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# Experimental Observations of Double Diffusive Natural Convection in Solar Ponds With Nonlinear Salinity Profiles

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## ABSTRACT

A solar pond can be used as a thermal energy source provided that convective instabilities do not occur. This paper experimentally examines the stability of a fluid layer with nonlinear salinity profiles. A nonlinear salt profile was set up in a 0.7m x 0.7m x 1.4m deep tank, and the water was heated by a solar radiation simulator. Three experiments were conducted, each over a time scale of about one week. An instability was produced in two of the experiments.

The instabilities occurred at the location of the weakest salinity gradient, and were confined to a narrow depth, as predicted by theory. A local length scale was used to produce a stability parameter, the ratio of thermal to solute Rayleigh numbers. It is shown that for nonlinear solute gradients, the appropriate length scale is based on the radius of curvature of the salinity distribution. With this choice of a length scale, good agreement was found between theory and experiment for the onset of an instability. However, only fair agreement was obtained for the disturbance frequency.

## NOMENCLATURE

### English

d	local length scale	(cm)
h	mixed sublayer thickness	(cm)
g	gravitational acceleration	(cm/s <sup>2</sup> )
G	salinity gradient, $\partial S/\partial z$	(%/cm)
G <sub>0</sub>	salinity gradient at inflection point	(%/cm)
K <sub>S</sub> <sup>0</sup>	thermal diffusivity	(cm <sup>2</sup> /s)
K <sub>T</sub> <sup>0</sup>	solute diffusivity	(cm <sup>2</sup> /s)
L <sub>T</sub>	total layer thickness	(cm)
p	oscillatory frequency of instability	(rad/sec)
Ra	thermal Rayleigh number, $g\alpha\Delta T_0^3/K_T$	(-)
Rs	solute Rayleigh number, $g\beta\Delta Sd^3/K_T$	(-)
S	salinity	(% by weight)
T	temperature	(C)
z	vertical coordinate	(cm)

### Greek

$\alpha$	coefficient of thermal expansion	(C <sup>-1</sup> )
$\beta$	coefficient of solutal expansion	(% <sup>-1</sup> )
$\lambda$	stability parameter, Ra/Rs	(-)
$\rho$	radius of curvature	(cm)
$\sigma$	Prandtl number, $\nu/K_T$	(-)
$\tau$	inverse Lewis number $K_S/K_T$	(-)
	kinematic viscosity	(cm <sup>2</sup> /s)

## INTRODUCTION

A solar pond can be used as a thermal energy source provided that convective instabilities do not occur. Convective instabilities will mix the fluid in the solar pond and reduce the high temperature at the bottom of the pond. Overmixing can be reduced by the use of a solute, such as sodium chloride, so that the upward density gradient is negative. However, a layer of fluid that is statically gravitationally stable may become unstable, a phenomenon described as double diffusive convection.

The problem addressed in this paper is the stability of a fluid layer with nonconstant salinity gradients, the specific application being the nonconvecting zone of a solar pond. The pond consists of: a bottom mixed layer in which the radiant energy is absorbed, a nonconvecting zone through which heat transfer is by conduction and radiation only, and top mixed layer. The objectives of this work are to experimentally determine the effect of a nonconstant salt gradient on the stability of the fluid, and on the location, size, and frequency of the resulting disturbance.

The stability of fluid layers with constant salt gradients has been extensively studied. Veronis [1,2] analyzed the stability of a fluid layer assuming a linear distribution of salt and heat with upper and lower boundaries which are dynamically free (the vertical gradient of the horizontal velocity is zero at the boundaries) and thermodynamically free (a constant flux of heat and salt across the boundaries

keeps the temperature and salinity fixed at the boundaries). He determined that, for a fixed nondimensional salinity difference, the saline Rayleigh Number,  $R_s$ , across the vertical extent of the fluid, the instability to infinitesimal perturbations will first occur as overstable oscillations at a critical value of the nondimensional temperature difference, the thermal Rayleigh Number  $R_a$ . Baines and Gill [3] performed a complete and detailed analysis of the diffuse-double-diffusive stability problem for all values of the thermal and saline Rayleigh Numbers. They found that the layer became unstable to small perturbations when

