to the actual admittance multiplied by the frequency. Thus, the unit of the IMPEDANCE in this study is given as ohms/(rad/sec) and the unit of the ADMITTANCE is given as mho*rad/sec.

II. SERIES COMPENSATED SYSTEM

First, a series connected system is considered. The total admittance of the system from point M can be written as follows:

\[ Y_s + Y_m' + Y_r' = 0 \]  \hspace{1cm} (1)

where:
\[ Y_r' = \text{admittance of the rotor branch} \]
\[ Y_m' = \text{admittance of the magnetizing branch} \]
\[ Y_s = \text{admittance of the load and stator branch} \]
The equilibrium of the real power is maintained by balancing the real power produced by the rotor branch and the real power absorbed by the stator and load branch. Similarly, the equilibrium of the reactive power is maintained by balancing the reactive power absorbed by the rotor branch, magnetizing branch, and the stator branch and the reactive power produced by the capacitor $C$. Equation 1 can be separated into two equations governing each reactive power and real power.

Real part of $(Y_s + Y_{m'} + Y_{r'}) = 0$ ... (2)

and

Imaginary part of $(Y_s + Y_{m'} + Y_{r'}) = 0$ ... (3)

The above equations can be written in final version as follows:

\[
\frac{R_x}{\omega} + \frac{(L_{1s} - \frac{1}{\omega^2C})}{\omega^2} = 0 \quad (4)
\]

\[
\frac{1}{L_{m'}} + \frac{1}{\omega^2C} \frac{L_{1s}}{\omega} - \frac{L_{1r}^2}{\omega^2} = 0 \quad (5)
\]

From the two equations presented above, it is obvious that for any fixed load ($R_L$ and $C$) and fixed frequency, the two variables governing the equilibrium will be the slip and the saturation level at the magnetizing branch. The saturation level affects the operating point of the system. The size of magnetizing inductance varies at different flux levels of the induction machine.

The operating point of the system can be found by solving the two equations above. For a given set of parameters ($R$, $L$ and $C$) and frequency, the slip can be computed by solving equation 4 above.

Using the slip found from equation 6 and the data given, the corresponding magnetizing inductance can be found. Thus, the operating flux corresponding to this flux level can be found from the non linear characteristics $L_{m'}$ versus the flux given in appendix 2. From the given information on frequency and the air-gap flux, the air-gap voltage $V_g$ can be found. After solving for $L_{m'}$ and $V_g$ from the slip computed above, the rest of the operating conditions can be solved. The equation to solve for $L_{m'}$ can be derived from equation 5:

\[
L_{m'} = \left( \frac{1}{\omega^2C} - \frac{L_{1s}}{\omega^2} - \frac{L_{1r}^2}{\omega^2} \right)^{-1} \quad (6)
\]

Another way to solve for the operating point of the system is by solving the frequency for a given slip and the parameter of the induction machine. The equation to solve for the frequency is

\[
A_2 \omega^4 + A_1 \omega^2 + A_0 = 0 \quad (8)
\]

\[
A_2 = R_x S L_{1s}^2 C^2 + R_x C^2 L_{1r}^2 S^2
\]

\[
A_1 = R_x C^2 R_{s}^2 + R_{s}^2 S R_x C^2
\]

\[
A_0 = R_{s}^2 S
\]

The solution for $L_{m'}$ can be found from equation 7 above. Thus, for any set of induction generator parameters, there is a corresponding operating point ($S$, $\omega$, $L_{m'}$ and $V_g$).
The impedance can be expressed in terms of its admittance as

\[ Z = \frac{1}{Y} \]

\[ = \frac{-G_1}{G_1^2 + B_1^2} + j \frac{B_1}{G_1^2 + B_1^2} \]

The real part of the impedance can be rearranged as follows:

\[ (G_1 - \frac{1}{2R_1})^2 + B_1^2 = \frac{1}{(2R_1)^2} \]

Thus, the equation describing the stator-load branch can be expressed as an equation of a circle with a radius of \(1/(2R_1)\), where \(R_1\) is the real part of the impedance of the stator-load branch. Assuming the operating condition is fixed at constant frequency, the size of the radius is constant for a constant resistive load. As the size of the imaginary part of the impedance \((X_1)\) is varied, the operating point travels along the perimeter of the circle. The size of \(X_1\) can be changed by varying the size of the capacitor \(C\). The stator load admittance is represented by a phasor starting from the origin to the point on the perimeter of the circle. The size of the current is proportional to the size of the stator admittance multiplied by the flux level. As the capacitor size is varied, the point corresponding to the new condition moves along the perimeter of the semi-circle.

