

SERI/TP-641-1114  
PREPRINT

SOLAR ENERGY RESEARCH INSTITUTE  
Solar Energy Information Center

OCT 27 1981

GOLDEN, COLORADO 80401

A COMPARISON OF ALGORITHMS  
FOR PREDICTING OPTICAL CONSTANTS

GARY J. JORGENSEN  
MICHAEL G. LANG

SEPTEMBER 1981

SUBMITTED TO THE JOURNAL  
OF THE OPTICAL SOCIETY OF  
AMERICA

**ARCHIVAL  
FILE**

PREPARED UNDER TASK No. 1003.00  
WAPA No. 281.81

**Solar Energy Research Institute**

A Division of Midwest Research Institute

1617 Cole Boulevard  
Golden, Colorado 80401

Prepared for the  
U.S. Department of Energy  
Contract No. EG-77-C-01-4042

CONFIDENTIAL

SECRET

CONFIDENTIAL

**NOTICE**

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States nor any agency thereof, nor any of their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for any third party's use or the results of such use of any information, apparatus, product, or process disclosed in this report, or represents that its use by such third party would not infringe privately owned rights.

## A comparison of algorithms for predicting optical constants

Gary J. Jorgensen and Michael G. Lang

Materials Research Branch  
Solar Energy Research Institute  
1617 Cole Boulevard  
Golden, Colorado 80401

Three programs were written to compute the optical constants of materials from normal incidence reflectance measurements. These include a Kramers-Kronig technique, a Fourier series algorithm, and an oscillator matching method. Each approach was used to calculate  $n$  and  $k$  for potassium bromide from measured reflectance data between 55-250  $\mu\text{m}$ . A 1% uncertainty in the reflectance data resulted in a 5%-10% uncertainty in the optical constants computed by the Kramers-Kronig analysis. The difficulty with this method of treating spectral bands beyond the measured region is also inherent to the Fourier series procedure. Although the Fourier technique correctly predicts the phase shift on reflectance, very low frequency data are needed to accurately compute the optical constants in the usual spectral region of interests. The oscillator matching method is a complex reflectance curve fitting routine which iteratively converges on  $n$  and  $k$  expressed as a sum of classical harmonic oscillators. As with the Kramers-Kronig analysis, the optical constants were found to be extremely sensitive to uncertainties in the measured reflectance, especially at high frequencies.



## INTRODUCTION

To predict how a material performs optically in a solar energy system, the optical constants must be known for the spectral and temperature range of intended use. Knowledge of  $n$  and  $k$ , the real and imaginary components of the complex index of refraction, allows the reflectance of mirrors, the transmittance of glazings, and the energy absorbed by solar receiver surfaces to be predicted. Knowledge of the optical constants also leads to a greater understanding of the underlying atomic structure in solids and the molecular interaction of thin film surfaces.

The determination of the optical constants of materials that are mostly opaque in the region of interest is accomplished using ellipsometry, Fourier-transform spectroscopy, or normal-incidence reflectance methods. The first two techniques are experimentally difficult but allow simple and accurate data reduction. Normal reflectance measurements, by contrast, are experimentally simple and widely available. However, obtaining values of  $n$  and  $k$  from these measurements is difficult, and significant inaccuracies may result unless data are reduced carefully.

This paper compares three methods of obtaining optical constants from reflectance data and explores their sensitivity to errors in the data. One common way in which the optical constants  $n$  and  $k$  are inferred from reflection data is through the use of a

Kramers-Kronig dispersion relation. This is an integral transformation using the reflectance data in the measured region of interest and an extrapolation procedure for unknown reflectance values outside that region. The choice of the extrapolation scheme can greatly influence the calculated values of  $n$  and  $k$ .<sup>1</sup> An extrapolation method is demonstrated in this paper which is not dependent upon an understanding of the nature of the material under question.

Another method, the oscillator matching technique, obtains  $n$  and  $k$  from dielectric constants assumed to be a sum of Lorentzian oscillators. The final oscillator parameters are determined by a nonlinear least squares procedure which fits the reflectance calculated from the dielectric constants to the measured reflectance. This method requires no extrapolation but does require an insightful initial choice of the oscillator strengths and frequencies. In addition to the preceding two methods, a Fourier series technique is briefly discussed.

## I. METHOD

In this section, three algorithms for predicting optical constants are considered. Several approaches are then outlined which allow the validity and the accuracy of the various techniques to be determined and compared.

### A. Kramers-Kronig Method

We wish to predict the optical constants  $n(\omega)$  and  $k(\omega)$  from measurement of normal incidence spectral reflectance  $R(\omega')$ , where the bandwidth defined by  $\omega$  is contained within the range  $\omega'$  (arbitrary units). From the reflectance measurement, the phase shift  $\theta(\omega)$  of the reflectance amplitude can be computed using a Kramers-Kronig method as in Ref. 2:

$$\begin{aligned}\theta(\omega_i) &= \frac{2\omega_i}{\pi} \int_0^{\infty} \frac{\ln R^{1/2}(\omega) - \ln R^{1/2}(\omega_i)}{\omega_i^2 - \omega^2} d\omega \\ &= \frac{\omega_i}{\pi} \int_0^a \frac{\ln R(\omega) - \ln R(\omega_i)}{\omega_i^2 - \omega^2} d\omega + \frac{\omega_i}{\pi} \int_a^b \frac{\ln R(\omega) - \ln R(\omega_i)}{\omega_i^2 - \omega^2} d\omega \\ &\quad + \frac{\omega_i}{\pi} \int_b^{\infty} \frac{\ln R(\omega) - \ln R(\omega_i)}{\omega_i^2 - \omega^2} d\omega\end{aligned}$$