$$R_a = \frac{\sigma + \tau}{\sigma + 1} R_s + \frac{27\pi^4}{4} \frac{(1 + \tau)(\sigma + \tau)}{\sigma} \quad (1)$$

where

$$R_a = \alpha g \Delta T d^3 / \nu K_T \quad (2)$$

$$R_s = \beta g \Delta S d^3 / \nu K_T \quad (3)$$

and  $\Delta T$  and  $\Delta S$  are the temperature and salinity differences across the length scale,  $d$ . The value of  $R_s$  for solar ponds is typically of the order of  $10^{10}$ , while  $\sigma \sim 7$ , and  $\tau \sim 0.01$ , so the linear gradient stability criterion for solar ponds becomes

$$\lambda = \frac{R_a}{R_s} \leq \lambda_{crit} = \left( \frac{\sigma + \tau}{\sigma + 1} \right) \quad (4)$$

The frequency of the neutral disturbances was found to be

$$p = \left[ \frac{1}{3} \frac{\sigma}{1 + \sigma} R_s \right]^{1/2} K_T / d^2 \quad (5)$$

$$= \left[ \frac{1}{3} (1 + \sigma)^{-1} g \beta \Delta S / \partial z \right]^{1/2}$$

Experimental verification of the above equation was reported by Wright and Loehrke [4]. Further aspects of the constant gradient case are reviewed in Huppert and Turner [5].

It is difficult to produce and maintain a constant salt gradient in a solar pond, so it is very likely that the salt gradient will assume some non-linear profile. The stability of fluid layers with nonlinear salt profiles has not been as extensively studied as the linear case. Zandgrando [6] observed thin isothermal sublayers in the nonconvecting zone of a solar pond. The behavior of the sublayers was complex. The sublayers would sometimes decay and disappear, or remain stationary, or evolve into larger sublayers. Zandgrando has also performed a numerical analysis of the marginal stability of a fluid layer using linearized governing equations, a symmetric cubic salinity profile and a linear temperature profile [7]. Her work indicates that an

instability will be produced at an inflection point where the dimensionless salinity gradient,  $G$ , has a minimum value, and that the instability is confined to a narrow band centered about the inflection point. In addition, for large  $R_s$ , the stability criterion becomes

$$\lambda \leq \lambda_{crit} = \frac{\sigma + \tau}{\sigma + 1} G_0 \quad (6)$$

where  $G_0$  is the minimum value of the salinity gradient.

A linearized asymptotic analysis by Walton [8] confirms the above points. In addition, the frequency of the oscillations for small  $\tau$  and large  $R_s$  is shown to be

$$p = \left[ \frac{\sigma}{1 + \sigma} G_0 R_s \right]^{1/2} \frac{K_T}{d^2} = \left[ (1 + \sigma)^{-1} g \beta \frac{\partial S}{\partial z} \Big|_0 \right]^{1/2} \quad (7)$$

for small  $\tau$ . Therefore for large  $R_s$ , the predicted effect of the non-linear salt profile is to multiply the constant gradient stability and frequency equations by the minimum local salinity gradient. As one would intuitively expect, the behavior of the fluid layer is governed by a local gradient, not the gradient averaged over the fluid layer.

The present experiment was designed to measure in detail the effects of a nonconstant salt gradient on the stability of a fluid layer and compare the experimental results with the above predictions. The fluid layer is heated until an instability occurs and a mixed layer forms. Measurements of the temperature and salinity profiles are made during the entire test period. The occurrence of an instability is detected by measurement of temperature oscillations and by the formation of an isothermal mixed layer.

#### EXPERIMENTAL APPARATUS

The experiments were performed in a square glass tank of height 1.22m and sides 0.61m, shown schematically in Figure 1. Heating of the tank was achieved by using four 1 KW quartz halogen lamps. Since the lamps were mounted on the sidewall of the room, the radiant energy was reflected into the tank by a mirror which was suspended from the ceiling directly above the tank. The radiant flux at the surface was estimated at about 1 KW/m<sup>2</sup>. The lamps were chosen to match the spectral distribution of the sun as closely as possible; however, the infrared radiation from the lamps was somewhat higher than actual solar radiation. A fan was used to cool the upper mixed layer.