**B. Conservation of Real and Reactive Power**

The conservation of real and reactive power is always preserved by following the same rule; that is, the total admittance is equal to zero as given by equation 1. The phasors of the current in the rotor, magnetizing, and stator branches are proportional to the size and direction of the admittance phasor. Thus, it can be observed both from the total admittance diagram and the equations given that the balance of real power will be maintained by the real part of \(Y_s\) and \(Y_r\) and the balance of reactive power will be maintained by the imaginary part of \(Y_m\). The flux level affects the size of \(Y_m\). Therefore, in a balanced system, the sum of the rotor, magnetizing, and stator-load admittance must be zero.

![Admittance diagram for the system at a single frequency](image_url)

The size of the current is affected by the saturation level of the air-gap flux as well as the size of the admittance in the individual branches. The total admittance diagram is given in Fig. 4. Important equations for the admittances are given as follows:

\[ Y_s = \frac{R_s}{R_1^2 + X_1^2} + j \frac{-X_1}{R_1^2 + X_1^2} \]

where \(X_1 = L_{1e} - \frac{1}{\omega^2 C}\)

\[ Y_r' = -j \frac{1}{L_{1e}} \]

\[ Y_{r'} = \frac{R_s^r}{\omega s} \frac{1}{(\frac{R_s^r}{\omega s})^2 + (L_{1e}^r)^2} + \frac{R_s^r}{\omega s} \frac{-L_{1e}^r}{(\frac{R_s^r}{\omega s})^2 + (L_{1e}^r)^2} \]
C. Effect of Varying the Capacitor

The effect of varying the capacitor will appear as an operating point moving along the semicircle describing the stator-load admittance. As the size of the capacitor increases, the imaginary part of the stator-load admittance increases until the maximum condition is achieved when the imaginary part of the stator-load is equal to the size of the radius of the stator-load semicircle (i.e. equals to $1/(R_L + R_s)$).

The increment of the size of susceptance is counteracted by the increment of the size of the magnetizing branch admittance, which corresponds to the level of air-gap flux saturation. The level of saturation translates to the size of the air-gap voltage, which in turn will affect the torque size and output power.

D. Effect of Varying the Resistive Load

The admittance diagram of the stator and the load presented in Figs. 3 and 4 are based on the assumption that the resistive load is constant. Thus, the size of the radius $1/2R_1 (R_1 = R_L + R_s)$ is constant. As the size of the capacitor is varied, the operating point travels along the perimeter of the semicircle. On the other hand if the size of the capacitor is constant while the size of the resistive load is varied the admittance diagram for the stator and load can be modified.

Consider the stator-load impedance in equation 10. The imaginary part of the stator impedance can be represented in terms of its conductance $G_L$ and susceptance $B_L$:

$$X_s = \frac{B_L}{G_L^2 + B_L^2}$$

The above equation can be simplified into a preferable form as follows:

$$G_L^2 + \left(\frac{B_L}{2X_s}\right)^2 = \left(\frac{1}{2X_s}\right)^2$$

$$X_s = \frac{1}{\omega^2 C}$$

Equation 18 is an equation of a circle that has a center at 0, 1/(2X_s) and a radius of 1/(2X_s). Thus, as can be seen from the admittance diagram, at constant frequency and a constant capacitor value the radius of the semicircle is constant. The higher the size of the capacitor, the larger the radius becomes. The operating point of the system travels along the perimeter of the semicircle as the size of the load is varied (refer to Fig. 5). As can be seen from the diagram, these operating points affect the slip and level of the air-gap flux.

