Hybrid RANS-LES of the Atmospheric Boundary Layer for Wind Farm Simulations

Preprint

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Hybrid RANS-LES of the Atmospheric Boundary Layer for Wind Farm Simulations

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Wind farm simulations often do not accurately represent wake-atmospheric boundary layer (ABL) interactions, blade boundary layer (BL) dynamics, and turbine-turbine interactions. In this work, we use Active Model Split (AMS), a new hybrid Reynolds-Averaged Navier Stokes (RANS)-large eddy simulation (LES) model, which is well suited to capture these effects because the model can (i) accurately simulate the ABL with the Coriolis effect, (ii) is accurate in adverse pressure gradients such as those near wind turbine blades, and (iii) has sufficiently low computational cost to simulate multiple turbines while resolving the blade BL. For simplicity and consistency we develop AMS to be used throughout the domain rather than in a zonal method. We implement our work in the massively parallel flow solver, Nalu-Wind, so that our model can access the compute resources needed for blade-resolved simulations of multiple wind turbines. To accomplish these aims, we modify the baseline AMS by changing the RANS contribution to SST $k - \omega$ with a length scale limiter, adding the Coriolis effect, and developing an appropriate wall treatment. We show that AMS of the ABL with the Coriolis effect matches LES reference results better than those obtained with RANS. We describe our plans to add buoyancy effects and wind turbines to our AMS simulations.

I. Nomenclature

HF	=	High Fidelity
ABL	=	Atmospheric Boundary Layer
BL	=	Boundary Layer
RANS	=	Reynolds-Averaged Navier Stokes
LES	=	large-eddy simulation
HRL	=	Hybrid RANS-LES

AMS	= Active Model Split
SST $k - \omega$	= Shear Stress Transport $k - \omega$
x_1, x_2, x_3	= streamwise, spanwise, and wall-normal directions of flow
u_1, u_2, u_3	= wind velocity in the x_1, x_2, x_3 directions
p	= pressure
k	= turbulent kinetic energy
ω	= specific dissipation rate
ε	= total dissipation rate
ϵ_{ijk}	= Levi-Civita tensor
ρ	= density
ν	= kinematic viscosity
μ	= dynamic viscosity
ν^e	= LES eddy viscosity
ν^R	= RANS eddy viscosity
τ_{ij}^e	= fluctuating subgrid stress = LES subgrid stress model
τ_{ij}^m	= mean subgrid stress
τ_{ij}^R	= RANS stress model
F_i	= AMS active forcing
F_c	= Coriolis force
F	= SST $k - \omega$ blending function
$\langle \cdot \rangle$	= mean quantity
$\cdot^>$	= resolved fluctuation
$\cdot^<$	= unresolved fluctuation
$\bar{\cdot} = \langle \cdot \rangle + \cdot^>$	= resolved quantity
$(\cdot) = \bar{\cdot} + \cdot^< = \langle \cdot \rangle + \cdot^> + \cdot^<$	= total quantity
$\tilde{\beta}$	= AMS mean subgrid stress scaling parameter
β	= SST $k - \omega$ model parameter
r	= AMS resolution adequacy indicator
T^R	= RANS time scale
l_t	= mixing length

II. Introduction

Wind energy production has been growing rapidly and must continue to accelerate to meet nations' targets for reducing greenhouse gas emissions. For example, the United States has installed 10-20 GW of wind each of the last five years and needs to install 20-35 GW of wind each year until 2050 to meet its targets of 100% clean electricity by 2030 and net-zero greenhouse gas emissions by 2050 [1]. Improving the efficiency of wind farms could reduce the cost of meeting these goals by over \$1 billion annually [2] by improving wind turbine design and wind farm siting, layout, and control. A key tool to improve the efficiency of wind farms is a high fidelity (HF) model of wind turbines in a realistic atmospheric boundary layer (ABL) that resolves the boundary layer (BL) over the wind turbine blades [3]. This HF model would make it possible determine which physics are important to include in the simpler, faster models used for wind farm optimization and control. It would also help tune these models for different wind farm operation conditions.

Options for this HF wind farm model include Reynolds-Averaged Navier Stokes (RANS) models, large eddy simulations (LES), and hybrid RANS-LES (HRL) models. RANS models have sufficiently low computational cost to simulate multiple turbines while resolving the blade BLs, but current RANS approaches poorly capture important flow dynamics, such as wake-ABL and turbine-turbine interactions [4]. While LES models can theoretically capture these dynamics, the associated computational cost makes blade-resolved simulations of multiple wind turbines with LES unaffordable [5]. For example, Ref. [6] used LES to simulate one wind turbine, but found that the grid resolution needed for accurate solutions was unaffordable for simulating multiple turbines. HRL models seek to combine the low computational cost of RANS with the high accuracy of LES, making them well suited to HF modeling of multiple wind turbines [3, 6, 7]. In Ref. [7], an HRL approach was used to fully capture a blade-resolved simulation of a single turbine in a realistic ABL.

The overarching motivation for this work is to develop an HRL model which is well suited to simulate multiple wind turbines in a realistic ABL. We seek an HRL model which is different from other approaches used to simulate wind

turbines in ABL flows in two main ways. First, for simplicity and to limit the use of ad hoc blending, we use a consistent model everywhere in the domain. This avoids the use of empirical blending between a distinct ABL model, primarily run as LES, and a near-blade model, using RANS or HRL, such as the blending in Ref. [7]. Second, this model is implemented in a massively parallel codebase, Nalu-Wind [3], so that it can take advantage of petascale and future exascale machines to run accurate and large scale simulations of wind turbines. To our knowledge, no HRL model that meets these two criteria has been used to simulate the ABL including the Coriolis effect. To meet this goal, in this work we focus on developing an HRL to accurately model the ABL including the Coriolis effect. In future work we will add buoyancy effects and wind turbines to our simulations.

We use the Active Model Split (AMS) framework [8], which is a recently developed HRL approach that is well suited for use as a non-zonal model in a blade-resolved ABL wind farm simulation. Many other HRL models exist, as surveyed in Refs. [8–10]. Some of them, such as Detached Eddy Simulations (DES) and Simplified Improved Delayed DES (SIDDES) [11, 12], have been used to model the ABL. We use AMS here, because of the systematic way it addresses difficulties faced in previous HRL modeling efforts, such as modeled-stress depletion, ensuring RANS and LES models are applied only for their intended purposes, and removing the need for ad-hoc blending in-between RANS and LES [8]. As a key part of its design, AMS can use any underlying LES model. To simplify the AMS construction, ideally the underlying LES model will be zero in expectation. Therefore, we choose to use the M43 model [13] used in the development of AMS [8] as our underlying LES model. AMS can also use any RANS model that has a transport equation for the turbulent kinetic energy, k . This includes two equations RANS models such as $k - \varepsilon$ and $k - \omega$.