The sidewalls of the tank were constructed of 12mm thick glass, joined together with RTV silicon rubber. The bottom and all sidewalls were insulated with 30 cm thick pieces of polyurethane. The choice of glass was an attempt to closely match the thermal conductivity of the tank wall to that of the fluid it would contain. A good correspondence between these thermal conductivities was essential to minimize conduction between upper and lower mixed layers. This further reduced lateral temperature gradients and

contributed to a stratified fluid layer such that sidewall boundaries could be ignored. A vertical salt distribution can be destabilized by a lateral heat flux, as described in [9]. It was experimentally verified that a sufficient amount of insulation was used because no sidewall instabilities were detected by the temperature instrumentation.

The temperature instrumentation consisted of a Minco platinum resistance temperature device (RTD) and a rake of eleven closely spaced thermistor probes. The RTD was used to determine temperature profiles in the tank at a given time. The thermistor rake was used to determine the temperature history in the region where the instability was expected to occur. The RTD was factory calibrated to .01 C and determined repeatable to within .005 C over the 50 C experimental temperature range expected. A one milli-amp current source was used to produce a voltage that could be read by the Hewlett Packard 3497A voltmeter. The total absolute uncertainty in the temperature measurement was estimated to be  $\pm 0.028$  C.

The thermistor rake consisted of eleven thermistor probes spaced one cm apart. The eleven probes and wiring harness were held in place by a slender delrin holder which was suspended from the top of the tank. The thermistor beads were glass encapsulated and had a maximum tip diameter of 1.5 mm. The glass beads were mounted at the end of a telescoping series of concentric glass capillary tubes. This arrangement contained the lead wires and provided 5 cm between the sensing tip and thermistor holder. Since it was essential to keep the thermistor and wiring connections dry, all joints were sealed with silicon rubber RTV. The thermistor time constant was estimated to be about 1.7 seconds.

The salinity measurements were performed using fluid withdrawal through 1.5 mm diameter hypodermic needles. The locations of the fluid withdrawal needles which are used to extract fluid samples were nonevenly spaced. In the region where the instability was expected, sampling ports were placed every 2 cm, and in other regions every 10 cm.

The salinity concentration was determined by weight analysis using a 15 and a 25 ml pycnometer flask that can be refilled to produce samples which contain the same volume. These samples were weighed and compared to a sample of distilled water at the same temperature to obtain the specific weight and subsequently the density of the sample. The salinity concentration by weight was obtained by double interpolation from a standard table [10].

The nonlinear salinity profile was set up using a technique called redistribution, first developed by Zangrando [6]. Initially a brine of approximately 15% salt by weight is prepared and is used to fill the tank to some predetermined level. Fresh water is injected through a diffuser which permits discharge only horizontally. The beginning location of the diffuser determined the height of the bottom mixed layer. A nonconstant salt gradient was created by varying the rate at which the diffuser was raised. Finally, about 30 cm of fresh water were carefully added to the top gradient region. This would become the upper mixed layer when the experiment was started. Due to the uneven raising of the diffuser, the initial salt profile contained many small (<1 cm) "kinks" or density interfaces. These small density interfaces disappeared after about 20 hours. Once

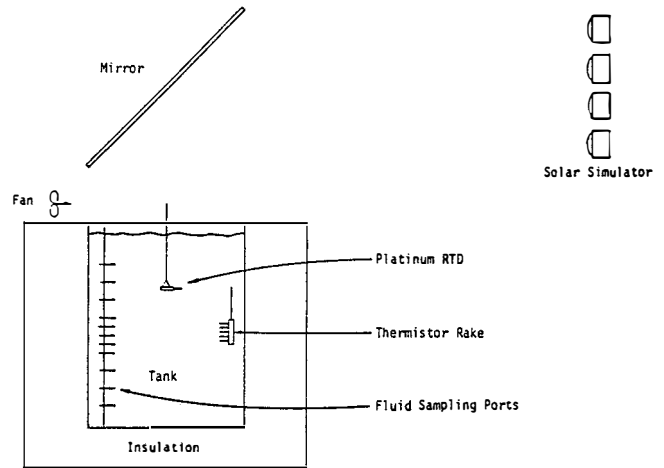


Fig. 1 - Experimental Apparatus

the smoothed salinity gradient was established the solar simulator and fan were turned on. This was considered the beginning of the experiment and all observations were referenced from this time.

