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A THOROUGH APPROACH TO MEASUREMENT UNCERTAINTY ANALYSIS APPLIED TO IMMERSED HEAT EXCHANGER TESTING

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ABSTRACT

Measurements are frequently taken with little or no regard for the uncertainties or errors of those measurements, which may lead to incorrect conclusions and decisions. Professional societies today, including ASME, are becoming more and more concerned with measurement uncertainty. This paper is developed in full support of that concern.

This paper discusses the value of an uncertainty analysis, discusses how to determine measurement uncertainty, and then details the sources of error in instrument calibration, data acquisition, and data reduction for a particular experiment. Methods are discussed to determine both the systematic (or bias) error in an experiment as well as to determine the random (or precision) error in the experiment. The detailed analysis is applied to two sets of conditions in measuring the effectiveness of an immersed coil heat exchanger. It shows the value of such analysis as well as an approach to reduce overall measurement uncertainty and to improve the experiment.

This paper outlines how to perform an uncertainty analysis and then provides a detailed example of how to apply the methods discussed in the paper. The authors hope this paper will encourage researchers and others to become more concerned with their measurement processes and to report measurement uncertainty with all of their test results.

INTRODUCTION

This paper grew out of the authors' concern for the quality of research results since many research papers and reports do not describe the uncertainties in either intermediate measurement results or final experiment results and conclusions.

Measurement error has been defined as the difference between the true value and the measured value (1). It includes both systematic (bias) and random (precision) errors. The true value might be considered as the value obtained if <u>every</u> source of measurement error could be known and eliminated or corrected (a physical impossibility).

Measurement uncertainty, as used here, provides an estimate of the largest error that may reasonably be expected in a particular measurement process and is a function of that measurement process. Errors larger than the stated uncertainty should rarely occur in actual practice (2).

Measurement uncertainty analysis refers to the specific methodology developed in recent years and is now becoming the adopted technique in a number of national and international standards (1,3,4,5,6).

The measurement uncertainty analysis described in this paper is applied to measuring the effectiveness of a heat exchanger immersed in a tank of water where the effectiveness is the ratio of the actual heat transfer to the theoretical maximum heat transfer for a given heat exchanger, flow rate, and temperature difference between the inlet of the heat exchanger and the storage tank. The tests were conducted with a single-wall, finned, bayonet heat exchanger with a heat transfer area of 0.9 m^2 installed at the bottom of a 409 ℓ plastic tank filled with water. Water was used as the heat transfer fluid inside the heat exchanger with tests conducted at flow rates of 5, 10, and 15 ℓ/m. Tests were begun with a cold tank and a high fluid temperature entering the heat exchanger. Tests were concluded when the tank temperature approached the heat exchanger inlet temperature within a few degrees.

Time, effort, and expense are required to perform a valid measurement uncertainty analysis for an experiment. However, a thorough uncertainty analysis is not only good practice but can save time, effort, and expense. Before beginning an experiment the experimenter should answer the following questions:

- How will the results be used?
- What accuracy is desired and what accuracy is actually required for the final result to be useful or meaningful?
- Where can improvements be made in the experiment or test to get the greatest improvement in the accuracy of the results?
- Below what accuracy is the experiment not worth performing?

The measurement uncertainty analysis methodology described here and in the references are helpful in determining beforehand whether or not the required accuracy for the final results can be achieved. This analysis also identifies sources of large inaccuracies that may be reduced to improve overall accuracy.

Publishing uncertainty statements and information not only gives integrity to the test result and allows the reader to understand the limits of the conclusions but also enables others who use the experimental results to propagate uncertainty information through their own results without any loss of integrity in the uncertainty analysis process.

HOW TO PERFORM A MEASUREMENT UNCERTAINTY ANALYSIS

A key point of this approach is to separately propagate to the final result the elemental random and systematic errors and the degrees of freedom for the measurement process. The random and bias errors are only combined in the final overall uncertainty calculation for a particular quantity or result.

Kline and McClintock (7) developed a useful method to estimate measurement error using a root-sum-square technique. However, their method does not adequately account for the separate nature of systematic errors, which generally do not fluctuate, and random errors, which often follow a certain statistical distribution. If there were no systematic errors or if they were always at a fixed value and "calibrated out" of the results, then these two approaches should give the same result. However, since systematic errors do exist and are not always constant it is important to report them with the test results and carefully describe how they were combined with the random errors. An example of a variable systematic error is a temperature probe downstream from the actual point of interest. If the closest point to install a temperature sensor is 1 m from the outlet (or cold side) of a heat exchanger, that probe will always sense a lower temperature than desired. The drop in temperature is related to the heat loss from that 1 m length of pipe (which should be well-insulated), which in turn is a function of the flow rate. Therefore, if a heat exchanger is tested at several flow rates, the systematic error will change with each flow rate.