$$= \alpha(\omega_1) + \phi(\omega_1) + \beta(\omega_1) \quad (1)$$

In general,  $R(\omega)$  is measured only in some finite bandwidth  $a < \omega < b$ . This allows  $\phi(\omega_1)$  to be directly computed numerically, but some assumptions must be made about the behavior of the reflectance at frequencies beyond the measured range to compute  $\theta(\omega_1)$ . Elaborate extrapolation schemes that attempt to match the supposed functional dependence with the locally measured behavior near  $\omega=a$  and  $\omega=b$  have been developed.<sup>1</sup> Another approach<sup>3</sup> is to assume that  $\ln R(\omega)$  varies slowly outside the measured region which allows the estimates:

$$\alpha(\omega_1) = \left[ A - \frac{\ln R(\omega_1)}{2\pi} \right] \ln \left[ \frac{\omega_1 + a}{\omega_1 - a} \right] \quad (2)$$

and:

$$\beta(\omega_1) = \left[ B + \frac{\ln R(\omega_1)}{2\pi} \right] \ln \left[ \frac{b + \omega_1}{b - \omega_1} \right] \quad (3)$$

If the phase is known at two frequencies within the measured spectrum, that is,  $\theta(c)$  and  $\theta(d)$  are known for  $a < c < d < b$ , then the constants A and B can be evaluated.



From the computed phase given by Eq. (1) and the measured reflectance, the optical constants can subsequently be obtained from:

$$n(\omega) = \frac{1 - R(\omega)}{1 + R(\omega) - 2 \sqrt{R(\omega)} \cos \theta(\omega)} \quad (4)$$

and:

$$k(\omega) = \frac{-2 \sqrt{R(\omega)} \sin \theta(\omega)}{1 + R(\omega) - 2 \sqrt{R(\omega)} \cos \theta(\omega)} \quad (5)$$

The Kramers-Kronig process is shown schematically in Fig. 1.

The drawback of this method is the requirement that  $\theta(c)$  and  $\theta(d)$  be known, for example, from independent ellipsometry measurements.

#### B. Fourier Series Algorithm

Another approach comparable in accuracy but offering faster computation times than the Kramers-Kronig method is to use a Fourier Series algorithm.<sup>4</sup> As outlined in Fig. 2,  $R(\omega)$  can be transformed to

$$\bar{R}(\Omega) = \left[ \frac{1}{2R(\omega)} \frac{dR(\omega)}{d\omega} \right]_{\omega = -\cot \frac{\Omega}{2}} \quad (6)$$

which can be expressed in terms of a Fourier sine series as

$$\bar{R}(\Omega) = \sum_{m=1}^{\infty} a_m \sin(m\Omega) \quad , \quad (7)$$

from which we have

$$a_m = \frac{2}{\pi} \int_{-\pi}^0 \bar{R}(\Omega) \sin(m\Omega) d\Omega \quad . \quad (8)$$

Then the phase can be written as

$$\theta(\omega) = \sum_{m=1}^{\infty} a_m \left[ \omega^{-2^{m-1}} I_m(\omega) - \sum_{k=1}^{[m/2]} \beta_{km} I_{m-2k}(\omega) \right] \quad , \quad (9)$$

where

$$[m/2] \equiv \begin{cases} 1/2 m & \text{for } m \text{ even} \\ 1/2 (m-1) & \text{for } m \text{ odd} \end{cases} \quad , \quad (10)$$

$$\beta_{km} = (-1)^k \binom{m}{k} 2^{m-2k-1} \binom{m-k-1}{k-1} \quad , \quad (11)$$

and

$$I_j(\omega) = \int_0^{\omega} \left( \frac{\omega^2 - 1}{\omega^2 + 1} \right)^j d\omega \quad . \quad (12)$$

In practice, it is hoped the sum over  $m$  can be truncated to some finite maximum in Eq. (9). The phase as so expressed is very sensitive to the coefficients  $a_m$  computed by Eq. (8). However, once  $\theta(\omega)$  is computed, the optical constants again follow from relationships (4) and (5). Although this technique requires no extrapolation in contrast to the Kramers-Kronig method, the transformation given by Eq. (6) necessitates  $R(\omega)$  be known for  $0 < \omega < \infty$ . In fact, the problem is accentuated by the extreme nonlinearity of transformation Eq. (6). The following table which maps  $\omega$  into  $\Omega$  illustrates the problem:

$\omega$	$\Omega$
0	$-\pi$
0.414	$-3\pi/4$
1	$-\pi/2$
2.414	$-\pi/4$
$\infty$	0.

Thus, the low-frequency reflectance measurements are drastically emphasized in the calculation of  $a_m$  (Eq. 8). For typical spectral measurements, a bandwidth  $a < \omega < b$  corresponds to an inordinately compressed spectrum, making accurate computation of  $a_m$  infeasible.