Temperature profiles were taken periodically by raising or lowering the RTD probe in 7 mm steps. Salinity profiles were determined by withdrawing 100 ml samples, from each of the 17 sampling ports. Fifty ml were used to purge the fluid withdrawal lines and 50 ml became the sample. The fluid was assumed to be withdrawn from a uniform layer. The pycnometer analysis was done after all samples came to room temperature. The uncertainty in the salinity measurements was  $\pm 0.0026$  wt.%. The thermistor rake was centered at the weakest point of the salinity gradient and data was collected at specified intervals of time on all eleven thermistors. The data acquisition system used was a Hewlett Packard 3054DL consisting of an HP 85 computer and a HP 3497A Data Acquisition and Control unit. Further information on the experimental apparatus is given in Gordon [11].

## RESULTS

Three experiments denoted as runs 1, 2, and 3 were conducted. An instability was produced in runs 1 and 3. The instabilities occurred at the location of the minimum salinity gradient, and were confined to a narrow depth, as predicted by theory.

The salinity profiles prior to instability are shown in Figure 2 for runs 1 and 3. In the nonconvecting zone, the salinity profile has a large gradient region, followed by a small gradient region followed by a large region again. In the mixed layers above and below the nonconvecting zone the salt concentration is uniform.

The temperature profiles are shown in Figures 3, 4, and 5 as a function of time. The temperature gradient in the nonconvecting zone increases as the temperature of the bottom mixed layer increases. An isothermal region at the 58 cm level can be seen in run 1 sometime after about 80 hours of heating, and also in run 3 after about 70 hours of heating. No isothermal region in the nonconvecting zone is evident in run 2. The presence of the isothermal region

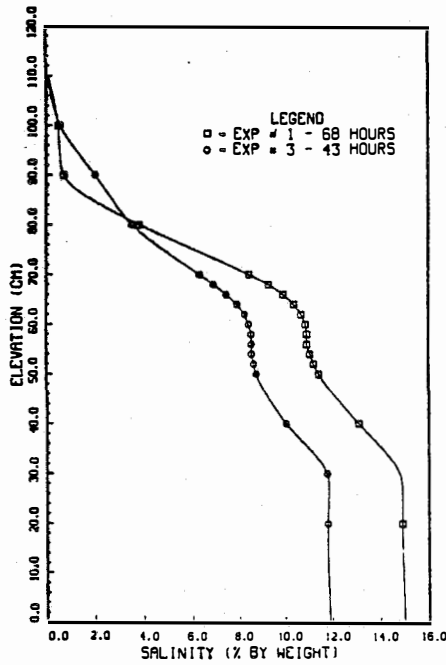


Fig. 2 - Salinity profiles prior to instability

indicates that an instability resulting in a mixed layer has occurred.

In order to compare theory and experiment, the length scale,  $d$ , is required, so that  $\Delta T$ ,  $\Delta S$ , and thus  $Ra$  and  $Rs$  can be calculated. For constant gradients, the relevant length scale is the distance between the top and bottom mixed layer. But for non-constant gradients, as discussed by Walton [8], the behavior of the fluid layer is independent of such a length scale, since in his analysis this length scale is infinite when compared to the vertical scale of the disturbance.

In this paper we propose a length scale based on radius of curvature arguments. The properties of the nonlinear salinity curves used in the foregoing analyses are the first and second derivatives of salinity with respect to depth. The first derivative is used to modify the constant gradient analysis, and the second derivative is used to locate the inflection point at which the instability is predicted to occur.

