Overview

This work focuses on using variations of the Frank-Wolfe (FW) algorithm [1] for solving unit commitment problems with high volumes of demand responsive loads on the power grid. We present a formulation of the unit commitment problem with demand responsive loads and subsequently show, through reformulation and relaxations, that variations of the Frank-Wolfe algorithm can be used to determine time series decisions for demand responsive loads. Using computational experiments on the IEEE Reliability Test System, we demonstrate that the timeseries of demand responsive load decisions obtained through our approach is near optimal and describe how large-scale parallel implementations of our approach can achieve a high degree of computational efficiency.

Problem Formulation and Structure

Like [2], we present a generic version of the UC problem, but with a set of demand responsive agents and their respective flexible loads, which we call the UC DR problem. This is a standard DCOPF power flow formulation where we add variables to represent demand responsive loads. The sets represent constraints on a demand responsive load.

\[
\min_{p, x, y} \sum_{t \in T} \left( c_f(p_t, x_t) + \sum_{\omega \in \Omega} c_g(y_{\omega t}) \right) + \sum_{t \in T} c_i(x_t, y_t) \\
\text{s.t.} \quad \sum_{p \in P_t} p_t + \sum_{e \in E_t} f_t - \sum_{e \in E_{\text{gen}}(t)} f_t - \sum_{e \in E_{\text{flow}}(t)} f_t = d_t = \sum_{a \in A_t} y_t^a + y_t^{\gamma t} - y_t^{\delta t} \quad \forall t \in T \\
E_{x_t} \leq f_t \leq F_x, \quad \forall e \in E \\
B_t(\theta_t - \theta_{t-1}) = f_t, \quad \forall e \in E, t \in T. \\
(p_{a t}, x_{a t}) \in \Pi_a, \quad \forall y \in G \\
(s_{a t}, z_{a t}) \in \Pi_a, \quad \forall a \in A 
\]

Solution Approach

Due to problem structure of the UC DR problem we can solve a relaxed version using variations of the FW algorithm. These relaxations can be used to approximate optimal flexible load profiles. They can also be used in a branch and bound scheme.

Our Approach:
1) Solve a relaxed version of the UC using a variation of the FW algorithm
2) Project each onto \( \Pi_a \), which we denote as \( s_a \)
3) Solve UC DR with \( s_a \) values fixed

Problem Reformulation

To apply a variation of the FW algorithm, we must reformulate the UC DR problem and relax certain constraints to obtain a relaxed version suitable for algorithms.

The formulation presented here is equivalent to the UC DR, however it casts the problem as a function of the feasible loads, with local constraints on each flexible load.

We must ensure our objective function is convex, as well as our constraints. Towards this end we consider a relaxed version of \( \ell_q(d) \) and use the convex hull of \( \Pi_a \).

Problem Relaxation

When all integer variables in (16) are relaxed it is well known that \( H \) becomes a convex piece-wise linear function of \( d \), see[3],[4]. We denote the resulting convex function as \( \tilde{H} \). We consider \( \Pi_a \) in the constraints to cast the relaxation.

This relaxation has a convex L-Lipschitz continuous objective function and a convex compact feasible region. This problem we solve with a parallelizable variation of the FW algorithm.

Proposed Algorithm (FW-Sample)

We apply a modified version of the FW based algorithm presented in [5], which minimizes the max over all sub-gradients of points in small neighborhood around the current iterate. Instead, we use a sample-based set \( \Pi_{\text{sample}} \) that provides only a rough approximation of all nearby sub-gradients. However, our results indicate that using this set results in good quality steps and is computationally efficient.

Highlighted in red are the parts of the algorithm that can be warm-started based on previous iterations or can be done in parallel. The other parts of the algorithm are extremely cheap to compute.

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References