### C. Oscillator Matching Technique

A third method of predicting optical constants from measured reflectance data is to use an oscillator matching technique.<sup>5</sup> The

program logic flow is diagramed in Fig. 3. In this case, the dielectric constants are calculated as a sum of Lorentzian oscillators given by:

$$\epsilon_1(\omega) = \epsilon_\infty + \sum_{j=1}^N \frac{s_j [1 - (\frac{\omega}{\omega_j})^2]}{[1 - (\frac{\omega}{\omega_j})^2]^2 + (\Gamma_j \frac{\omega}{\omega_j})^2} \quad (13)$$

and

$$\epsilon_2(\omega) = \sum_{j=1}^N \frac{s_j (\Gamma_j \frac{\omega}{\omega_j})}{[1 - (\frac{\omega}{\omega_j})^2]^2 + (\Gamma_j \frac{\omega}{\omega_j})^2} \quad (14)$$

where

$\omega_j$  = frequency of  $j^{\text{th}}$  Lorentzian oscillator (arbitrary units)

$\Gamma_j$  = oscillator linewidth normalized by  $\omega_j$

$s_j$  = oscillator strength normalized by  $\omega_j^2$

$\epsilon_\infty$  = high frequency dielectric constant.

Starting with initial guesses for the above  $3N+1$  parameters, the optical constants can then be expressed in terms of the dielectric constants as:

$$n(\omega) = \{1/2[(\epsilon_1^2(\omega) + \epsilon_2^2(\omega))^{1/2} + \epsilon_1(\omega)]\}^{1/2} \quad (15)$$

and

$$k(\omega) = \varepsilon_2(\omega)/2n(\omega) \quad , \quad (16)$$

from which a computed reflectance can be obtained:

$$R_c(\omega) = \frac{[1 - n(\omega)]^2 + k^2(\omega)}{[1 + n(\omega)]^2 + k^2(\omega)} \quad . \quad (17)$$

$R_c(\omega)$  is then compared with the measured reflectance data using a  $\chi^2$  convergence criteria. A Davidon linear minimization algorithm<sup>6</sup> is used to improve the values of the oscillator parameters, and Eqs. (13)-(17) are repeated until convergence is achieved. The greatest difficulty with this technique is its sensitivity to the initial guesses for the unknown oscillator parameters.

#### D. Approach

The three identified optical constants predictor algorithms are applied to optical spectra to determine the sensitivity of the predicted values of  $n(\omega)$  and  $k(\omega)$  to erroneous or incomplete knowledge of the measured reflectance. Two cases are considered. First, an analytical expression for  $R(\omega)$  is chosen that is defined for all  $0 < \omega < \infty$  and that can be solved exactly by either the Fourier series or the Kramers-Kronig methods. The exact solutions

for  $n(\omega)$  and  $k(\omega)$  are then compared with the results for  $R(\omega) \pm 1\%$ . The accuracy of the computer program versions are then tested by comparing their results with the analytical case.

A second approach is based on amplitude spectroscopy reflectance measurements of potassium bromide between 55-250  $\mu\text{m}$ .<sup>7</sup> An asymmetric Michelson interferometer uses a Fourier-transform technique to directly provide both the magnitude and the phase of reflectance measurements. Equations (4) and (5) are then used to obtain  $n(\omega)$  and  $k(\omega)$  directly from measurement. The Kramers-Kronig and oscillator matching techniques are then applied to this data assuming  $\pm 1\%$  and  $\pm 2\%$  error bands on the magnitude of the reflectance measurement. The results are then compared to the experimentally determined optical constants.

## II. RESULTS

### A. Analytical Reflectance

To gain insight into the sensitivity of optical constants to errors in reflectance measurements, an analytical form for the reflectance was chosen as

$$R(\omega) = e^{-2/(1+\omega^2)} . \quad (18)$$

This is similar to the case studied by King<sup>4</sup>, except that Eq. (18) provides more realistic bounds on  $R(\omega)$  (see Fig. 4).

Both the Kramers-Kronig and the Fourier series methods are based upon calculating the phase from the measured reflectance. Direct application of either Eqs. (1) or (6)-(9) results in

$$\theta(\omega) = -\omega/(1+\omega^2) , \quad (19)$$

or, expressing the phase in terms of the reflectance,

$$\theta(R) = \frac{\ln R \sqrt{-\left(\frac{2}{\ln R} + 1\right)}}{2} . \quad (20)$$

It is now possible to perturb the reflectance given in Eq. (18) and observe the effect upon the calculation of phase from Eq. (20) and the resultant optical constants. Changing  $R(\omega)$  by  $\pm 1\%$  gives rise to errors in the phase that range from minimum disagreement near  $\omega = 1$  to increasingly poor accord at higher frequencies (Fig. 4). Using Eq. (4) to calculate  $n(\omega)$ , the offset in reflectance is balanced by the corresponding phase inaccuracy. Figure 5 shows that  $n(\omega)$  is little affected by errors in  $R(\omega)$ . However,  $k(\omega)$  is more strongly dependent upon the phase (Eq. 5) and exhibits dramatic sensitivity (Fig. 6) to variations in the reflectance at frequencies where the phase is affected as well. Errors as high as 30%-40% in  $k(\omega)$  occur at the higher frequencies shown. Thus, even when the reflectance is known for all frequencies ( $0 < \omega < \infty$ ) and the phase can be expressed analytically rather than computed numerically, small errors in reflection values can severely distort the predicted optical constants.