The radius of curvature is

$$\rho(z) = \frac{[1 + (\frac{\partial S}{\partial z})^2]^{3/2}}{|\partial^2 S / \partial z^2|} \quad (8)$$

Walton [8] suggests that the radius of curvature of the salinity distribution at the inflection point be

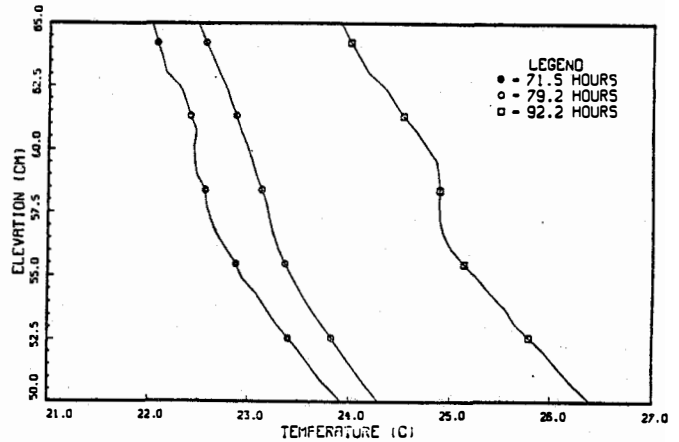


Fig. 3 - Temperature profiles for run 1

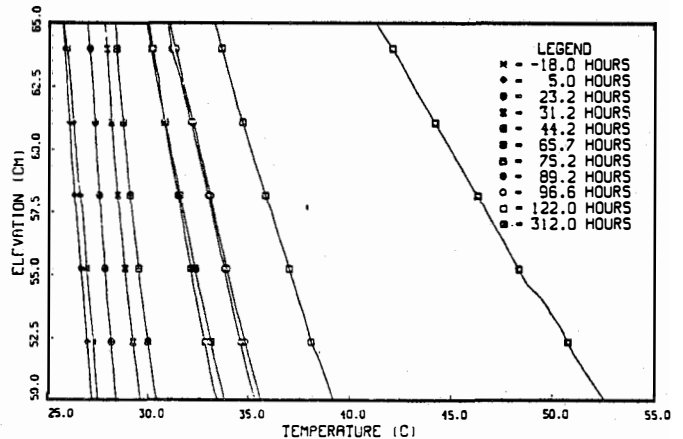


Fig. 4 - Temperature profiles for run 2.

used as the length scale, however, this is not suitable since the inflection point has an infinite radius of curvature. If the radius of curvature  $\rho(z)$  does not just monotonically decrease on both sides of the inflection point, but has both minimum and maximum values, then the distance between the two minimum values on either side of the inflection point can be used to define a local length scale. This local length scale defines a region of smaller salinity gradient which is more susceptible to instabilities than the region above and below.

A fifth order curve was fitted to the salinity data, and the inflection point and the radius of curvature calculated from the resulting equation. The curve fit and the radius of curvature are shown in Figures 6 and 7. For run 1 the calculated critical point is  $Z = 57.4$  cm. The instability was observed at about 55 cm. For run 3, the calculated critical point is  $z = 55.7$  cm. The instability was observed at about 58 cm. The local length scale,  $d$ , is 12.4 cm for run 1 at 68 hours and 13.0 cm for run 3 at 43.5 hours.

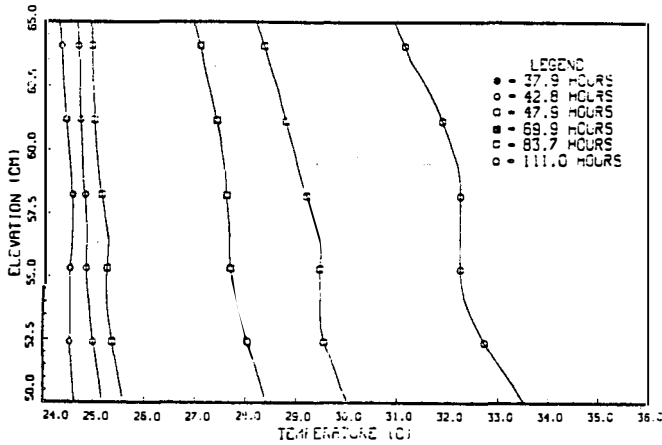


Fig. 5 - Temperature profiles for run 3

Since the diffusivity of salt is two orders of magnitude less than the diffusivity of heat, the salinity profile is assumed to remain constant relative to the changing temperature profile. Thus, once the length scale is chosen,  $\Delta S$  and  $G_0$  are fixed,  $\Delta T$  can be determined as a function of time, and thus  $\lambda$  and  $\lambda_{crit}$  can be computed as a function of time.