Another example is the self-heating of sensors, such as thermistors, where the amount of self-heating, an asymmetrical systematic error, is dependent upon the resistance of the thermistor, which is temperature dependent.

The following discussion presents a systematic approach to determine experimental error and separately propagate systematic and random errors.

- First, make a complete, exhaustive list of every possible elemental error source for each and every measurement. Remember the acronym CAR. This represents the three major areas into which errors will be grouped for logical evaluation: calibration, acquisition of data (including sensor installation), and reduction of data.

- Next, either measure or estimate the random uncertainty. One way is to determine the standard deviation from a large number of readings or use the standard deviation of the mean of several groups of measurements. By using the test or experiment data acquisition system to measure this, the random uncertainty of your entire system can be measured. These measurements should be taken under the expected test conditions of temperature, flow, pressure, etc., and at steady-state conditions. The random uncertainty of several resistant temperature detectors (RTDs) was measured in still water to be about 1 mK. Only later was it determined that two of the RTDs fluctuated about ±0.5 K in moving water. Remember, this test only provides a measurement of how the response of the sensor will fluctuate at a given set of conditions; it does not determine how close the response of the sensor is to the actual conditions; i.e., what the systematic error may be.

- Next, measure or estimate the systematic (or bias) error of each portion of the measurement process. Actual calibration data is the best source of information, although manufacturer's specifications may be used if the instrument has been calibrated according to the manufacturer's schedule. It is useful to perform a system end-to-end check with NBS traceable standards. For example, put temperature sensors in a calibrated, well characterized (known stability and measured temperature gradients) stirred temperature bath along with a temperature standard. A stirred bath minimizes temperature gradients and sensor self-heating effects. Compare the response of the temperature standard with the temperature calculated by the data acquisition system. This approach will include the effects of relay closures, lead lengths, actual measurement current to the sensor, calibration constants, curve fitting, and outside interferences. Although it may seem cumbersome, it provides a thorough check of the entire measurement system, including calibration, data acquisition, and data reduction. By using this approach, one erratic data acquisition channel and incorrect calibration constants for one sensor were detected (the sensor was a replacement for another, but the new calibration constants were not used).

- After all the random and systematic uncertainties are determined, propagate them separately into a total systematic error and a total random error for each particular measurement (or sensor). The total random error for a particular measurement, R_J, is calculated as the root-sum-square of the individual random errors, R_i, as

$$R_J = \left(\sum_{i=1}^{N} R_i^2 \right)^{1/2} , \tag{1}$$

and likewise, the total systematic or bias error, B_J, is calculated from the individual systematic errors, B_i, as

$$B_J = \left(\sum_{i=1}^{N} B_i^2 \right)^{1/2} , \tag{2}$$

where N is the number of sources of error and J is a particular measurement (such as temperature). Now the total systematic error and total random error are known for a particular measurement (or sensor). References (1) and (2) present methods to combine errors if some are an order of magnitude or larger than others. For asymmetrical systematic errors, such as those caused by sensor self-heating, the positive and negative limits are calculated separately. If this happens, then a positive systematic error and a negative systematic error are calculated and propagated separately. These are the total errors for a physical measurement or parameter. They are used to calculate the error or uncertainty in a final result or calculated quantity.

- Once the total random error and the total systematic error are known for a given measurement they can then be determined for particular calculated results. Once again, the uncertainties are separately propagated. This time the uncertainties or errors are multiplied by a sensitivity factor calculated by

$$B = \left[\sum \left(\frac{\partial Q}{\partial J} B_J\right)^2 \right]^{1/2} , \tag{3}$$

where B is the total systematic error for the calculated result, Q, and B_J is the total systematic error for that measurement. For example, the systematic error in measuring a volume

$$(V = \pi r^2 \ell) \tag{4}$$

would be

$$B = \left[(\pi r^2 B_\ell)^2 + (2\pi r \ell B_r)^2 \right]^{1/2} , \tag{5}$$

where B_ℓ is the systematic error of the length and B_r is the systematic error of the radius and each includes the systematic error from calibration, acquisition of data, and reduction of data. This would be repeated separately for the random error. The total random and systematic uncertainties are then combined to give an overall measurement uncertainty. One can either add the results as

$$U_{99} = \pm\left(B + t_{95}\frac{R}{\sqrt{N}}\right) , \tag{6}$$

or the two types of uncertainties can be root-sum-squared as

$$U_{95} = \pm \left(B^2 + \frac{(t_{95}R)^2}{N}\right)^{1/2} , \tag{7}$$

where t_{95} is from the two-tailed, student's "t" table and accounts statistically for the sample size used to calculate the random error and N is the number of degrees of freedom of the system. Either way is acceptable, but the procedure must be stated. The two procedures give approximately the same result if the random error is much larger than the systematic error. If not, U_{99} is about 35% larger than U_{95}, where the actual result is expected to be within $\pm U_{99}$ 99% of the time but within $\pm U_{95}$ only 95% of the time.