#### B. Sensitivity Analysis of Analytical Reflectance

It is interesting to compare the analytically known optical constants derived from Eqs. (18) and (19) with those predicted by the various numerical algorithms. For the case being considered, the dielectric constants behave as

$$\epsilon_1(\omega) \sim 4(1-\omega^2) \quad , \quad (21)$$



and

$$\varepsilon_2(\omega) \sim 4\omega \quad (22)$$

Since the dependence of the dielectric constants implies a Lorentzian oscillator of infinite linewidth and strength, the oscillator matching technique was not used. A spectral reflectance curve was generated using Eq. (18) for  $0 < \omega < 10$  and input to the Fourier series and Kramers-Kronig computer programs. The optical constants predicted by the numerical techniques are compared with those obtained using Eqs. (15) and (16).

The plots in Figs. 7 and 8 clearly illustrate the care that must be exercised when using the Fourier series approach. The discrepancy between the predicted and the actual values of  $n(\omega)$  and  $k(\omega)$  is due to the inaccuracy in the computation of the phase. This in turn is a result of the sensitivity of the phase to the computed Fourier coefficients  $a_m$ . In the present case, the error in  $a_m$  is attributable to the numerical implementation of the transformation in Eq. (6). For typical spectral reflectance measurements within a finite bandwidth (especially at higher frequencies), even greater imprecision is introduced.

Results of the Kramers-Kronig analysis are more encouraging. In general, the greatest deviations are expected at the ends of the predicted spectrum due to the estimation of  $\alpha(\omega_i)$  and  $\beta(\omega_i)$  in Eq. (1). Here, the lower frequency limit is  $a = 0$  so

that  $\alpha(\omega_1) = 0$  and excellent agreement is evident in the low-frequency range. Behavior at the upper frequency end is slightly sensitive to the two values  $\omega = c, d$  where the phase is assumed to be independently known. For this case,  $\theta(c=.5) = -.4$  and  $\theta(d=7) = -.14$  were used.

For a particular spectral measurement, the accuracy in the ultraviolet region will in general be different from that at infrared wavelengths. Thus, the analysis of data subjected to a constant offset in reflectance ( $\pm 1\%$ ) may not be indicative of the results encountered for realistic measurements. To investigate the sensitivity of the numerical techniques to this effect, the reflectance spectrum given by Eq. (18) was differentially offset by  $\pm 1\%$  in the UV and  $\mp 1\%$  in the IR. The reflectance was assumed unchanged for visible ( $.3 \mu\text{m} < \lambda < .8 \mu\text{m}$ ) wavelengths. These spectra were then input to the Kramers-Kronig program and the results were compared to those associated with constant, full-spectrum offsets of  $\pm 1\%$ .

Errors in the calculation of  $k(\omega)$  were identical for both the differential and the full-spectrum offsets in reflectance. The calculated errors in  $n(\omega)$  in the differential-offset case were bounded by those resulting in the constant-offset case for infrared wavelengths. Slight perturbations were introduced in  $n(\omega)$  near the visible (cross-over) part of the spectrum by the differential-offset analysis that were not present in the constant-offset treatment. Based on these results, it was concluded that the constant-offset approach represented typical error bounds encountered in realistic measurements.

### C. Measured Reflectance

As further validation and investigation the numerical algorithms were applied to published data. Bell<sup>7</sup> measured both the magnitude and the phase of reflectance from potassium bromide using an amplitude spectroscopy technique. The reflectance amplitude curve was digitized and input to the Kramers-Kronig and the oscillator matching routines. Because of previously discussed problems with finite bandwidths, the Fourier series approach was not considered.

The oscillator matching technique fits a curve to the measured reflectance data. Figure 9 shows the success of this procedure using four Lorentzian oscillators having the converged upon parameters:

$$\epsilon_{\infty} = 2.429$$

i	$\omega_i$	$\Gamma_i$	$s_i$
1	74.9	0.833	1.379
2	114.0	2.066	0.035
3	152.8	0.093	0.270
4	168.0	0.041	0.010

Additional oscillators do not improve the agreement between the observed and the calculated reflectance in the evaluation of  $\chi^2$ . Comparison of the corresponding predicted optical constants with

those obtained from the measured reflectance and phase reveals excellent accord (Fig. 10). Agreement between calculated and experimentally determined values of  $n(\omega)$  and  $k(\omega)$  is evident except at the respective peaks, which are slightly overestimated by the fitting algorithm.

The accuracy of the Kramers-Kronig approach is a function of how well the phase is computed from the measured reflectance. Using the data given in Fig. 9 (solid curve) as input to the Kramers-Kronig program, the phase was calculated and plotted as Fig. 11. Best agreement was found in the mid-spectral range; discrepancies at the upper and lower frequency limits may be expected, as discussed earlier. As was the case with the oscillator matching technique, the predicted optical constants conform very well to those derived from measurements (Fig. 12) for the Kramers-Kronig analysis.

#### Sensitivity Analysis of Measured Reflectance

Having validated the precision of the two optical constants predictor algorithms under consideration, it is now possible to investigate the sensitivity of these techniques to errors in the measured reflectance. For comparison purposes, the results displayed in Figs. 10 and 12 were taken as the baseline cases. The numerical analyses were repeated for each algorithm with the reflectance data (Fig. 9) offset by  $\pm 1$  and  $\pm 2\%$ . Differences

between these results and the baseline are then indicative of the accuracy required in the reflectance measurement to assure a given desired precision in the predicted optical constants.