The values of  $G_0$  are 0.126 for run 1 and 0.133 for run 3. The equivalent  $G_0$  for a linear gradient has a value of 1. Therefore, according to equation (6), the fluid layer with a nonconstant salinity gradient will become unstable at lower values of the stability parameter  $\lambda$  than equivalent constant gradient fluid layers with the same overall salinity gradient.

The values of  $\lambda$  and  $\lambda_{crit}$  are plotted in Figure 8 for runs 1 and 3 as a function of time. The theoretical stability parameter  $\lambda_{crit}$ , as defined by Walton [8], changes with time since the average temperature in the tank is increasing, and thus  $\sigma$  and  $\tau$  are changing. The uncertainty in  $\lambda$  is dominated by uncertainty in the tabulated property data, and is estimated to be about 10%. Upon comparison of the experimental  $\lambda$  with the critical  $\lambda$ , the onset of instability is predicted to be about 94 hours for run 1 and 79 hours for run 3. In Figure 9, the experimental values of  $\lambda$  are always less than the critical value of  $\lambda$ , and thus no instability is predicted, which is confirmed by the experimental results.

In Figure 10 the thermistor output for run 3 is shown. The initial disturbance is oscillatory, in agreement with linearized theory. It is very interesting to note that the oscillations occur a number of hours before the time of instability predicted by Figure 8. The period of the temperature oscillations range from 100 to 600 seconds, with the average oscillation period being about 200 seconds. The oscillation period predicted by the constant gradient theory, equation (5), is 55 seconds; and the oscillation period predicted by the nonconstant gradient theory, equation (7), is 87 seconds. The resulting mixed layer in runs 1 and 3 is relatively

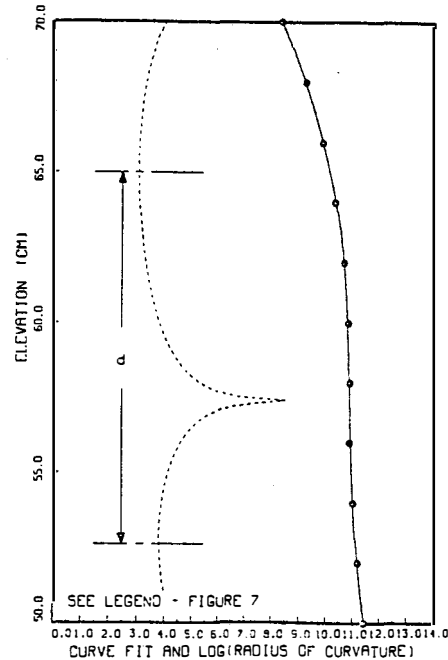


Fig. 6 - Salinity curve fit for run 1, 68 hours

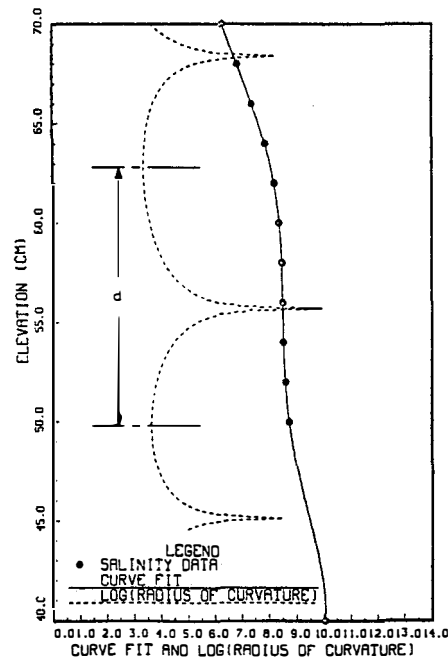


Fig. 7 - Salinity curve fit for run 3, 43.5 hours

thin, as predicted by theory. For run 1, the mixed layer thickness,  $h$ , is about 3.5 cm, so  $h/L = 0.057$ , and  $h/d = 0.28$ . For run 3, the mixed layer thickness is 4.7 cm, so  $h/L = 0.068$ , and  $h/d = 0.36$ . Once the mixed layer forms, it will grow in time, as shown in Figures 3 and 5, and eventually the entire depth of the solar pond will be well mixed.