For the choice of the underlying RANS model to use in the AMS model framework, we seek to choose one that will be accurate in adverse pressure gradients, such as those near wind turbine blades, as well as in the freestream of the ABL. The $k - \varepsilon$ model is often used to simulate the ABL, as in Ref. [14]. However, it is inaccurate in regions with adverse pressure gradients [15, 16]. $k - \omega$ is more accurate in adverse pressure gradients, but is overly sensitive to the freestream value of ω [15, 16]. Therefore, the SST $k - \omega$ model, which blends between $k - \varepsilon$ (away from walls) and $k - \omega$ (near walls), using each model where it is accurate, is a more ideal fit. Previous work suggests that when simulating the ABL with Coriolis, RANS eddy viscosity models require a length scale limiter [12, 14, 17, 18]. Therefore, we add a length scale limiter, presented in Ref. [19] and described in detail in a future publication, in our SST $k - \omega$ model.

In Section III, we introduce the basic AMS framework and the modifications to the underlying RANS and LES models needed to use it to simulate the ABL. An emphasis is placed on modeling changes to address the Coriolis effect and to develop an appropriate wall treatment. In addition, we briefly discuss Nalu-Wind and how AMS was implemented efficiently in a massively parallel code. In Section IV, results are compared between AMS, RANS, LES, and experimental measurements of the ABL including the Coriolis effect and the effect of the length scale limiter in the SST $k - \omega$ AMS simulations is studied. Finally, we summarize our findings and describe future work in Section V.

III. Active Model Split for Atmospheric Boundary Layer Simulations

Here we describe the Active Model Split (AMS) hybrid RANS-LES (HRL) model and modify it for simulations of the Atmospheric Boundary Layer (ABL), including the Coriolis effect. Sections III.A and III.B introduce AMS and the large eddy simulations (LES) model used in AMS, respectively. We then explain how to modify AMS to simulate the ABL by: choosing an appropriate Reynolds-Averaged Navier Stokes (RANS) contribution to AMS (Section III.C), adding the Coriolis effect (Section III.D), handling the wall treatment for the ABL (Section III.E), and selecting appropriate model constants (Section III.F). Finally, Section III.G describes the massively parallel implementation of AMS.

A. Active Model Split Formulation

For convenience, we summarize AMS as presented in Ref. [8]. The momentum equation for AMS is

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} + \frac{\partial}{\partial x_j} (\tau_{ij}^m + \tau_{ij}^e) + F_i, \quad (1)$$

where x_i is the distance from the origin in the i -th direction, t is time, $\bar{u}_i$ is the resolved velocity, $\bar{p}$ is the resolved pressure, ν is the kinematic viscosity, ρ is the density, τ_{ij}^m is the mean subgrid stress, τ_{ij}^e is the fluctuating subgrid stress, and F_i is an active forcing term. The resolved velocity is $\bar{u}_i = \langle u_i \rangle + u_i^>$, where $\bar{\cdot}$ is a resolved quantity, $\langle \cdot \rangle$ is a mean, and $\cdot^>$ is a resolved fluctuation. The mean, $\langle \cdot \rangle$, is defined at the end of this section, with Eqns. 4 and 5. AMS, like LES, solves for the resolved velocity which does not include the unresolved (subgrid) fluctuations, $u_i^<$, where $\cdot^<$ is an

unresolved fluctuation. The total velocity is $u_i = \langle u_i \rangle + u_i^> + u_i^<$.

The momentum equation for AMS, Eqn. 1, has three key characteristics. First, AMS splits the subgrid stress into two terms, one intended to capture the mean subgrid stress, τ_{ij}^m , and one intended to capture the fluctuating subgrid stress, τ_{ij}^e , which is responsible for energy transfer from resolved to unresolved scales. The mean subgrid stress, τ_{ij}^m , is a scaled RANS stress model. The fluctuating subgrid stress, τ_{ij}^e , is an LES subgrid stress model acting only on the resolved fluctuations. This split is made so RANS stress models can be used for their intended purpose: capturing the effect of fluctuations on the mean flow behavior, and LES subgrid stress models can be used for their intended purpose: transferring energy from resolved to unresolved scales. Furthermore, the split allows the mean subgrid stress and energy transfer to be modeled independently. Finally, since both the RANS and LES subgrid stress models are active everywhere in the domain, the splitting method removes the need for RANS/LES zones or functions to blend in-between RANS and LES models.

In regions where AMS resolves some turbulent kinetic energy, the mean subgrid stress (modeled by the RANS stress model) must be reduced correspondingly. This is done by scaling the RANS contribution to AMS, τ_{ij}^R :

$$\begin{aligned}\tau_{ij}^m &= \alpha(2 - \alpha)\tau_{ij}^R, \\ \alpha &= \tilde{\beta}^{1.7}, \\ \tilde{\beta} &= \max\left(1 - \frac{k_{res}}{k}, \beta_{lim}\right), \\ k_{res} &= \frac{1}{2}\langle u_i^> u_i^> \rangle,\end{aligned}\tag{2}$$

where k_{res} is the average resolved turbulent kinetic energy and k is the total turbulent kinetic energy found from the RANS k equation. In regions where the grid is unable to support resolved content, $k_{res} \rightarrow 0$ so $\tilde{\beta} \rightarrow 1$, and thus the mean subgrid stress equals the RANS stress model. In regions where the grid supports all turbulent content (DNS limit), $\frac{k_{res}}{k} \rightarrow 1$ so $1 - \frac{k_{res}}{k} \rightarrow 0$ but $\tilde{\beta}$ is limited to maintain reasonable behavior in near-wall regions. This scaling is derived and further explained in Ref. [8].