Using the oscillator matching technique, significant errors in the prediction of the optical constants are introduced by error in the measured reflectance. Figure 13 is demonstrative of this effect. At the lower end of the measured frequency band, a 1% offset in reflectance results in a 3% change in  $n(\omega)$ . The same error produces a 9% variation of  $k(\omega)$  at the higher frequencies. Both optical constants exhibit a 5% inaccuracy at their respective peaks. These effects appear to be linear; a 2% displacement in reflectance doubles the errors in the optical constants quoted above; a 3% offset increases the errors threefold.

Unlike the results for the oscillator matching technique, application of the Kramers-Kronig method to  $R(\omega) \pm 1\%$  does not produce symmetric perturbations in the optical constants (Fig. 14). Poorest agreement is found at those frequencies where the measured reflectance is small. However, the magnitude of the errors in the optical constants is roughly the same as those resulting from the oscillator matching approach.



### III. SUMMARY

The sensitivity of the calculation of phase to the numerical implementation of transformation Eq. (6) and the limits of integration given in Eq. (8) render the Fourier series technique impractical as an optical constants predictor method for typical measurements of interest. Similar extrapolation problems inherent to the Kramers-Kronig approach are avoided by the present formulation which approximates the contribution to the phase by the wings using Eqs. (2) and (3). This approximation introduces less error than that associated with the measured data.<sup>8</sup> Application of this method to both analytical and measured reflectance data shows that the overall inaccuracy of the computed optical constants is considerably greater than the error bounds of the reflectance. This is contrary to the pervasive assumption found in the literature that the optical constants can be calculated to the same or better precision as that evidenced in the original measurement. These results are not, however, as discouraging as those reported by Zanzucchi<sup>9</sup>, who asserted that a 1% error in reflectance can produce a 30% inaccuracy in the optical constants.

Errors in the optical constants associated with the oscillator matching analysis are equivalent to those achieved by the Kramers-Kronig technique. Extreme care must be exercised with either approach, both in measuring the reflectance and in choosing the extrapolation algorithm for the Kramers-Kronig technique and

in choosing the initial oscillators for the oscillator matching method.

The oscillator matching method should have the additional advantages of: 1) the physical insight into atomic and molecular structure that it provides for infrared measurements; and 2) the ability to determine optical constants of thin films as well as bulk materials.



#### IV. REFERENCES

1. G. Leveque, J. Phys. C - Solid State Phys. 10, 4877 (1977).
2. G. Andermann, A. Caron, and D. A. Dows, J. Opt. Soc. Am. 55, 1210 (1965).
3. D. M. Roessler, Brit. J. Appl. Phys. 16, 1119 (1965).
4. F. W. King, J. Phys. C - Solid State Phys. 10, 3199 (1977).
5. H. W. Verleur, J. Opt. Soc. Am. 58, 1356 (1968).
6. P. Eykhoff, "System Identification: Parameter and State Estimation", Wiley Interscience, 164 (1974).
7. E. E. Bell, Aspen International Conference on Fourier Spectroscopy, 1970. 71 (1971).
8. D. M. Roessler, Brit. J. Appl. Phys. 17, 1313 (1966).
9. P. J. Zanzucchi, et al., J. Electrochem. Soc.: Solid-State Science and Technology. 125, 299 (1978).



## Figure Captions

- FIG. 1. Kramers-Kronig method.
- FIG. 2. Fourier series algorithm.
- FIG. 3. Oscillator matching technique.
- FIG. 4. Analytical reflectance and phase.  $R = e^{-2/(1 + \omega^2)}$  .
- FIG. 5. Sensitivity of  $n$  to errors in  $R$ .  $R = e^{-2/(1 + \omega^2)}$  .
- FIG. 6. Sensitivity of  $k$  to errors in  $R$ .  $R = e^{-2/(1 + \omega^2)}$  .
- FIG. 7. Predicted and actual values of  $n$ .
- FIG. 8. Predicted and actual values of  $k$ .
- FIG. 9. Reflectance of potassium bromide -- oscillator matching technique.
- FIG. 10. Optical constants of potassium bromide -- oscillator matching technique.
- FIG. 11. Phase of potassium bromide -- Kramers-Kronig method.
- FIG. 12. Optical constants of potassium bromide -- Kramers-Kronig method.
- FIG. 13. Sensitivity of optical constants to change of  $\pm 1\%$  in  $R$  for potassium bromide -- oscillator matching technique.
- FIG. 14. Sensitivity of optical constants to change of  $\pm 1\%$  in  $R$  for potassium bromide -- Kramers-Kronig method.