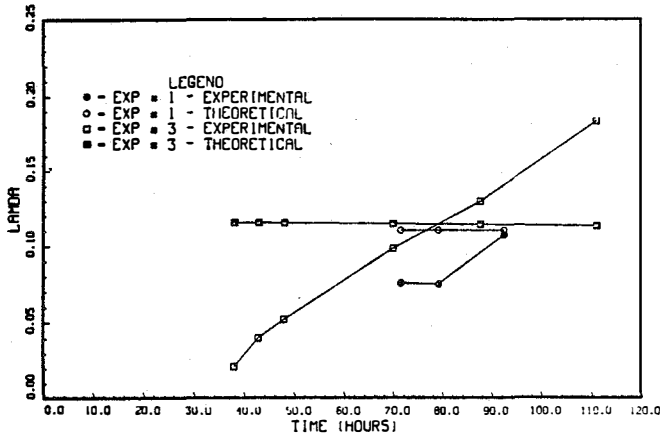


Fig. 8 - Lamda versus time for runs 1 & 3

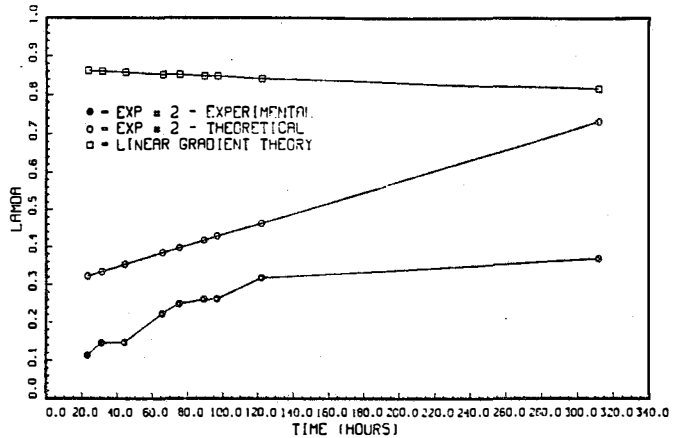


Fig. 9 - Lamda versus time for run 2

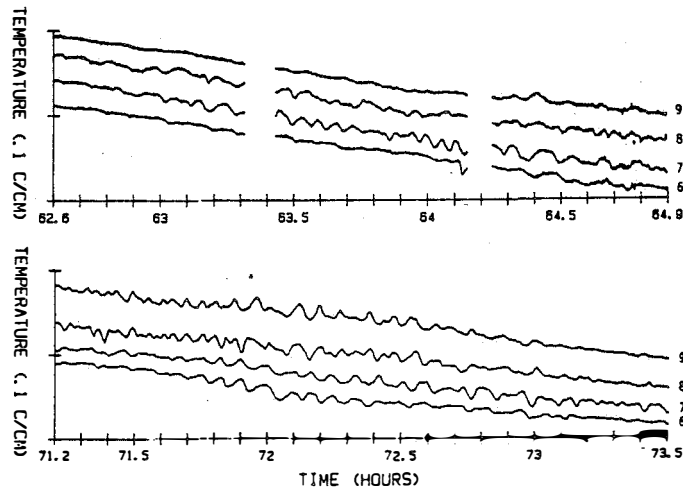


Fig. 10 - Thermistor output for run 3.  
#6=57.1 cm, #7=56.2, #8=55.1 cm, #9=54.1 cm

CONCLUSIONS

The main conclusion of the experiment is that fluid layers with nonconstant salinity gradients will become unstable at lower values of the stability parameter  $\lambda = Rs/Ra$  than equivalent constant gradient fluid layers with the same overall average salinity gradient. The local conditions govern the local behavior of the fluid layer.

The relevant salinity gradient for the nonconstant case is the local salinity gradient at the inflection point. A length scale which gives good agreement between linearized theory and the experimental results is the distance between the minimum radii of curvature of the salinity profile.

The instability occurs at the point of minimum

salinity gradient, i.e., the inflection point of the salinity profile. The instability is initially oscillatory, and results in the formation of a thin mixed layer. The onset and location of the instability is predicted fairly closely by linearized theory with nonconstant gradients. The predicted and measured oscillatory frequencies are in fair agreement.

ACKNOWLEDGMENTS

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