The second key characteristic of AMS is a resolution adequacy indicator, r . r determines what regions can support additional resolved fluctuations. r takes into account the anisotropy in the grid through the anisotropic grid metric tensor, $\mathcal{M}_{ij}$ and the anisotropy in the turbulence through the subgrid production tensor, $\mathcal{P}_{ij}^{SGS}$, and an approximation of the wall normal velocity variance, $\langle \bar{v}^2 \rangle$, inspired by the Durbin $\langle \bar{v}^2 \rangle - f$ model [20]. Balancing these contributions, the resolution adequacy parameter is:

$$\begin{aligned}r &= \left(\frac{2}{3\tilde{\beta}\langle \bar{v}^2 \rangle}\right)^{3/2} \max_{\lambda} \left(\mathcal{P}_{ik}^{SGS} \mathcal{M}_{kj}\right), \\ \mathcal{M}_{ij} &= \mathcal{J}_{ij}^T \mathcal{J}_{ij}, \\ \langle \bar{v}^2 \rangle &= 5 \frac{v_t^R}{T^R}, \\ T^R &= \frac{l_t}{\sqrt{k}}, \\ l_t &= \frac{k^{1/2}}{\beta^{*1/4}\omega},\end{aligned}\tag{3}$$

where λ denotes the eigenvalues of $\left(\mathcal{P}_{ik}^{SGS} \mathcal{M}_{kj}\right)$, $\mathcal{J}_{ij}$ is the tensor mapping the unit cube to a grid cell, v_t^R is the eddy viscosity from the RANS model discussed in Section III.C, T^R is the time scale from the RANS model, l_t is the mixing length, and β^* is an SST $k - \omega$ model constant. We discuss the choice of time scale, T^R , further in Section IV.D. Ref. [8] provides additional details on the resolution adequacy indicator.

The third key characteristic of AMS is that it includes an active forcing term, F_i , which generates resolved fluctuations in regions of the grid identified by r . Since AMS doesn't reduce the underlying RANS stress until resolved turbulent content is present, the active forcing term introduces initial resolved turbulence in desired regions. This balances the reduction in mean subgrid stress, τ_{ij}^m , with an increase in resolved turbulence, preventing modeled-stress depletion. Ref. [8] describes the exact form of the forcing function.

Calculating several of the terms in AMS, such as k_{res} , requires defining an averaging operation, $\langle \cdot \rangle$. AMS uses a causal time average for $\langle \cdot \rangle$. For a quantity at timestep $n + 1$, ϕ^{n+1} , the causal time average is

$$\frac{\partial \langle \phi^{n+1} \rangle}{\partial t} = \frac{1}{T^{R,n}} \left(\phi^{n+1} - \langle \phi^n \rangle \right), \quad (4)$$

where $T^{R,n}$ is the RANS time scale evaluated at timestep n . Discretizing the causal time average, we see that it's a weighted average of the instantaneous value at the current timestep and the causal time averaged value at the previous timestep

$$\langle \phi^{n+1} \rangle = \frac{\Delta t}{T^{R,n}} \phi^{n+1} + \left(1 - \frac{\Delta t}{T^{R,n}} \right) \langle \phi^n \rangle, \quad (5)$$

where Δt is the timestep size.

B. Fluctuating Subgrid Stress Model: M43 Large-Eddy Simulation

AMS can use any LES model for the fluctuating subgrid stress. In the AMS framework, the LES model acts only on fluctuating quantities and ideally has zero mean subgrid stress, $\langle \tau_{ij}^e \rangle = 0$ [8]. The zero mean subgrid stress for the LES model fully separates the action of the LES and RANS models: the LES model controls only energy transfer and doesn't contribute to the mean subgrid stress. Many eddy viscosity LES models, such as Smagorinsky [21], don't satisfy this requirement. We use the M43 model developed in Ref. [22], which does satisfy this requirement. Furthermore, the M43 model was developed to handle high-aspect-ratio grid cells and the resulting anisotropy in resolution. This is important for our overarching motivation, to apply this model to wind farms with resolved boundary layers. To resolve boundary layers with reasonable computational cost, cells near surfaces are orders of magnitude smaller in the wall-normal direction than the streamwise and spanwise directions. While the LES model contribution to AMS is expected to decrease near the wall, it might still have a non-negligible contribution to cells with anisotropic resolution.

M43 explicitly captures anisotropy by using an anisotropic model for the stress tensor and a tensor for the eddy viscosity:

$$\begin{aligned} \tau_{ij}^e &= \nu_{jk}^e \frac{\partial u_i^>}{\partial x_k} + \nu_{ik}^e \frac{\partial u_j^>}{\partial x_k}, \\ \nu_{ij}^e &= C(\mathcal{M}) \langle \varepsilon \rangle^{1/3} \mathcal{M}_{ij}^{4/3}, \end{aligned} \quad (6)$$

where the coefficient C depends on $\lambda(\mathcal{M})$ and $\langle \varepsilon \rangle$. The total dissipation rate, ε , is calculated from the RANS ω and k .

C. Mean Subgrid Stress Model: Reynolds-Averaged Navier Stokes SST $k - \omega$ Model

AMS can use any RANS model that acts only on mean quantities and has a k transport equation [8]. The mean quantities requirement is needed because RANS models are designed and calibrated to act only on mean quantities. When RANS models act on quantities that include fluctuations they can behave poorly [8]. The k equation is needed so that the AMS mean subgrid stress scaling parameter, $\tilde{\beta}$, as in Eqn. 2, represents the ratio of the resolved turbulent kinetic energy (tke) to total tke. The $k - \varepsilon$ model satisfies this requirement. Furthermore, $k - \varepsilon$ is often used to simulate the ABL and has been modified to capture attributes of the ABL such as the Coriolis effect [14, 18]. However, $k - \varepsilon$ is inaccurate in regions with adverse pressure gradients [15, 16], such as those near wind turbine blades. The $k - \omega$ model represents the effects of adverse pressure gradients more accurately but is overly sensitive to the freestream value of ω [15, 16].