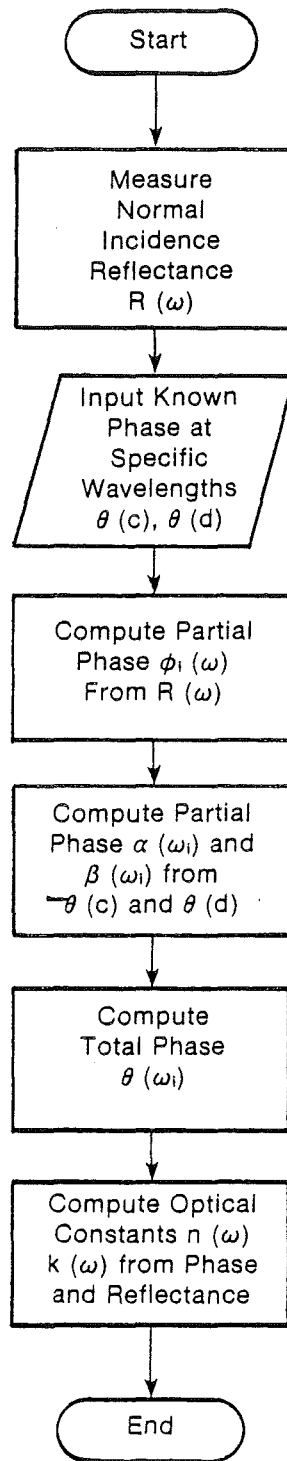


Figure 1

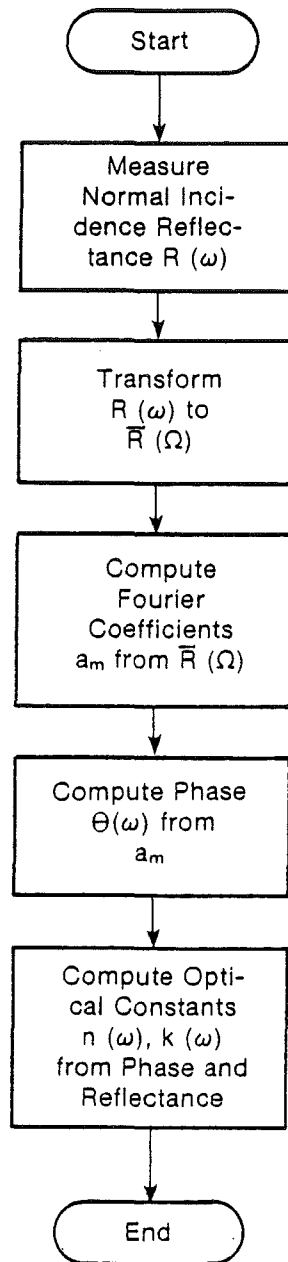


Figure 2

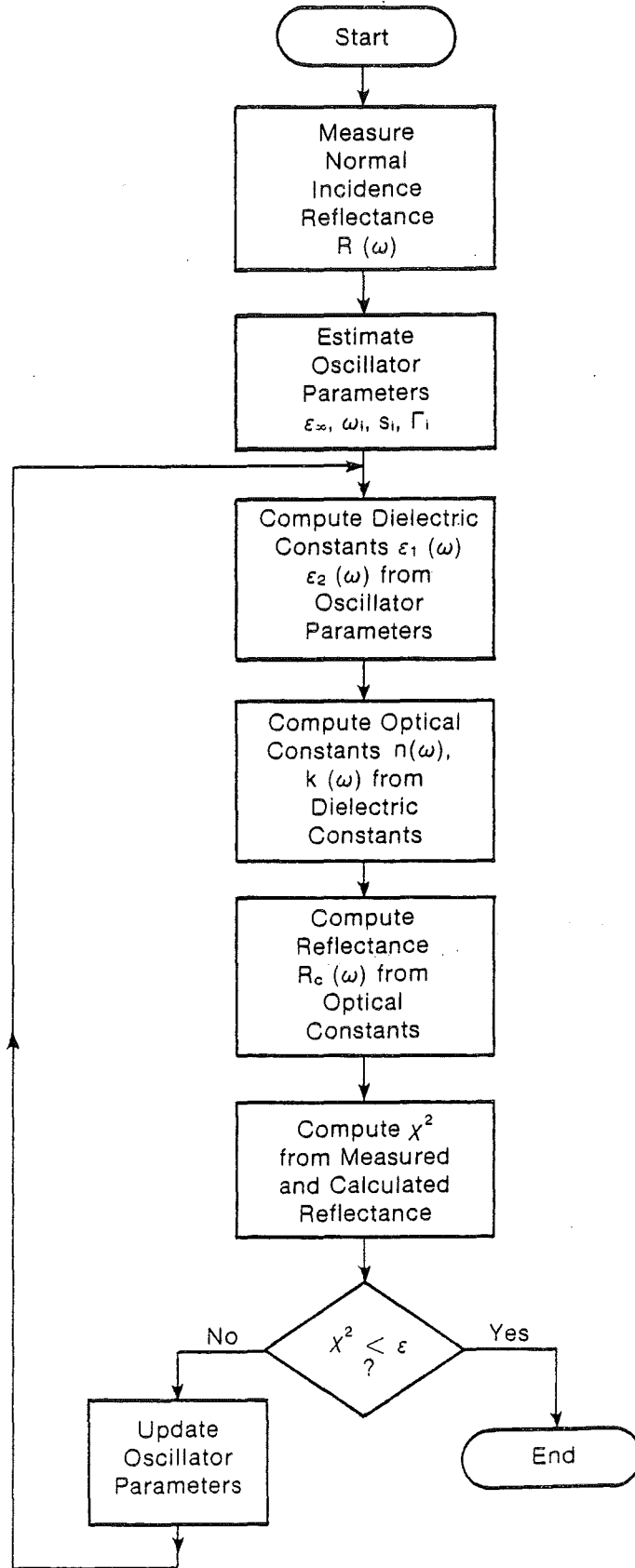


Figure 3

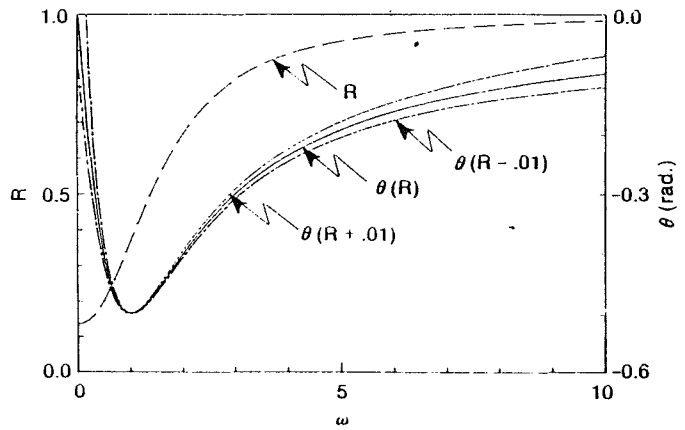


Figure 4