SOURCES OF HEAT EXCHANGER TEST MEASUREMENT ERROR

As discussed earlier, a convenient method for identifying the sources of error in a test or research experiment is to group them into the three broad categories of sources: the calibration process, sensor installation and data acquisition, and data reduction. Other groupings might be appropriate in some situations. Table 1 lists sources of elemental error for measuring the effectiveness of an immersed heat exchanger. Within each category, the error sources are classified as to their nature, whether they cause scatter in the data (random or precision errors), or whether they cause fixed offsets in the data (systematic or bias errors).

The elemental errors were derived from various sources. When possible they were measured or derived from manufacturers' specifications. When this was not possible they were calculated or estimated using the best information available.

First, the sources of error in the calibration process were considered. These errors came from the standards used to establish the known temperatures, the circulating liquid calibration baths in which the temperature sensors were calibrated, the instrumentation to measure the sensors during the calibration, self-heating errors in the sensors during calibration, etc. The errors that resulted from the curve-fitting process used with the calibration data to establish the polynomial expressions (used in the data acquisition computer to calculate temperatures from measuring the resistance of the sensors) are included in the data reduction category of errors.

Four-wire RTDs were used where possible to eliminate the effect of lead resistance. The RTD leads were short-circuited near the sensor to ensure that a proper four-wire measurement was being made. This led to the discovery that the instruction manual used for the digital voltmeter (DVM) contained an error in its discussion of four-wire RTD measurements. After consulting with the DVM manufacturer, the correct method to make four-wire resistance measurements was determined.

Table 1. Elemental Errors (mK)

Error Sources	Systematic	Random	Degrees of Freedom
Calibration			
Triple point of water	±0.2		
Reproducibility in DQT[1] calibration	-0, +1.0		
DQT calibration resolution	±0.5		
DQT stability	±10.0		
DQT linearity and hysteresis	±16.0		
Bath stability	±3.0		
Bath gradients	±3.0		
RTD resistance measurement	±12.5	±3.8	>30
RTD stem conductance	±5.0		
RTD self-heating	±0.1		
DVM[2] thermal EMF's	±2.5		
Thermistor, factory calibration	±100.0		
Acquisition			
RTD:			
DVM, 4 wire ohms	±3.2	±0.9	>30
Self-heating	±0.1		
Stability	±125.0		
Repeatability		±5.0	>30
Installation	-10.0		
Thermistors:			
DVM, 2 wire ohms	±7.0	±0.1	>30
Self-heating	-0, +170.0		
Stability	±25.0		
Repeatability		±5.0	>30
Temp. effects of lead resistance	±0.1		
Reduction			
RTD curve fitting to calibration data (worst residual)	±2.1		
Computation of temperature from polynomial		±5.0	>30

[1]DQT, HP 2804A digital quartz thermometer

[2]DVM, HP 3056A digital voltmeter

The heat exchanger RTDs were about 1 m from the actual inlet and outlet of the heat exchanger. Although the sensor and pipe outlets were well insulated, an energy balance showed that the temperature of the fluid dropped about 0.010 K at a flow rate of 5 ℓ/m and dropped about 0.003 K at 15 ℓ/m. This was a systematic error that was always negative. Since these errors were in the same direction, they cancelled for calculation of the heat transfer from the heat exchanger and in the numerator of the effectiveness calculation. Stainless steel, sheathed RTDs 30 cm long and 0.3 cm in diameter were immersed directly into the center of the fluid stream using tees. The sensitive tips of the RTDs were upstream in the flow. They were well insulated to minimize conduction losses along the sheath and to the supporting structure.

Very small thermistors with a very short time constant (measured as 18 ms using the plunge into water technique) to avoid disturbing the convective flow and allowing detection of very rapid changes in the convective currents were used to measure the tank temperature.

The thermistors came with 2 m leads calibrated by the manufacturer. The tank temperature thermistor had a 2 m lead resistance of 46.79 ohms and a resistance from additional leads of 3.40 ohms. The lead resistance was accounted for in the data acquisition program. The change in resistance of the leads because of varying temperature of the environment was less than 1 mK.