Since we aim to develop an AMS hybrid model that is accurate in both the ABL and near wind turbine blades, we use the SST $k - \omega$ model. SST $k - \omega$ blends between $k - \varepsilon$ (away from walls) and $k - \omega$ (near walls), using each model in the region where it is most accurate [15, 16]. SST $k - \omega$ satisfies the AMS requirements to act only on mean quantities. However, the k equation in SST is not actually a representation of the true total tke, as desired for AMS [8]. The sensitivity of AMS to k is a new area of research since prior work used the $\langle \bar{v}^2 \rangle - f$ RANS model [20]. In this initial work, we neglect the difference between k , as defined by the SST RANS model, and the total tke. Under this approach we don't modify SST $k - \omega$ or the use of k in AMS to correct for the difference in-between k and total tke. SST $k - \omega$ uses the Boussinesq approximation [16] for the mean subgrid stress model

$$\tau_{ij}^R = 2\nu_T^R \langle S_{ij} \rangle - \frac{2}{3} k \delta_{ij}, \quad (7)$$

where $\langle S_{ij} \rangle = \frac{1}{2} \left(\frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right)$. The $(2/3)k\delta_{ij}$ term is scaled by $\tilde{\beta}$ and then absorbed into the pressure term. The SST $k - \omega$ eddy viscosity [23] is

$$\nu_T^R = \frac{a_1 k}{\max(a_1 \omega, \langle S \rangle F_2)}, \quad (8)$$

where a_1 is a constant, $\langle S \rangle = \sqrt{2\langle S_{ij} \rangle \langle S_{ij} \rangle}$, and F_2 is a blending function. The eddy viscosity is closed with two equations, one for k and for ω . The k equation is

$$\frac{\partial k}{\partial t} + \frac{\partial \langle k u_i \rangle}{\partial x_i} = \frac{1}{\rho} P - \beta^* k \omega + \frac{\partial}{\partial x_i} \left[(\nu + \sigma_k \nu_T^R) \frac{\partial k}{\partial x_i} \right], \quad (9)$$

where σ_k is a model parameter and P is production. The ω equation is

$$\frac{\partial \omega}{\partial t} + \frac{\partial \langle \omega u_i \rangle}{\partial x_i} = \frac{\gamma}{\mu_T^R} P - \beta \omega^2 + \frac{\partial}{\partial x_i} \left[(\nu + \sigma_\omega \nu_T^R) \frac{\partial \omega}{\partial x_i} \right] + 2(1 - F) \sigma_{w2} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}, \quad (10)$$

where $\mu_T^R = \rho \nu_T^R$; γ , β , and σ_ω are model parameters; and F is a blending function. The model parameters are calculated from the $k - \omega$ and $k - \varepsilon$ model constants by

$$\phi = F \phi_1 + (1 - F) \phi_2, \quad (11)$$

where ϕ_1 denotes the model constant from $k - \omega$ and ϕ_2 denotes the model constant from $k - \varepsilon$. The blending function, F , is one near the wall ($k - \omega$) and zero away from the wall ($k - \varepsilon$). The blending function, F , is

$$F = \tanh \left(\left(\min \left(\max \left(\frac{l_t}{\beta^* 3/4 y}, \frac{500 \mu}{\rho y^2 \omega} \right), \frac{4 \bar{\rho} \sigma_{w2} k}{C D_{k\omega} y^2} \right) \right)^4 \right), \quad (12)$$

where y is the distance from the wall and $C D_{k\omega}$ is a parameter that depends on the k and ω gradients.

RANS eddy viscosity models require a length scale limiter to simulate the ABL with the Coriolis effect [12, 14, 17, 18]. Without a limiter, the mixing length grows with height. The large mixing length away from the ground smooths away the Coriolis effect, preventing it from fully developing. We use the SST $k - \omega$ length scale limiter that we presented in Ref. [19]:

$$\begin{aligned} \gamma^* &= F \gamma_1^* + (1 - F) \gamma_2^*, \\ \gamma_i^* &= C_{\varepsilon i}^* - 1 \text{ for } i \in \{1, 2\}, \\ C_{\varepsilon 1, i}^* &= C_{\varepsilon 1, i} + (C_{\varepsilon 2, i} - C_{\varepsilon 1, i}) \frac{l_t}{l_e} \text{ for } i \in \{1, 2\}, \\ C_{\varepsilon 1, i} &= \gamma_i + 1 \text{ for } i \in \{1, 2\}, \\ C_{\varepsilon 2, i} &= \frac{\beta_i}{\beta^*} + 1 \text{ for } i \in \{1, 2\}, \\ l_e &= 0.00027 G / f_c, \\ f_c &= 2 \Omega \sin \theta, \end{aligned} \quad (13)$$

where G is the geostrophic (freestream) wind speed, Ω is the Earth's angular velocity, and θ is the latitude. The limiter works by backing out the $C_{\varepsilon 1, i}$ and $C_{\varepsilon 2, i}$ constants from γ_i , β_i , and β^* for each of the $k - \omega$ ($i = 1$) and $k - \varepsilon$ ($i = 2$) models. Then for each model, we replace $C_{\varepsilon 1, i}$ with $C_{\varepsilon 1, i}^*$. $C_{\varepsilon 1, i}^*$ uses the ratio of the mixing length to the maximum mixing length to limit the growth of the mixing length. The method to limit $C_{\varepsilon 1}$ comes from Ref. [14, 17], who derived it for $k - \varepsilon$. We modify it here to use the SST $k - \omega$ mixing length. The expression for the maximum mixing length, l_e , comes from Ref. [24]. We then calculate γ_i^* from $C_{\varepsilon 1, i}$ and combine the $k - \omega$ and $k - \varepsilon$ contributions, γ_1^* and γ_2^* , into γ^* . Finally, we replace γ with γ^* in the production term of the ω equation, Eqn. 10, so $\frac{\gamma}{\mu_T^R} P$ becomes $\frac{\gamma^*}{\mu_T^R} P$. The derivation and a detailed analysis of the SST $k - \omega$ mixing length limiter was presented in Ref. [19] and will be published in a future work.

D. Coriolis Effect

The Coriolis effect introduces a velocity component perpendicular to the geostrophic wind due to the rotation of the earth. To capture this effect we add a Coriolis forcing term to the right-hand side of the AMS momentum equation, Eqn. 1. The Coriolis forcing term is the vector

$$(F_C)_i = -2\epsilon_{ijk}\Omega_j\overline{u_k}, \quad (14)$$

where ϵ_{ijk} is the Levi-Civita tensor.

E. Boundary Conditions

To develop AMS for simulations of the ABL we select appropriate boundary conditions and consider their interaction with AMS. For the bottom boundary, we use the wall boundary condition from Ref. [11], which models surface roughness with the sand grain roughness approximation:

$$\begin{aligned} \overline{u_i} &= 0, \\ k &= \frac{u_\tau^2}{\sqrt{\beta_*}}, \\ \omega &= \frac{u_\tau}{\sqrt{\beta_*}\kappa z_0}, \end{aligned} \quad (15)$$

where u_τ is the friction velocity, z_0 is the aerodynamic roughness height, and κ is the Von Kármán constant. These boundary conditions only yield physically meaningful solutions above the sand grain roughness height, $k_s = \frac{1}{30}z_0$. Adding resolved fluctuations near the wall where the solution is not physically meaningful can introduce noise that propagates throughout the domain. Therefore, when using these boundary conditions for AMS, we force AMS to only use the RANS model contributions by setting $F_i = 0$, $\beta = 1$, and $r = 1$ below the sand grain roughness height, k_s .

