Prediction of Stochastic Blade Responses Using a Filtered Noise Turbulence Model in the FLAP Code

R. W. Thresher
W. E. Holley
A. D. Wright

November 1988

Prepared for the Eighth ASME Wind Energy Symposium
Houston, Texas
22-25 January 1989

Prepared under Task No. WE811202

Solar Energy Research Institute
A Division of Midwest Research Institute
1617 Cole Boulevard
Golden, Colorado 80401-3393

Prepared for the
U.S. Department of Energy
Contract No. DE-AC02-83CH10093
ABSTRACT

The wind turbine structural dynamics model, FLAP (Force and Loads Analysis Program), has been modified to include turbulent wind fluctuations based on a filtered-noise model. The importance of including the effects of turbulent wind fluctuations in the structural-loads predictive model has long been recognized. These stochastic loads are the dominant fatigue loads for many structural components in horizontal-axis wind turbines.

The turbulence field at the rotor plane is approximated by interpolating functions, which allow the velocity field to vary quadratically for velocity components normal to the rotor plane and linearly for in-plane velocity components. The velocity field represented in this fashion is constructed to vary randomly with time and space and give the proper correlation between spatial locations and velocity components. For the normal velocity components, the spectral representations of these velocity fluctuations approximate those observed from a rotating turbine blade up to a frequency of two times per rotor revolution (2P). For the less-important in-plane components, the spectral representations approximate those observed from a rotating turbine blade up to 1P.

The response spectra calculated from these time series were then compared with the experimental measurements obtained from the field test. Comparing the simulation results with actual test measurements generally shows good agreement.

It takes about an hour to run a 450-revolution FLAP simulation using an 80386-based personal computer (PC) running at 20 MHz. The required knowledge of the actual turbulence characteristics is modest. The mean wind speed and the turbulence intensity are easily computed from time-series wind data. As discussed here, the integral scale of the turbulence can be estimated from calculations of the longitudinal wind spectrum. This type of simulation should become part of the process of designing wind turbines. It produces time-series results that can be used to determine peak loads, and it can be rainflow-counted for estimating fatigue damage rates. In addition, the computational and data input requirements are within the means of even the smallest design team.

INTRODUCTION

Accurately predicting wind turbine blade loads and resulting stresses is important for predicting the fatigue life of components. There is a clear need within the wind industry for validated codes that can predict not only the deterministic loads from the mean wind velocity, wind shear, and gravity, but also the stochastic loads from turbulent inflow. The FLAP code has already been validated for predicting deterministic loads (1,2). This paper concentrates on validating the FLAP code for predicting stochastic turbulence loads using the filtered-noise turbulence model, developed in (3), as input.

THE FILTERED-NOISE TURBULENCE MODEL

Because the blades of a horizontal-axis wind turbine rotate through the wind velocity field, fluctuations in the velocity seen from a moving blade occur at frequencies that are multiples of the rotation rate (1P, 2P, etc.). This effect can be understood by considering a wind velocity field that varies over the rotor disk but does not vary with time. As a wind turbine blade moves through this field, it encounters dif-
fferent velocities. After one complete revolution, the cycle is repeated, leading to a periodic velocity fluctuation. In the actual case, the time varying velocity field is convected past the wind turbine by the mean flow so that the fluctuations are no longer exactly periodic. However, the rotation frequency of the rotor is usually much greater than those typical frequencies in the fluctuating velocity observed at a stationary point; consequently, the eddy structure in the turbulence will remain highly coherent for several rotor revolutions as it passes the wind turbine. Therefore, the wind velocity seen by the rotating blade will have strong fluctuations at a band of frequencies near the integer multiples of the rotation rate. The spectral density will locally appear as a wide band process with an amplitude sinusoidally modulated at multiples of the rotation frequency. This rotational effect was described in (4) and was modeled by Kristensen and Frandsen (5), whose work was based on the earlier work of Rosenbrock (6).

There are essentially two approaches to modeling the three-dimensional correlation structure of atmospheric turbulence. In the first, or "hydrodynamic" approach, the fluctuating flow field is assumed to be homogeneous, isotropic, and incompressible. The theory of such a field is described in (7); the correlation structure or its Fourier transform, the spectral density, is completely described by a single function, usually given in the frequency domain by the energy spectrum. von Karman (8) suggested a particular form for the energy spectrum that fits the inertial subrange described in (9) and approaches a constant spectral density at low frequencies. This theory is widely used in aircraft flight control and structural-dynamics applications (10).