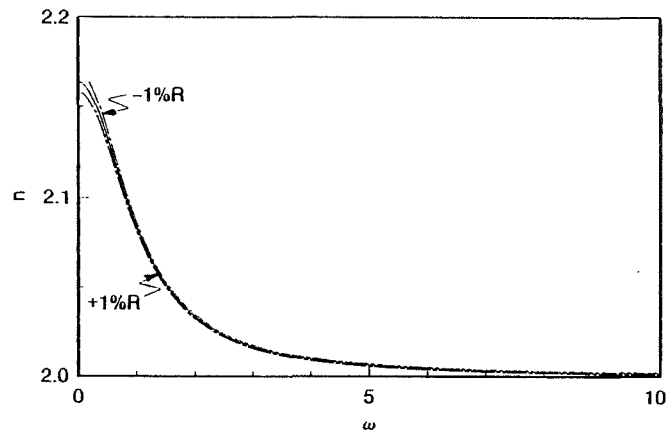


Figure 5

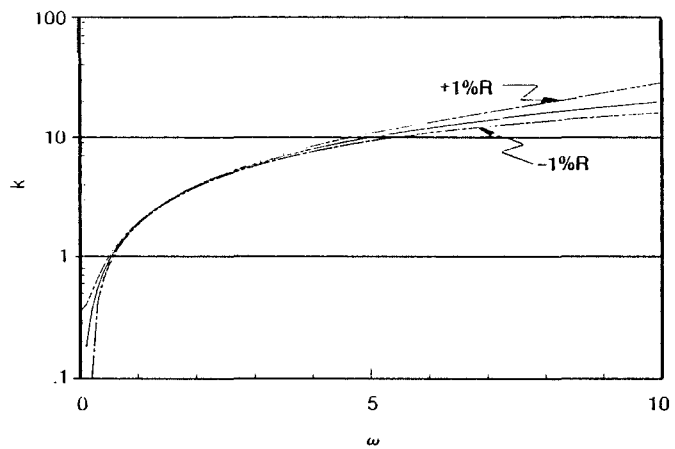


Figure 6

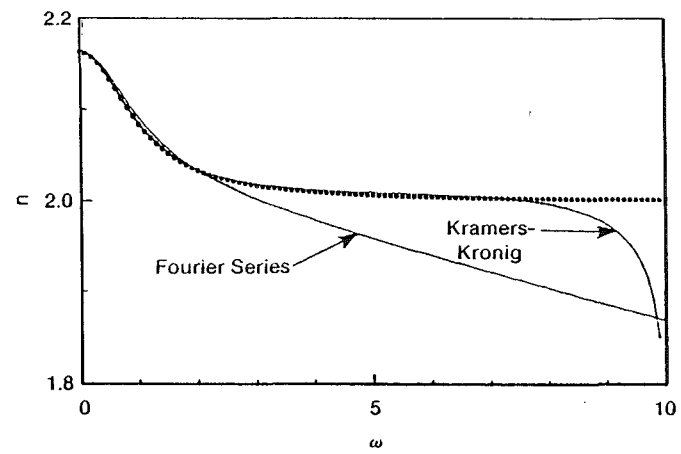


Figure 7



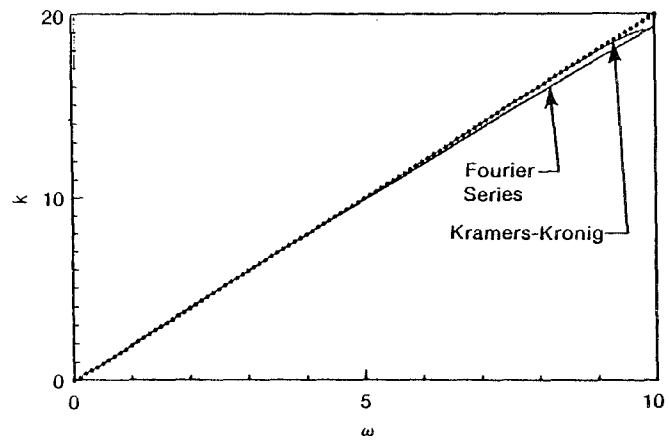


Figure 8.