In the streamwise and spanwise directions we use periodic boundary conditions. For the top boundary we use a symmetry boundary condition which enforces that the boundary-normal velocity is zero. In addition to these boundary conditions we force the velocity to the desired freestream velocity near the top of the domain by adding a source term to the momentum equation.

F. Model Constants

The appropriate values for the SST $k - \omega$ model constants depend on the type of simulation. Here we use the same values as previous SST $k - \omega$ simulations of the ABL, such as Refs. [11, 12, 25], which were tuned in Ref. [25] for atmospheric flows. The SST $k - \omega$ model constants are given in table 1.

Table 1 SST $k - \omega$ model constants

$\beta_1 = .0236$	$\gamma_1 = .3255$	$\sigma_{k1} = .85$	$\sigma_{\omega1} = .5$	$\kappa = .40$	$\beta^* = 0.03$
$\beta_2 = .0276$	$\gamma_2 = .3011$	$\sigma_{k2} = 1.0$	$\sigma_{\omega2} = .67$	$a_1 = .31$	$c_1 = 10.0$

We use the AMS model constants from Ref. [8]. The forcing factor, which scales the magnitude of the AMS active forcing, is $C_F = 8$. The RANS time scale coefficient is $C_\phi = 1$.

G. Implementation in a Massively Parallel Flow Solver

We implement AMS in Nalu-Wind [3], which is an open-source, incompressible, massively parallel flow solver for simulations of wind turbines in wind farms, available at <https://github.com/Exawind/nalu-wind>. This implementation scales on current petascale systems, such as the second fastest supercomputer in the world, Summit [26], and is designed to scale on next-generation exascale systems. Nalu-Wind is developed through the Exascale Computing Project, a Department of Energy effort to develop applications, software, and hardware that is ready to make use of exascale computing [27].

Nalu-Wind uses highly efficient solvers from the Trilinos [28] and HYPRE [29] software packages. These packages parallelize code with MPI, accelerators, and vectorization. In this work we run our simulations on CPUs without

accelerators. We solve the u , k , and ω equations with the Trilinos Tpetra GMRES solver with an SGS preconditioner. The pressure Poisson equation uses the HYPRE GMRES solver with the boomerAMG preconditioner. We fix the pressure by setting it to zero at the node nearest (100, 250, 1)m. We time step using the second order three state scheme, BDF2. The basic time stepping algorithm was modified to change the projection time scale and use under-relaxation in the pressure update. Ref. [30] provides additional details on the solvers and time stepping algorithm.

IV. Simulations of the Atmospheric Boundary Layer including the Coriolis Effect

We present results using the AMS model described in Section III to simulate the ABL including the Coriolis effect. Section IV.A describes the setup of our test case and Section IV.B presents the results of our test case, comparing AMS to RANS, LES, and measurements. We evaluate the effect of including a length scale limiter in the SST $k - \omega$ contribution to AMS in Section IV.C. Finally, we discuss the choice of the time scale used in AMS causal time averaging (Eqns. 4 and 5) when using SST $k - \omega$ in Section IV.D.

A. Atmospheric Boundary Layer Simulation Setup

To evaluate AMS for simulations of the ABL including the Coriolis effect, we use AMS as described in Section III, SST $k - \omega$ [23] with a length scale limiter as described in Section III.C, and the k^{SGS} LES model [31] to simulate the Leipzig test case as described in Ref. [14]. We compare the AMS simulations to the RANS SST $k - \omega$ simulations, LES, and measurements from Ref. [32], who reanalyzed the original measurements from Ref. [33]. We also compare to previous works that simulated the Leipzig test case with $k - \varepsilon$ including a length scale limiter [14] and LES [34]. In accordance with the Leipzig test case described in Ref. [14], the ABL is set in an empty domain that is (1000, 1000, 3000)m in the streamwise, spanwise, and wall-normal directions. The geostrophic wind speed is 17.5m/s in the streamwise direction and the latitude is 51° .

We initialize the AMS, RANS, and LES simulations to the Monin-Obukhov (MO) log wind profile [35],

$$u(z) = \frac{u_\tau}{\kappa} \ln \left(\frac{z - z_0}{z_0} \right). \quad (16)$$

The MO profile is an approximation of the wind magnitude in neutral conditions in the atmospheric surface layer (ASL). The ASL is roughly the bottom 100m of the ABL. The MO profile is the expected wind magnitude profile in the absence of the Coriolis effect and buoyancy [19]. The simulations use the streamwise and spanwise periodic boundary conditions and the symmetry top boundary condition described in Section III.E. The AMS and RANS wall boundary conditions are as described in Section III.E. LES uses a wall function that specifies the surface heat flux, q_s , and the surface shear stress, τ_s as:

$$\begin{aligned} \tau_s &= \frac{\kappa \rho u_\tau}{\ln(z/z_0)}, \\ u_\tau &= \frac{\kappa u(z)}{\ln(z/z_0)}. \end{aligned} \quad (17)$$

The mesh used for AMS and RANS is uniform in the streamwise and spanwise directions with cell size 15m and is stretched in the wall-normal direction as described in Ref. [11], with a maximum cell size of 15m. The LES mesh is uniform with cell size 10m in each direction. RANS uses a timestep of $\Delta t = 100s$, resulting in a max Courant number of $C = \frac{u \Delta t}{\Delta x} \approx 125$. Our RANS implementation tolerates large Courant numbers because it doesn't resolve grid-dependent turbulence and the time-stepping algorithm has been modified to be stable with a Courant number above one as summarized in Section III.G and further described in Ref. [30]. For LES and AMS we use a timestep of $\Delta t = 0.7s$, resulting in a max Courant number of $C \approx 1$, as required since LES and AMS resolve grid-dependent turbulence. The LES Courant number is somewhat larger than the AMS one because AMS uses a larger mesh at the top of the domain where the maximum Courant number is located.