In the second, more empirical approach, the spectral densities of the velocity components are given by an empirical form (such as in (11)), and the coherencies between velocities at different points are given by the Davenport model (12). This approach is often used in dynamic analyses involving wind excitations of large structures such as towers, smoke stacks, and bridges (13). It has also been used for wind turbine structures by Sundar and Sullivan (14) and by Veers (15). Dragt (16) also used the Davenport coherence model but introduced an expansion of the periodic, rotating coherence function in Fourier series. Madsen (17) also used the Fourier series approach but modified the form of the Davenport coherence function to eliminate the dips in the rotating spectra at multiples of the rotor passage frequency described by Dragt. These dips have not been observed experimentally, and they result from the inadequacy of the Davenport model for frequencies below the limit of the inertial subrange (as pointed out by Kristensen and Jensen (18)). These approaches have provided useful results but lead to certain difficulties. First, the empirical parameters for the coherence function must be determined for each site, requiring multiple correlations among spatially separated anemometers. Second, only the longitudinal velocity component (normal to the rotor disk) is modeled. Third, the incompressible flow condition is not satisfied. For these reasons, the hydrodynamic approach using the von Karman isotropic model was adopted as the basis for the turbulence model reported here.

Series Approximation for the Rotor Disk Turbulence Field

Consider a disk in the vicinity of the rotor of a horizontal-axis wind turbine. The coordinate system used is shown in Figure 1. The undisturbed wind velocity is assumed to vary a small amount over the region

![Fig. 1. Rotor disk coordinate'system](image)
small influence on blade loads, they may contribute significantly to the rotor yaw loads in some situations. The two in-plane velocity components are thus approximated by

\[ \nu_x = V_x(t) + \left[ \gamma(t) - \gamma_c(t) \right] \sin \psi + \left[ \gamma(t) - \gamma_c(t) \right] \cos \psi \]

\[ \nu_z = V_z(t) + \left[ \gamma(t) + \gamma_c(t) \right] \sin \psi + \left[ \gamma(t) + \gamma_c(t) \right] \cos \psi \]

(4)

The six terms appearing in Eq. (4) are given by equations similar to Eq. (3) and can be found in (20).

Once all twelve terms are specified, it is possible to compute the correlation relations among them using the von Karman correlation model. As an example, consider the autocorrelation function for the term \( V_y(t) \). Direct application of the first of Eqs. (3) gives

\[ \mathbb{E}[V_y(t+\tau)V_y(t)] = \frac{\alpha^2}{\pi^2} \int \int [f(\xi) + \frac{1}{2} f'(\xi)] \frac{2}{\xi} \, dA_1 \, dA_2 \]

where

\[ \xi^2 = \eta^2 + \psi^2 \]

\[ \eta^2 = r^2 + \rho^2 - 2r \rho \cos(\psi-\phi) \]

\[ \rho \phi = \text{polar coordinates in } D1 \text{ (Disk 1)} \]

\[ \rho \phi = \text{polar coordinates in } D2 \text{ (Disk 2)} \]

\[ f(\xi) = \text{von Karman correlation function:} \]

\[ \sigma^2 \theta \left( \frac{L}{\alpha L} \right)^{1/3} K_{1/3} \left( \frac{\xi}{\alpha L} \right) \]

\[ f'(\xi) = \frac{d}{d\xi} f(\xi) \]

\[ \alpha = \frac{\Gamma(1/3)}{\sqrt{\pi} \, \Gamma(5/6)} \quad (=1.339) \]

\[ \beta = \frac{2^{2/3}}{\Gamma(1/3)} \quad (=0.5925) \]

\[ \sigma = \text{standard deviation of the velocity components} \]

\[ L = \text{integral scale parameter} = \int f(\xi) \, d\xi \]

\[ K_{1/3}(\cdot) = \text{modified Bessel function of order } 1/3 \]

\[ \Gamma(\cdot) = \text{gamma function}. \]

The terms D1 and D2 refer to the positions of the rotor disk relative to the air mass at times \( t \) and \( t + \tau \), respectively. Unfortunately, the integrations needed in Eq. (5) cannot be carried out analytically. However, if the velocities are scaled by the standard deviation \( \sigma \), time is scaled by the ratio \( L/V \), and length is scaled by the radius \( R \) of the rotor disk, the resulting correlation function becomes a nondimensional family depending on the single parameter \( R/L \).