RESULTS AND CONCLUSIONS

Theoretically, only three sensors are required to determine the effectiveness of an immersed heat exchanger: the temperature of the fluid entering the heat exchanger, the temperature of the fluid leaving the heat exchanger, and the temperature of the tank.

The effectiveness for an immersed heat exchanger is

$$\varepsilon = \frac{\text{actual heat transfer}}{\text{theoretical max. heat transfer}}$$

$$= \frac{(\dot{m}C_p)_{Hx}\,(T_{Hx,i} - T_{Hx,o})}{(\dot{m}C_p)_{min}\,(T_{Hx,i} - T_T)} . \tag{7}$$

The minimum heat capacitance, $(\dot{m}C_p)_{min}$, is on the heat exchanger side and, hence, equal to $(\dot{m}C_p)_{Hx}$ since the storage tank has a very large thermal capacitance. $T_{Hx,i}$ is the temperature of the fluid entering the heat exchanger, $T_{Hx,o}$ the temperature of the fluid leaving the heat exchanger, and T_T the tank temperature. The fluid in the tank was well mixed throughout the tests since it was heated from the bottom. Therefore, equation (8) is written as

$$\varepsilon = \frac{T_{Hx,i} - T_{Hx,o}}{T_{Hx,i} - T_T} . \tag{8}$$

Using equation (4), the total systematic error is

$$B_\varepsilon = [(\frac{\partial\varepsilon}{\partial T_{Hx,o}} B_{T_{Hx,o}})^2 + (\frac{\partial\varepsilon}{\partial T_{Hx,i}} B_{T_{Hx,i}})^2 + (\frac{\partial\varepsilon}{\partial T_T} B_{T_T})^2]^{1/2} , \tag{9}$$

which equals

$$B_\varepsilon = [(\frac{1}{T_{Hx,i} - T_T})^2 B^2_{T_{Hx,o}} + (\frac{T_{Hx,o} - T_T}{(T_{Hx,i} - T_T)^2})^2 B^2_{T_{Hx,i}} + (\frac{T_{Hx,i} - T_{Hx,o}}{(T_{Hx,i} - T_T)^2})^2 B^2_{T_T}]^{1/2} , \tag{10}$$

where the Bs are the total systematic errors for that measurement. The same equation is used to calculate the total random error for the heat exchanger effectiveness except that the individual random measurement errors, R_J, are used instead of B_J.

Properly combining the uncertainties (including using methods of combining large and small errors) from each elemental source of error using equations (1) and (2) leads to the results shown in Table 2.

The uncertainties need to be calculated for the total positive systematic error and the total negative systematic error (since these are some asymmetrical systematic errors) as well as the total random error. After that the two types of errors are combined. For a high temperature difference between the heat exchanger inlet and tank the following conditions existed during the test:

$$\dot{m} = 0.164 \text{ kg/s } (9.85\ \ell/\text{m})$$

$$T_T = 30.80\ ^\circ\text{C}$$

$$T_{Hx,i} = 69.68\ ^\circ\text{C}$$

$$T_{Hx,o} = 48.43\ ^\circ\text{C}$$

$$\varepsilon = 54.65\% .$$

These values are put into equation (10), which results in

$$B_\varepsilon = [0.000662 B^2_{T_{Hx,o}} + 0.000136 B^2_{T_{Hx,i}} + 0.000198 B^2_{T_T}]^{1/2} . \tag{11}$$

It is clear that at these conditions the accuracy of $T_{Hx,o}$ is more important than the other two temperatures and would be the first one to improve if higher accuracies were required. This basic equation is used to calculate both systematic errors (positive and negative) and the random error. The sensor uncertainties from Table 2 are used in equation (11) to give

$$B^+_\varepsilon = \begin{cases} [19.97 \times 10^{-6} + 4.10 \times 10^{-6} + 17.24 \times 10^{-6}]^{1/2} \\ 0.0064 \end{cases} \tag{12}$$

Table 2. Total Sensor Measurement Uncertainty (mK)

Measurement	Bias Limit	Random Uncertainty
Heat exchanger inlet temperature	±173.7	±8.1
Heat exchanger outlet temperature	±173.7	±8.1
Tank temperature	+295.1, −125.2	±5.0

It is quite clear that the uncertainties associated with $_{,o}$ and T_T are the greatest contributors to the uncertainty for the total positive systematic error.