Each simulation is run on Eagle [36], one of the 100 fastest supercomputers in the world [26]. For each of our runs we use 8 nodes, each with 36 CPUs. The RANS simulation ran for 20,000 timesteps and converged after 19,100 timesteps. Here we define converged to mean that the model quantities averaged over the streamwise and spanwise directions and $O(10)$ timesteps are no longer changing. There can still be fluctuations about the mean. The RANS simulation took 6 hours and 20 minutes. We report the results averaged over every 100 timesteps from 19,100 to 20k.

The AMS and LES simulations ran for 40k timesteps and converged after 35k timesteps. We report the results averaged over every 100 timesteps from 35-40k. AMS took 15 hours and 30 minutes, while LES took 11 hours and 20 minutes. The AMS and LES run times are comparable, even though AMS uses a wall boundary condition and finer mesh near the wall whereas LES uses a wall function and coarse mesh. This is in line with our expectation that AMS will have lower computational cost than LES when resolving near-wall regions, such as in our planned blade-resolved simulations of wind turbines.

B. Atmospheric Boundary Layer Simulation Results

We present results for the AMS simulation of the ABL with the Coriolis effect using the Leipzig test case setup described in Section IV.A. Section III describes AMS, including our choice for the LES contribution: M43 (Section III.B) and the RANS contribution: SST $k - \omega$ with a length scale limiter (Section III.C.) We compare our simulations with AMS to those with our SST $k - \omega$ RANS and k_{SGS} LES. Before making this comparison, we validate our SST $k - \omega$ RANS and k_{SGS} LES by comparing them to reference $k - \epsilon$ RANS [14] and LES [34] of the Leipzig test case in Figures 1 and 2, respectively. We also compare AMS to the Monin-Obukhov (MO) log wind profile, described in Section IV.A. Our results are averaged in the streamwise and spanwise directions and over time as described in Section IV.A. The slight fluctuations in the velocity and turbulent kinetic energy profiles presented in this section indicate that for fully smooth profiles we would need to average over an even larger time window. The larger fluctuations in the mixing length indicate that for fully converged simulations we would need to run even longer or add damping to the top of the domain; the fluctuations exit the top of the domain very slowly.

Fig. 1 Streamwise, $\overline{u_1}$, (left), and spanwise, $\overline{u_2}$, (right) wind speed components for our AMS model (dark orange, dashed line), our k_{SGS} LES model (orange, dot-dashed line), our SST $k - \omega$ RANS model with length scale limiter (dark blue, solid line), a reference $k - \epsilon$ RANS model with length scale limiter [14] (blue, small dots), and measurements [32] (light grey, large dots.)

Figure 1 shows the streamwise, $\overline{u_1}$, and spanwise, $\overline{u_2}$, wind velocity components. Figure 2 shows the magnitude of the wind velocity, $|\overline{u}|$, and the mixing length, l_t . First we consider the performance of our baseline models: SST $k - \omega$ RANS and k_{SGS} LES. Figures 1 and 2 show that our RANS and LES velocity profiles closely match those from previous simulations of the Leipzig test case with RANS [14] and LES [34]. The mixing length profile for our RANS model is similar to that for the reference RANS model. The difference in magnitude between our RANS and the reference RANS model is relatively small given that the mixing length depends on the turbulent kinetic energy, k , which, as previously discussed, represents different quantities in our SST $k - \omega$ model and the reference $k - \epsilon$ model. Figure 2 shows that both our RANS and LES simulations match the Monin Obukhov log wind profile in the ASL (the bottom ~ 100 m of the ABL), as expected. Figures 1 and 2 show that our RANS simulations closely match the measured wind velocity.

The LES simulations match the measured wind velocity magnitude and the general profile shape, but have a larger boundary layer height than the measurements. The reference LES shows the same behavior in Figure 2. The discrepancy in-between LES and the measurements of the Leipzig test case matches previous analysis [37] which concluded that while the Leipzig measurements were reported as taken during neutral conditions [33], they were likely actually taken in slightly stable conditions. Stable conditions would decrease the boundary layer height, as shown by the measurements. Thus to match the measurements we would need to run LES in slightly stable conditions, such as those suggested in Ref.

Fig. 2 Wind speed magnitude (left) for our AMS model (dark orange, dashed line), our k_{SGS} LES model (orange, dot-dashed line), a reference LES model [34] (light orange, small dots), our SST $k - \omega$ RANS model (dark blue, solid line), the reference Monin-Obukhov (MO) profile (dark grey, dashed line), and measurements [32] (light grey, large dots.) Mixing length (right) for our AMS, our SST $k - \omega$ RANS, and the reference $k - \epsilon$ RANS [14] (blue, small dots) models.

[37]. The discrepancy in-between RANS and LES likely comes from the fact that LES includes buoyancy but was run in neutral conditions whereas RANS does not include buoyancy. The fact that RANS without buoyancy aligns with measurements for slightly stable conditions likely stems from the way the model constants were tuned in previous studies or a coincidence of model errors canceling. To better understand the RANS/measurements-LES discrepancy and to further develop AMS, in future work we will add buoyancy effects to SST $k - \omega$ and run SST $k - \omega$, AMS, and LES in slightly stable conditions.

Given how well the reference and our RANS and LES models simulate the ABL, were our overarching motivation just to model the ABL with the Coriolis effect, these results indicate that RANS or LES could accomplish that task. However, our overarching motivation is actually blade-resolved simulations of multiple wind turbines in the ABL. As described in Section II, RANS models don't accurately capture some of the important effects in simulations of multiple wind turbines and LES models are too computationally expensive for blade-resolved simulations of multiple wind turbines. Thus, although RANS and LES models can accurately simulate the ABL with the Coriolis effect, developing AMS to do the same is still an important initial step in our overarching goal.

With our baseline RANS SST $k - \omega$ and LES k_{SGS} models validated, we now compare AMS to these baseline models. Figures 1 and 2 show that AMS closely matches the streamwise and spanwise velocity and velocity magnitude profiles from LES—much more closely than RANS does. Figures 2 and 3 show that the AMS mixing length, specific dissipation rate, ω , and turbulent kinetic energy, k , are similar to that of SST $k - \omega$. The primary difference in-between SST $k - \omega$ and AMS is in the velocity profiles; AMS matches LES better than SST $k - \omega$ does. As explained above, to better understand the discrepancy in-between the SST/measurements and LES/AMS results, in future work we will add buoyancy to SST $k - \omega$ and run SST $k - \omega$, AMS, and LES in slightly stable conditions matching those in which the measurements were taken.