To more fully understand how to use these results, consider the case of filtered white noise. A stochastic differential equation describing filtered noise is given by

\[ \dot{x} + ax = bw \]

(6)

where

\( w = \text{the white-noise excitation} \]

\( x = \text{the filtered response} \]

The development and discussion of the turbulent inflow modeling presented here have been by necessity brief. The reader interested in a more detailed development is referred to (3) and (20).
sequence f(k) is found to be sequentially uncorrelated with Gaussian statistics. Thus, f(k) can be chosen to
be independent random samples from a Gaussian distribution with unit variance. The coefficient \( \Gamma \) is then
given by

\[
\Gamma = b[(1-q^2)a^2L/(2avV)]^{1/2}
\]

where the noise power spectral density has been chosen to be \( a^2L/V^2 \).

**Computer Simulation**

The discrete time model given by Eq. (12) is readily simulated on the computer. Two factors must be considered. First, because \( a \) is nearly 1 for most simulation time steps, the coefficient \( \Gamma \) will be relatively small. Thus, as time progresses, the state variable will reflect the sum of many random increments, which, because of the law of large numbers \( (23) \), means that the state variable will be asymptotically Gaussian for a sequence \( \xi(k) \) from any reasonable distribution. Thus, the \( \xi(k) \) need only be roughly Gaussian for \( x(k) \) to have statistics that are much more nearly Gaussian. The second factor to consider is that many computer simulation codes allow \( A \) to vary from time to time. Examination of Eqs. (12) and (13) shows that \( a \) and \( \Gamma \) must be recomputed whenever \( A \) is changed.

Using the discrete time model for computer simulation depends on the ability to generate sequences of independent random samples from a roughly Gaussian distribution. This roughly Gaussian distribution can be obtained by summing three random samples from a uniform distribution. Suppose \( \epsilon \) is uniform on the interval \([0,1]\). Then, define

\[
\xi = 2(\epsilon_1 + \epsilon_2 + \epsilon_3) - 3
\]

where \( \epsilon_1, \epsilon_2, \) and \( \epsilon_3 \) are three independent samples from the uniform distribution. Simple results from probability theory give the mean and variance:

\[
E[\xi] = 2E[\epsilon_1] + E[\epsilon_2] + E[\epsilon_3] - 3 = 0
\]

\[
E[\xi^2] = \delta E[\epsilon_1^2] + E[\epsilon_2^2] + E[\epsilon_3^2] = 1
\]

The independent samples from the uniform distribution can be generated on the computer using the linear congruence method. Understand that any computer procedure for generating random numbers cannot be truly random because a specific deterministic procedure is always used. Therefore, these methods are often called "pseudo-random" number generators. In particular, the linear congruence method utilizes modular arithmetic. Thus, consider

\[
n(k+1) = Mod[c \cdot n(k), m]
\]

where

\[
n(k) = a \text{ sequence of integers}
\]

\[
c = \text{the multiplier}
\]

\[
m = \text{the modulus}
\]

The function Mod(\( \cdot, \cdot \)) is the remainder when the first argument is divided by the second. When the sequence \( n(k) \) is examined, we find that only \( m \) different values of \( n \) are possible. Thus, the sequence will repeat at least every \( m \) times. To obtain a long period of repetition, Whitney \( (24) \) suggests that \( c > \sqrt{m} \) and \( m \) must be a prime number. Uniform real numbers for the interval \([0,1]\) are generated by the fraction

\[
\epsilon = \frac{n(k)}{m-1}
\]
For the particular program written here, the following values have been implemented:

\[ m = 231 - 1 = 2147483647 \]
\[ c = 46341 \]

Computation is made with double-precision, floating-point arithmetic to prevent unwanted periodicities from round-off error.

The actual FORTRAN implementation of this algorithm is quite simple. Figure 2 is a flow chart that shows the calculation scheme, and Figures 3 through 6 show the FORTRAN code used to model the turbulence in the FLAP code. With this turbulence simulation scheme, the FLAP code can run a 456 revolution simulation for the Howden turbine; this can be done in 50 minutes on a Compaq 386/20 PC using a single-blade mode shape calculating loads at 10° increments. A run of 456 revolutions takes about 10 minutes of operational time for the Howden machine, so the computer simulation takes 5 times longer than the actual run time.

**The FLAP Code**

The FLAP code is a PC-based model for predicting the dynamic loads and flapping motion of an individual wind turbine blade. It accounts for the blade bending deformation about the smallest blade inertia axis. The rotor is assumed to operate at a constant speed, and the hub is allowed to move in a prescribed yawing motion. Rotors that are tilted and yawed relative to the mean wind can be analyzed. FLAP can be used to model simple teetering-rotor hubs, but not a delta-3 hinge or underslung rotors.