The same procedure is followed for the total negative systematic error, which leads to

$$B_{\varepsilon}^{-} = 0.0052 \qquad (13)$$

and for the total random error, which leads to

$$R_{\varepsilon} = \pm 0.0002 \ . \qquad (14)$$

The largest contributor to both B_{ε}^{-} and R_{ε} is $T_{Hx,o}$. It is important to realize that the systematic errors are much larger than the random errors.

At this point the different types of errors are combined using equations (6) and (7) to get

$$U_{95}^{+} = \left[(0.0064)^2 + \frac{(1.96 \times 0.0002)^2}{1}\right]^{1/2} = +\ 0.0064 \qquad (15)$$

$$U_{95}^{-} = \left[(-0.0052)^2 + \frac{(1.96 \times 0.0002)^2}{1}\right]^{1/2} = -0.0052 \qquad (16)$$

for U_{95} and

$$U_{99}^{+} = 0.0064 + \frac{(1.96)(0.0002)}{1} = +0.0068 \qquad (17)$$

$$U_{99}^{-} = -0.0052 - \frac{(1.96)(0.0002)}{1} = -0.0056 \qquad (18)$$

for U_{99}.

For this example the overall uncertainty using U_{95} is

$$\varepsilon = 0.5465 +0.0064, -0.0052 \qquad (19)$$

or an error range from -1.0% to +1.2%. The error range using U_{99} is also -1.0% to + 1.2%.

Higher errors would be expected at smaller temperature differences. For a low temperature difference between the heat exchanger inlet and tank the following conditions existed during the testing:

$$\dot{m} = 0.165 \text{ kg/s } (9.89\ \ell/\text{m})$$

$$T_T = 55.40\ ^{\circ}\text{C}$$

$$T_{Hx,i} = 60.37\ ^{\circ}\text{C}$$

$$T_{Hx,o} = 58.67\ ^{\circ}\text{C}$$

$$\varepsilon = 34.33\%.$$

When these values are put into equation (10), the result is

$$B_{\varepsilon} = [0.04048 B_{T_{Hx,o}}^2 + 0.01753 B_{T_{Hx,i}}^2 + 0.00474 B_{T_T}^2]^{1/2} \ . \qquad (20)$$

At the low temperature differences the accuracy of $T_{Hx,o}$ is the most important contributor for both the systematic and random errors. Following the same procedures as before, the results are

$$B_{\varepsilon}^{+} = +0.0465$$

$$B_{\varepsilon}^{-} = -0.0427$$

$$R = \pm 0.0020$$

and

$$U_{95} = +0.0466, -0.0428$$

$$U_{99} = +0.0504, -0.0466.$$

This leads to the result that the overall uncertainty using U_{95} ranges from -12.5% to 13.6% of the measured effectiveness. The uncertainty range increases using U_{99} to range from -13.6% to 14.7% of the measured effectiveness. This higher uncertainty is expected at the lower temperature difference since the sensor error is a larger fraction of the temperature differences.

However, for back-to-back tests the systematic error cancels and the uncertainty is $t_{95}R$, which is equal to ±0.0004 or ±0.1% for the first example and ±0.004 or ±1.2% for the second example.

Since our final systematic errors were asymmetrical, we calculated positive and negative uncertainties. These results state that the actual effectiveness should rarely be beyond these uncertainty limits. It also shows to what extent the heat exchanger effectiveness can be measured confidently. Changes in heat exchanger effectiveness of 0.1% to 1%, which is the random uncertainty component of the result, can be measured if the instrumentation is not altered (sensors untouched, DVM left on, same environmental conditions, etc.)

An end-to-end analysis of the measurement system was conducted using two, digital quartz thermometer probes as reference temperatures in a stirred ice bath. The results were checked against systematic errors determined in the pretest analysis. The measurement system showed the response of $T_{Hx,i}$ to be 0.023 K below and $T_{Hx,o}$ to be equal to the digital quartz thermometers near the ice point of water. Each sensor had 100 consecutive readings taken to determine the random error of the measurement system for that sensor. The random error was 2.3 mK for the heat exchange inlet RTD, 2.2 mK for the heat exchanger outlet RTD, and 0.7 mK for the tank thermistor. These results show that the errors are well within the expected range of errors. However, the end-to-end check includes some built-in systematic errors in the standards that could not be determined. The results of the end-to-end check are well within the detailed calculations and are a necessary and useful check to show that there were not any gross errors in the experiment or uncertainty analysis.

Sensor failure, including severe degradation or catastrophic failure, will cause the result to exceed the uncertainty limits. In addition, the measurements must be in the proper location.

This uncertainty analysis was useful in detecting unsteady sensors and channels, increasing confidence in the results, and permitting others to make meaningful decisions by knowing the range of possible errors in the data.

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