Analysis of the AMS parameters, $\tilde{\beta}$, r , and F_i indicate that for this simulation AMS is operating mostly like a RANS model. Specifically, the AMS mean subgrid stress scaling parameter, $\tilde{\beta}$, remains near one, so the RANS contribution remains at nearly full force rather than being reduced to balance added resolved turbulent kinetic energy (tke). The resolution adequacy indicator, r , remains near zero, indicating that the grid could support additional resolved content, however the active forcing, F_i , is relatively small, indicating that relatively few resolved fluctuations are being generated. These AMS diagnostics are currently being investigated to better understand what aspects of AMS lead it to align with LES rather than RANS. It may be that the AMS causal time averaging described in Section III.A has a relatively large effect. We start to analyze this idea in Section IV.D and will further analyze the effect of causal time averaging and buoyancy in future work.

Fig. 3 Specific dissipation rate, ω , (left) and turbulent kinetic energy, k , (right) for the Leipzig test case for our AMS model (dark orange, dashed line), our k_{SGS} LES model (orange, dot-dashed line), and our SST $k - \omega$ RANS model (dark blue, solid line.) The ω profiles collapse onto one line.

C. Effect of SST $k - \omega$ Length Scale Limiter on Active Model Split

The results presented in Section IV.B used a length scale limiter for SST $k - \omega$ RANS and AMS. The effect of the length scale limiter on SST $k - \omega$ was presented in Ref. [19] and will be described in a future publication. Here, we analyze the effect of the length scale limiter on AMS by comparing simulations using AMS with and without a length scale limiter. Figure 4 shows that AMS without a length scale limiter develops little Coriolis effect: the streamwise velocity component shows no bulge around 1500m and the spanwise velocity component is nearly zero. Furthermore, the streamwise velocity profile is much steeper than for LES. In contrast, AMS with a length scale limiter has a gradually increasing streamwise velocity profile with a bulge and a large spanwise velocity component, very similar to those of LES.

Fig. 4 Streamwise, $\overline{u_1}$, (left), and spanwise, $\overline{u_2}$, (right) wind speed components for the Leipzig test case for our AMS model with a length scale limiter (dark orange, dashed line), our AMS model without a length scale limiter (light blue, tight dot-dashed line), and our k_{SGS} LES model (orange, dot-dashed line.)

Figure 5 shows that the mixing length limiter has the expected effect on the mixing length. The mixing length for AMS without a limiter grows large away from the wall. This excessive mixing prevents the Coriolis effect from fully developing. Adding the length scale limiter to AMS limits the growth of the mixing length away from the wall. Figure 5 also shows that without the mixing length the SST $k - \omega$ blending function, F , remains near one throughout the domain, keeping it in $k - \omega$ mode even away from the wall where $k - \omega$ is overly sensitive to the freestream value of ω . This occurs because the blending function depends on the ratio of the mixing length to the distance from the wall, $l_t/(\beta^{*3/4}y)$ (Eqn. 13.) Without the limiter the mixing length grows large away from the wall so that even though the wall distance, y , is large, $l_t/(\beta^{*3/4}y)$ is large. Therefore F equals $\tanh$ of a large number, which is one, so $F \approx 1$ everywhere in the domain. Adding the limiter allows the blending function to transition from one ($k - \omega$) near the wall to zero ($k - \varepsilon$)

away from the wall as intended.

Fig. 5 Mixing length (left) and SST $k - \omega$ RANS blending function (right) for the Leipzig test case for our AMS model with a length scale limiter (dark orange, dashed line) and our AMS model without a length scale limiter (light blue, tight dot-dashed line.)

Figure 6 shows that the turbulent kinetic energy, k , for AMS with a length scale limiter is much more similar to the k of LES than for AMS without a limiter. AMS without a limiter generates a k orders of magnitude too large. The specific dissipation rate, ω , for AMS without the limiter is similar but larger to that of AMS without the limiter.

Fig. 6 Specific dissipation rate, ω , (left) and turbulent kinetic energy, k , (right) for the Leipzig test case for our AMS model with a length scale limiter (dark orange, dashed line), our AMS model without a length scale limiter (light blue, tight dot-dashed line), and our k_{SGS} LES model (orange, dot-dashed line.)

The results presented in this section show that adding the length scale limiter to AMS allows AMS to better model the ABL with the Coriolis effect. In particular, the limiter makes the velocity and tke profiles of AMS much more similar to those of LES. It does so by preventing the mixing length from growing excessively large away from the ground. In addition, without the limiter, the SST $k - \omega$ blending function doesn't accomplish the intended behavior of switching in-between $k - \omega$ near the wall and $k - \varepsilon$ away from the wall. Adding the limiter restores this behavior. These results indicate that the length scale limiter is an important component of not just SST $k - \omega$ but also AMS when simulating the ABL with the Coriolis effect.

D. Active Model Split Time Scale When Using SST $k - \omega$

The AMS results presented in Sections IV.B and IV.C used the RANS time scale described in Eqn. 3, copied here for convenience, $T^R = l_t / \sqrt{k}$. In this section, to emphasize the difference in-between this time scale and alternate time scales we will discuss, we add the subscript, l_t , so T^R becomes $T_{l_t}^R$. This time scale differs from the RANS time scale

used in previous AMS studies [8] with $\overline{v^2} - f$ for the RANS contribution: $T_\varepsilon^R = k/\varepsilon$. The analogous time scale for SST $k - \omega$ would be $T_\omega^R = 1/(\beta^* \omega)$, using the definition $\omega = \varepsilon/(\beta^* k)$ from SST $k - \omega$. Inserting the definition of the mixing length, l_t , into T_ε^R , we see $T_\omega^R = 1/(\beta^* l_t^3 \omega)$. Thus the time scale used here, $T_{l_t}^R$, and the time scale that would be consistent with previous studies, T_ω^R , only differ by a constant factor $1/\beta^* l_t^3$. In this section we evaluate the effect of using $T_{l_t}^R$ versus T_ω^R as the RANS time scale in AMS. For both versions of AMS, $T_{l_t}^R$ and T_ω^R , we use the SST $k - \omega$ length scale limiter.