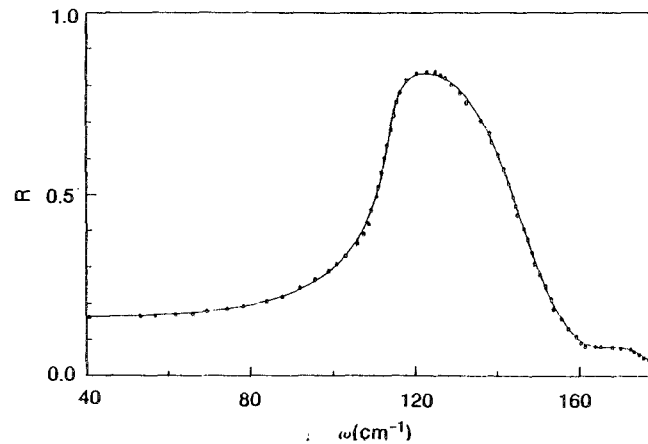


Figure 9

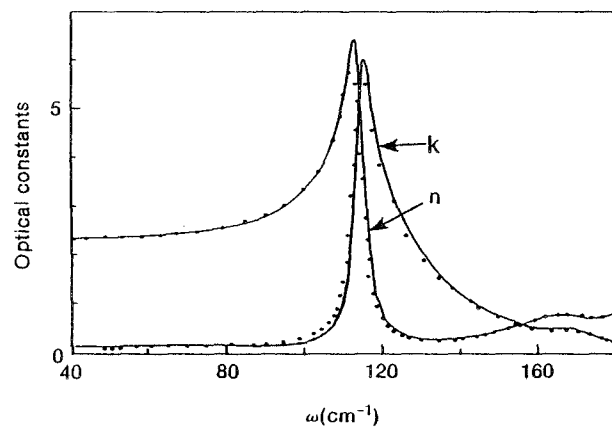


Figure 10

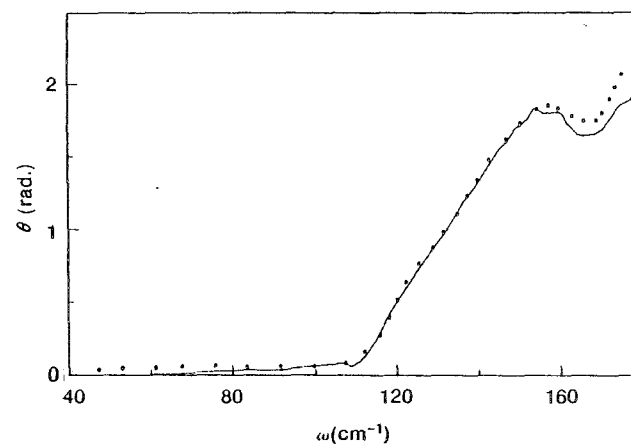


Figure 11

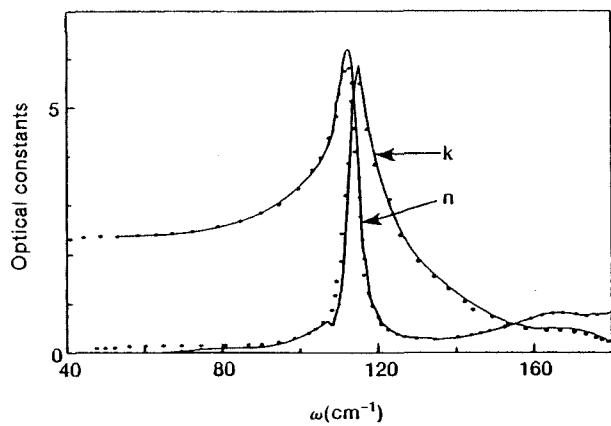


Figure 12

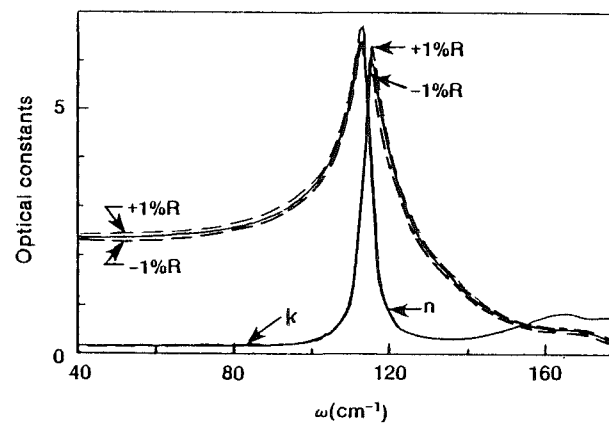


Figure 13

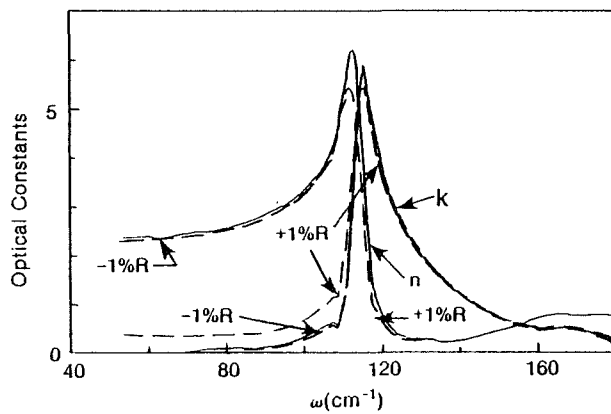


Figure 14