E. Steady State Calculation

From the equations derived above, the steady-state calculation is performed to compute the characteristics of the induction machine operated in the generating mode. The magnetizing inductance saturation is given in appendix 1, and the parameter of the induction machine is presented in the appendix 2. In the calculation, the value of the resistive load is kept constant for two values of capacitor, and the value of the capacitor is kept constant for two values of resistor. Fig. 6 shows the air-gap voltage versus speed. It is clear that, in general, the capacitor increment related to a higher voltage. This is similar to Fig. 5a, where as the size of the capacitor is increased at constant resistive load, the size of the imaginary part of the admittance increases, which correlates to the increase of the saturation level. Similarly, if the size of the capacitor is kept constant while decreasing the size of the resistive load $R_L$, the air-gap voltage increases. Fig. 5b shows, a decrease in resistance load will increase the air-gap voltage. Fig. 7 shows the stator current versus speed. This graph relates to Fig. 6, where the air-gap voltage and the size of the load impedance affect the size of the stator current. Fig. 8 shows the output power versus speed. This figure relates to the size of the stator current (Fig. 7) and the size of the resistive load. The computed torque speed characteristics shown in Fig. 9 can be approximated from Fig. 8, where the torque is approximately equal to output power divided by speed.
III. PARALLEL COMPENSATION

An equivalent circuit of a parallel compensated induction generator is shown below. The principle of excitation for the parallel compensated system is the same as in the series compensation, that is, the balance of real and reactive power must be maintained.

![Equivalent circuit of parallel compensation](image)

The total admittance of the system is given below:

\[
Y_s + Y_m' + Y_r' = 0 \quad (19)
\]

The above equation can be expanded into the equation for imaginary and real parts as shown in the following two equations.

\[
\frac{R_s}{\omega} + \frac{R_r}{S \omega} = 0 \quad (20)
\]

\[
\frac{1}{L_m} - \frac{L_s}{(R_s/\omega)^2 + (L_1)^2} - \frac{L_{1a}}{(R_{1a}/S \omega)^2 + (L_{1a})^2} = 0 \quad (21)
\]

where

\[
R_{1a} = R_s + \frac{R_{Ls}}{((\omega CR_L)^2 + 1)}
\]

\[
L_{1a} = \frac{CR_L^2}{((\omega CR_L)^2 + 1)}
\]

The real and imaginary parts of the admittance are given above. Thus for a given parameter set of the induction machine and the operating frequency, the slip can be solved from the real part of the admittance.
where

\[ a_2 = R_L L_{12}^{12} \omega^2 + R_s L_{12}^{12} \omega^2 + R_s \omega 4 R_L^2 C^2 L_{12}^{12} \]

\[ a_1 = R_s R_L^2 \omega^2 R_s C^2 - 2 R_s \omega^2 R_s^2 C^2 + 2 R_L R_s + L_{12}^1 \omega^2 + R_s R_L^1 + R_L^1 R_s \]

\[ a_0 = R_L R_s^{12} + R_s R_L^{12} + R_s \omega^2 R_L^2 C^2 R_L^{12} \]

After solving the above equation for the slip, the magnetizing inductance \( L_m' \) can be solved by using the equation 21. The flux level corresponding to the calculated \( L_m' \) can be found from the magnetization curve shown in appendix 2.

To illustrate parallel compensation in circle diagram approach, a simplification is to be made. For a parallel compensated system, the equivalent circuit can be simplified by assuming that the stator resistance and stator leakage inductance is negligible. The stator-load admittance depends solely on the load impedance. The stator admittance diagram becomes very simple. The variation in the resistive load and the capacitor size can be described as straight lines. The equation governing the stator-load admittance can be written in simplified form as:

\[ Y_s = \frac{\omega}{R_L} + j \omega^2 C \]  

(23)

At a constant frequency, for a constant resistive load \( R_L \), the operating point moves along the straight line parallel to the imaginary axis as the size of capacitor is varied. Similarly, for a constant size of capacitor, the operating point moves along the straight line as the size of the resistance load is varied. The rest of the equation (i.e. for the magnetizing branch and rotor branch) is the same as in series compensation.

A. Effect of Varying the Capacitor

Increasing the size of the capacitor moves the operating point on the stator-load admittance vertically, thus increasing the imaginary part of the stator load admittance. The increment is balanced by the increment of mostly the magnetizing branch admittance, which means increasing the saturation level. Thus, at any frequency, increasing the size of the capacitor will increase the air-gap flux (i.e. the air-gap voltage and the output voltage). This also means that more power will be dissipated by the resistive load and, consequently, the torque will increase.