The model operates in the time domain, and the blade acceleration equation is integrated via a modified Euler trapezoidal predictor-corrector method. The method incorporates a set of low-order relations, is self-starting and stable, and allows frequent step-size changes. The procedure is entirely automated within the computer program. Results of the blade loads analysis are printed in tabular form and include the deflection, slope, velocity, flapwise shear and moment, edgewise shear and moment, blade tension, and blade twisting moment for any point along the blade axis.

The program, written in FORTRAN 77, is in the public domain and was developed for easy end-user modification and customization. The code contains its own documentation through the extensive use of comments within the program. Readers interested in more information concerning the FLAP code should consult (1), (2), and (22).

**Comparison of Simulations with Field Test Measurements**

The FLAP code was initialized to model the Howden 330-kW horizontal-axis wind turbine located near Palm Springs, Calif., in San Gorgonio Pass. The turbine, manufactured by James Howden and Company, is a three-bladed, upwind machine with a rigid hub and wood/epoxy blades. It is rated at 330 kW in a hub-height wind speed of 32.4 mph (14.5 m/s) and was designed to operate in cut-in to cut-out wind speeds of 13.4 to 62.6 mph (6.0 to 28.0 m/s), respectively. The rotor diameter is 85.3 ft (26 m), and the rotor speed is 42 rpm. The blades are tapered and twisted, with a maximum chord of 4.8 ft (1.47 m) and a maximum twist of 16°; the blade tapered to a 2.6-ft (0.8-m) chord and 0° twist at the blade tip. The blade airfoil section is a GA(W)-1, 17% thick. The rotor axis centerline is 79.1 ft (24.1 m) above the ground, and the rotor coning angle (cone) is 0°. The tower diameter is 5.9 ft (1.8 m), and the distance from the yaw axis to the rotor plane is 11.5 ft (3.5 m).

A detailed description of the modeling inputs used for the aerodynamic and structural-dynamic parameters are given in (2). For the turbulence simulations reported here, only a single flap mode shape is used, and the natural frequency for that mode is set at 1.40 Hz (or 2P); this modeled the deterministic blade responses quite well (2). However, the gravity excitation and the wind-shear input are both set to zero for the wind load. The mean wind input is necessary to establish the mean angle of attack, about which vortex shedding causes perturbations. This approach assumes that the deterministic and stochastic response can be summed in a simple linear manner, and that there are no interaction effects. However, the validity of this assumption has never been verified.

In this paper, turbulence loads computed by FLAP are compared to measurements for three different 10-min data cases, which were taken from (26). Operating conditions for these three cases are shown in Table 2.

To compute the proper turbulence filter coefficients (using equations from Table 1) and parameters for the discrete time turbulence evolution (described in (12) and (13)), the mean wind speed \( V \), standard deviation of wind speed \( \sigma \), and integral scale \( L \) are needed. The 10-min mean is taken for \( V \), and the standard deviation and integral scale are obtained by curve fitting using the least-squares method. The wind-speed spectral density to the von Karman theoretical spectral density function, which is given by

\[
S(\omega) = \frac{1}{\pi} \frac{(\sigma L / V)^2}{\left[1 + (\alpha L / V)^2\right]^{3/2}}
\]

For the curve fitting, the quantities \( (\sigma^2 L / V) \) and \( (\alpha L / V) \) are treated as unknown parameters. Using the 10-min mean for \( V \) allows \( \alpha \) and \( L \) to be computed from the curve-fit parameter values.

The hub-height wind spectrum is computed using two 5-min time-series segments and then averaging the spectral estimates. The wind time-series data were electronically filtered at 1.2 Hz to eliminate unwanted noise above the anemometer cut-off frequency. The time series were then digitized at 40 Hz for convenience and then decimated to about 6 Hz before the spectra were computed. The curve fitting is done on the average of the two 5-min spectra, but no smoothing is employed before the curve fitting.

Table 2 summarizes the standard deviation and integral scales obtained for the three data cases. Figures 7 through 9 show the computed wind spectra and the results of the curve fitting; also shown is the corresponding wind spectrum computed from the FLAP simulation for the wind as observed from the tip of the rotating blade. This comparison shows the difference between experimental and simulated results for the low-frequency region, as well as the rotational effects at 1P and 2P. The most striking feature of these curves is the large data scatter for the experimental spectral estimates. This scatter should be expected because the significant wind fluctuations are not Gaussian.