Figure 7 shows that the streamwise and spanwise velocity profiles from AMS with $T_{l_t}^R$, as shown in Sections IV.B and IV.C, match the profiles from LES better than the profiles from AMS with T_ω^R do. The profiles from AMS with T_ω^R have some of the characteristic of the LES profile, such as larger boundary layer height. This indicates that for both RANS timescales, AMS captures some of the LES behavior better than RANS does. However, for the streamwise velocity component, AMS with T_ω^R doesn't show the bulge at $z \approx 1000 - 1500\text{m}$ that RANS, LES, and AMS with $T_{l_t}^R$ do. This indicates that AMS with T_ω^R doesn't properly model the ABL, in particular the Coriolis effect whereas AMS with $T_{l_t}^R$ does.

Fig. 7 Streamwise, $\overline{u_1}$, (left), and spanwise, $\overline{u_2}$, (right) wind speed components for the Leipzig test case for AMS with $T_{l_t}^R$ (dark orange, dashed line) and T_ω^R (grey, short dashed line), our SST $k - \omega$ RANS model (dark blue, solid line), and our k_{SGS} LES model (orange, dot-dashed line.)

Figure 8 shows that the mixing length and turbulent kinetic energy for AMS with T_ω^R and $T_{l_t}^R$ are similar. The specific dissipation rate, to avoid redundancy, is nearly identical for AMS with T_ω^R , AMS with $T_{l_t}^R$, and SST $k - \omega$. The SST $k - \omega$ blending function is also similar for AMS with T_ω^R and $T_{l_t}^R$. The primary difference is in the velocity profiles.

Fig. 8 Mixing length (left) and turbulent kinetic energy (right) for the Leipzig test case for AMS with $T_{l_t}^R$ (dark orange, dashed line) and T_ω^R (grey, short dashed line), our SST $k - \omega$ RANS model (dark blue, solid line), and our k_{SGS} LES model (orange, dot-dashed line.)

While T_ω^R and $T_{l_t}^R$ differ by only a constant factor, Figure 9 shows that this leads to very different timescales. The time scale for AMS with T_ω^R grows very large relative to $T_{l_t}^R$. The large timescale for AMS with T_ω^R means that the mean quantities update very slowly, as shown by the causal time average equation, Eqn. 5. Because of this slow update, the ABL with Coriolis effect doesn't develop fully in AMS with T_ω^R . The results in this section indicate the importance of using a time scale consistent with the SST $k - \omega$ length scale limiter when running AMS with SST $k - \omega$ using a length scale limiter.

Fig. 9 RANS time scales from AMS with $T_{l_t}^R$ (dark orange, dashed line) and T_ω^R (grey, short dashed line.)

V. Conclusions

We present a Hybrid RANS-LES (HRL) model that is well suited for blade-resolved simulations of multiple wind turbines in a realistic atmospheric boundary layer (ABL). The HRL model, Active Model Split (AMS) can resolve the blade boundary layer (BL) at reasonable cost because of the lower cost of HRL relative to large-eddy simulation (LES.) AMS can resolve the blade BL at reasonable cost because of the use of the SST $k - \omega$ Reynolds-Averaged Navier Stokes (RANS) model rather than a $k - \varepsilon$ RANS model, which is inaccurate near adverse pressure gradients such as those near wind turbine blades. We describe aspects of AMS that distinguish it from other HRL models and make it well suited for simulations of the ABL. In particular, AMS controls the mean and fluctuating subgrid stresses independently, uses active forcing to introduce resolved fluctuations, considers grid and turbulence anisotropy when determining where the grid can support additional resolved fluctuations, and is compatible with anisotropic LES models such as the M43 model used in this work.

We describe how to modify AMS for simulations of the ABL by using the SST $k - \omega$ RANS model with a length scale limiter, adding the Coriolis effect, using a wall treatment consistent with both ABL modeling and AMS, and using model constants appropriate for ABL simulations. We explain our implementation of AMS in Nalu-Wind, an open-source, incompressible, massively parallel flow solver for simulations of wind turbines in wind farms. Nalu-Wind scales on current petascale systems and is expected to scale on future exascale systems. While we don't need that level of scalability for the ABL simulations presented in this work, we will need it for our future work when we add multiple blade-resolved wind turbines to the ABL. By implementing AMS in Nalu-Wind we prepare for the next step in this work which requires massive compute power.

We use this implementation of AMS to simulate the ABL including the Coriolis effect. We show that AMS models the ABL with the Coriolis effect well. AMS generates velocity profiles much more similar to those of LES than RANS does. We analyze the effect of the length scale limiter, originally introduced for SST $k - \omega$ RANS simulations, on AMS. We find that the limiter is important for not only SST $k - \omega$ but also AMS simulations of the ABL. The velocity and turbulent kinetic energy profiles from AMS with the limiter match the profiles from LES much closer than the profiles from AMS without the limiter. In addition, without the limiter the SST $k - \omega$ blending function doesn't show the intended behavior of blending between $k - \omega$ near the wall and $k - \varepsilon$ away from the wall. Adding the limiter restores the intended behavior of the blending function. We also analyze the effect of the choice of time scale in AMS when using SST $k - \omega$ with a length scale limiter. We find that the time scale should be consistent with the length scale limiter. Time scales that work for AMS with other RANS models, such as $\overline{v^2} - f$, but do not take into account the length scale

limiter can lead to poor behavior for AMS using SST $k - \omega$ with a length scale limiter.

Our results demonstrate that the AMS model presented in this work simulates the ABL with the Coriolis effect well. The velocity profiles from AMS match those from LES much closer than those from RANS do. Furthermore, our results indicate the potential of using AMS with SST $k - \omega$. Previous work with AMS used $\overline{v^2} - f$ and suggested that SST $k - \omega$ might not be well suited for use as the RANS contribution to AMS [8]. This work indicates that AMS with SST $k - \omega$ can perform well, in this case simulating an ABL with the Coriolis effect very similarly to LES. As part of this initial work using AMS with SST $k - \omega$, we evaluate and ultimately show the importance of several aspects of AMS with SST $k - \omega$ for ABL simulations: the use of a length scale limiter in SST $k - \omega$ and the choice of the time scale used in AMS. In future work we will further investigate AMS with SST $k - \omega$ for ABL simulations with the Coriolis effect, focusing on understanding how AMS performs so similarly to LES while acting mostly in RANS mode. We will also add buoyancy effects to SST $k - \omega$ and run SST $k - \omega$, AMS with this SST $k - \omega$, and LES in slightly stable conditions to better understand the discrepancy we observed in-between RANS/measurements and LES/AMS. Finally, the ExaWind project at the National Renewable Energy Laboratory will use AMS for a blade-resolved simulations of multiple wind turbines in the ABL including the Coriolis and buoyancy effects.

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