B. Effect of Varying the Resistive Load

The effect of varying the resistive load will move the operating point horizontally. Thus increasing the load resistance, the size of the real part of the stator admittance will be reduced. Consequently, this will correspond to a lower slip and lower output power, which translates into a lower output torque. On the other hand, if the size of the resistive load is reduced, the size of the real part of the admittance will be larger, which corresponds to the higher output power and higher torque.

Theoretically, the lowest minimum resistance at any frequency will be limited by the radius of the semicircle describing the rotor admittance (i.e. it depends on the size of \( 1/L_m' \)).

C. Steady-State Calculation

The steady-state calculation was based on the complete equivalent circuit shown in Fig. 10a or Fig. 10b. The effect of varying the capacitor can be seen from the steady-state calculation as well. It is shown in Fig. 12 that the air-gap voltage increases as the capacitor size is increased. The trend is consistent with the circle diagrams representing the parallel compensation, where as the size of the capacitor increases at constant resistor load the locus of the imaginary part of the admittance moves vertically in the positive direction. To compensate for this move, the magnetizing admittance moves vertically in the negative direction \( (Y_m' \) increases), which corresponds to a higher flux level and higher air-gap voltage. By referring to both Fig. 10 and Fig. 12, the basic concept is clearly revealed.

On the other hand, for the same size of capacitor, if the resistive load is increased \( (R_L \) is smaller) the imaginary part of the stator admittance is practically not affected. Thus, the flux

\[ \frac{\text{Air-Gap Voltage-Speed}}{} \]

Fig. 12: Air-gap voltage for parallel compensated system
level is not directly affected by the change of the size of the resistive load.

In Fig. 13, the stator current $I_s$ is shown. The stator current flows in the stator winding. It is the sum of the current entering the resistive load and the current entering the capacitor. Since the current in the resistive load and the current in the capacitor are not in phase, the stator current $I_s$ is not the algebraic sum of the resistive and capacitive current. Instead, they are a vectorial summation. Thus, for the two different resistive loads at the same capacitor size, the difference between the two is not apparent. For the same load but two different capacitors, (20 ohms at two different capacitor) stator currents are separated apart because the air gap voltage for the larger capacitor is higher, which will affect the resistive and capacitive currents. In Fig. 14, the output power (electrical output) is shown. The output power for a parallel load can be found directly from the air-gap voltage shown in Fig. 12 and the size of the resistive load. The output power is equal to the voltage squared over the resistive load. Thus, the output power can be predicted from the air-gap voltage. Fig. 15 shows the torque versus speed, which can be derived from the power-versus-speed characteristic. The torque is approximately proportional to the power divided by the speed; thus the trends for different loads can be predicted from the power-speed characteristics.

For both series and parallel compensation, as the speed increases the frequency also increases and slip remains small. Thus, the stator current variation is also reasonably acceptable, allowing the induction generator to operate over a wide range of rotor speeds.

IV. CONCLUSION

Using of circle diagram approach helps clarify the concept of self excitation both for series and parallel compensation. From the circle diagram, it is easy to see how the operating condition of the induction generator changes as the parameters are changed.

From the discussions presented above, both the series compensation and parallel compensation can be implemented for isolated operation. With the correct choice of capacitor sizes, operation over a wider range of speed can be realized. As can be seen from the graphs, in isolated operation, induction generator has a wider speed range with a relatively low slip.

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REFERENCES

APPENDIX 1

Magnetizing Inductance of the Induction Machine

![Magnetizing Inductance Graph]

APPENDIX 2

<table>
<thead>
<tr>
<th>Baldor Squirrel Cage Induction Machine</th>
<th>Parameter Values (at 60 Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5 HP, 220 Volts 20 Amps</td>
<td>Rs = 0.199 ohms; Rr' = 0.121 ohms</td>
</tr>
<tr>
<td>175 rpm (motor)</td>
<td>Xls = Xlr' = 0.904 ohms</td>
</tr>
<tr>
<td>60 Hz, 3phase, 1.15SF</td>
<td>Xm' = refer to appendix 1</td>
</tr>
</tbody>
</table>