<table>
<thead>
<tr>
<th>Data Case</th>
<th>( V ) (ft/s)</th>
<th>( \sigma V )</th>
<th>Power (kW)</th>
<th>( \sigma V ) (ft/s)</th>
<th>( L ) (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-7</td>
<td>34.3</td>
<td>0.18</td>
<td>169</td>
<td>6.2</td>
<td>291</td>
</tr>
<tr>
<td>3-5</td>
<td>31.7</td>
<td>0.13</td>
<td>119</td>
<td>3.6</td>
<td>336</td>
</tr>
<tr>
<td>17-1</td>
<td>55.6</td>
<td>0.13</td>
<td>298</td>
<td>6.6</td>
<td>544</td>
</tr>
</tbody>
</table>

\( \sigma V \) = turbulence intensity = \((\text{standard deviation of the wind speed}) + (\text{mean wind speed})\)
Compute the turbulent wind velocity fluctuations

IF (T/shows) THEN
  XX = (XX+10)*XX
  RETURN
ENDIF

Do = 0.5, 1.0, 1.5, 2.0

Compute the spectrum during the 10-min time period of interest.

In this case, the unsmoothed spectral estimates have been plotted in order to compute the average atmospheric spectrum.

For (1) and (4) (SIGMA = standard deviation of wind speed)

spectral estimates have been plotted unsmoothed. Usually, wind spectra are heavily smoothed by averaging several adjacent estimates or by averaging several spectral estimates from consecutive time periods. In this case, the unsmoothed spectral estimates have been used for curve fitting to obtain the best estimate for the spectrum during the 10-min time period of interest, rather than an estimate for the average atmospheric spectrum.
The FLAP simulation results shown in Figures 7 through 9 have been smoothed by averaging three adjacent estimates; this is done primarily to reduce the data volume prior to plotting. In each of the three cases, the simulated wind spectrum agrees reasonably well with the target theoretical spectrum and the experimental data. The influence of the rotational effect is clearly seen at frequencies of 1P and 2P. The modeling neglects rotational effects at higher frequencies. Figure 10 plots the FLAP simulated lateral and vertical spectra for data case 12-7 and compares them with the theoretical von Karman spectrum that was the target.

The FLAP simulated rotor bending moments at 10% and 65% span are compared with experimental results in Figures 11 through 13. The comparison between predicted and measured results is generally quite good. The magnitudes of the major features are predicted quite well, even for the resonance at 2P. Cases 12-7 and 3-5 are for about the same wind speed; however, case 12-7 has a turbulence intensity of 18%, while case 3-5 has a turbulence intensity of about 13%. For these two cases, the pitch control system is inactive. In case 17-1, which is a high-wind-speed case, the pitch control system is active and the tips are constantly moving to control power. The FLAP code cannot model the control actions, so the agreement between predicted and measured results is not expected to be particularly good. Figure 13 shows a considerable discrepancy for the 65% span location, which is closer to the pitchable tip; however, the effect seems to average out at the 10% span. The experimental data show a response peak at about 3P that may be a tower mode excited by turbulence. The FLAP code does not model...
CONCLUSIONS

The filtered-noise turbulence model has been incorporated into the FLAP code to predict turbulence-induced blade bending loads. A comparison of simulation results with experimental measurements has demonstrated that the resulting stochastic loading is predicted quite well for a rigid-hub wind turbine. The filtered-noise turbulence model as implemented in the FLAP code has proved that these simulations can be done efficiently on a personal computer. This brings the ability to estimate turbulence-induced loads within the means of all wind turbine designers.

A word of caution to potential users: The filtered-noise turbulence model as currently developed underestimates rotational wind inputs above frequencies of 2.5P. Therefore, for blades having a first flap frequency above 2.5P, or for blades with a lightly damped natural frequency above 2.5P, the resulting simulated bending moments are likely to be underpredicted. For the comparisons presented in this paper, the Howden turbine had a first flap frequency at 2P, which resulted in favorable comparisons.

The success of applying the methods described in this paper to the problem of computing a lifetime loading histogram depends on knowing the mean wind probability distribution and associated joint probabilities for both the standard deviation and the integral scale. These data are not available, even for the three major California wind sites. In addition, the spectral characteristic of the California wind sites has not been well documented. Some feel that the spectral characteristics should resemble the text-book empirical formulas; the results of this paper don’t contradict that assertion. However, the joint probabilities that define the percentage of the time that a turbine operates in high wind and high turbulence (versus high wind and low turbulence) or any other combination of conditions are still unknown.

Ultimately, the results of analyses like the one presented here must be repeated for a variety of turbulence conditions that reflect the expected lifetime environment for the turbine. The resulting loading histograms must be weighted and superposed to obtain the lifetime loading histogram from which a fatigue life estimate can be made. Although the ability to
compute turbulence-induced loads is a major step forward, it is only a single step toward the final goal of a complete structural design methodology.

REFERENCES


23. Ibid, pp. 263-266.

