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## Preprint

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### NONLINEAR MODEL PREDICTIVE CONTROL BASED ON REAL-TIME ITERATION SCHEME FOR WAVE ENERGY CONVERTERS USING WEC-SIM

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#### ABSTRACT

One of several challenges that wave energy technologies face is their inability to generate electricity cost-competitively with other grid-scale energy generation sources. Several studies have identified two approaches to lower the levelised cost of electricity: reduce the cost over the device's lifetime or increase its overall electrical energy production. Several advanced control strategies have been developed to address the latter. However, only a few take into account the overall efficiency of the power take-off (PTO) system, and none of them solve the optimisation problem that arises at each sampling time on real-time. In this paper, a detailed Nonlinear model predictive control (NMPC) approach based on the real-time iteration (RTI) scheme is presented, and the controller performance is evaluated using a time-domain hydrodynamics model (WEC-Sim). The proposed control law incorporates the PTO system's efficiency in a control law to maximise the energy extracted. The study also revealed that RTI-NMPC clearly outperforms a simple resistive controller.

#### **1 INTRODUCTION**

A wave energy converter (WEC) is a device that converts the energy carried by the ocean waves into electrical energy through a power take-off (PTO). WECs can be classified into oscillating bodies or oscillating water columns based on their primary operating principle [1]. Today, a broad spectrum of concepts for wave energy conversion have been proposed and investigated.

One of several challenges that wave energy technologies face is their inability to generate electricity cost-competitively with other grid-scale energy generation sources, such as natural gas and wind [2]. The levelised cost of electricity (LCOE) is defined as the ratio of the total cost to total electrical energy produced over the lifetime of a wave energy converter, which is commonly reported in U.S. dollars per kilowatt-hour units [3]. Several studies have identified two ways to reduce the LCOE for ocean wave energy: reduce the cost over the device's lifetime or increase the device's overall electrical energy production [4–7].

Several control strategies for wave energy converters can be found in literature [8-10] and can be divided into two groups: passive control and active control. A passive

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controller implements a force that opposes the movement of the point absorber, and the energy flow is unidirectional, from the ocean to the grid. Resistive control [8, 11–13] is an example of this type. On the other side, active controllers involve a bidirectional energy flow from the sea to the grid and vice versa. Model predictive control (MPC) and spectral and pseudospectral methods [9, 12, 13] belong to this category. This paper offers a solution for the latter: an advanced control strategy to significantly improve energy capture efficiency.

In [14], the performance of a reactively controlled single point absorber of a Wavestar WEC with a nonideally efficient PTO was studied for regular waves, and the performance of regular and irregular waves was studied in [11]. For regular and irregular waves, partial reactive control was suggested in [15] as a causal suboptimal control approach for a heaving single-body wave energy converter, along with studies of the impact of the actuators' efficiency in the annual mean absorbed power.

In [16], an MPC approach was described that explicitly considers the efficiency of the PTO system. However, this controller, similarly to the one presented by the same authors in [17, 18], cannot be used for real-time implementation with small sampling times ( $T_p \leq 50 \text{ ms}$ ) since they are based on an offline solution [18]. Similar, the MPC algorithm presented in [17, 18] uses a discrete objective function that weights the instantaneous power value over the prediction horizon. The weightings are determined offline using an iterative optimisation approach based on repeated simulations of the WEC model over a set of sea states (a Nelder-Mead optimisation algorithm is used).

The major contribution of this paper is the implementation of a nonlinear model predictive control (NMPC) approach based on the real-time iteration (RTI) scheme [19] to incorporate the PTO system's efficiency when solving the optimal control problem (OCP) at each time step in a control policy that aims to maximise the amount of energy extracted from the ocean waves.

This research builds on previous work in [20]. A key extension is that the assumptions about the incident wave moment at the current time step and for a prediction horizon window are removed in this paper. Here, wave excitation moment is estimated using a Kalman filter, and the vector of future wave excitation moment is predicted using an autoregressive (AR) model. A further important novelty is that the simulation is performed on WEC-Sim to provide a more realistic/accurate simulation.

The structure of this paper is outlined as follows. Section 3 presents the time-domain modelling of the wave energy converter used in this work. Section 2 formulates the general objective of any energy-maximising control strategy. A detailed description of the modelling, prediction, optimisation and real-time iteration to implement the proposed RTI-NMPC scheme are presented in Section 4. The results of the simulations are presented in Section 5. Finally, Section 6 contains conclusions, summarises the paper's contribution and describes future work.

#### 2 PROBLEM FORMULATION

The main objective of a wave energy converter controller is to transfer as much energy as possible from the ocean waves to the grid for a broad range of sea states. The electrical energy  $E_e$  absorbed by the grid over a time horizon T, is defined as:

$$E_e = -\int_t^{t+T} P_e(\tau) d\tau = -\int_t^{t+T} \Gamma(\tau) P_m(\tau) d\tau \qquad (1)$$

where  $P_e$  denotes the electrical power delivered to the grid,  $P_m$  the raw hydromechanical power absorbed by the PTO system,  $\Gamma$  the overall efficiency of the PTO system and  $\tau$  is the variable of integration.

The negative sign in Eq. (1) is because the energy is drawn from the WEC and thus the maximisation of the energy absorbed corresponds to a minimisation of the control objective [21].

The instantaneous hydromechanical absorbed power is given by:

$$P_m(t) = M_{pto}(t)\dot{\theta}(t)$$
(2)

where  $M_{pto}$  is the PTO moment and  $\dot{\theta}$  represents the angular velocity of the arm.

Finally, to use standard nomenclature,  $M_{pto}(t_k)$  is replaced by  $u_k$ , and the discrete-time optimisation problem is

given by:

minimise 
$$J = \sum_{i=1}^{N_p} \gamma_{k+i} u_{k+i-1} \dot{\theta}_{k+i}$$
 (3a)

s.t. 
$$\underline{x}_{k+1} = f(\underline{x}_k, u_k, w_k)$$
 (3b)

$$U_{min} \le u_k \le U_{max} \tag{3c}$$

where  $N_p$  is the prediction horizon, Eq. (3b) represents the WEC dynamics with the states  $\underline{x} = [\theta \ \dot{\theta} \ \underline{r}]^T$ , the variables in  $\underline{x}$  are defined in Section 3,  $u_k$  the control input,  $w_k$ the discrete-time value for the excitation moment  $M_{exc}(t_k)$ , and  $\gamma_k$  the specific value for the PTO efficiency at time instant  $t_k$ ; that is,  $\Gamma(t_k) = \gamma_k$ .

*Remark 1:* equation (3a) considers the velocity  $\dot{\theta}$  and the control input *u* at different time steps (k + i and k + i - 1). This is chosen to ensure causality of the solution as discussed in [22].

#### 3 WAVE ENERGY CONVERTER MODELLING

The WEC chosen for testing the nonlinear controller strategy is a scaled model of a single device based on the Wavestar concept [23] used in the WEC Control Competition (WECCCOMP) [24]. In this point-absorber WEC, a hemisphere acts as a floater, and it is coupled to a rotating arm hinged at a fixed reference point A. (See Fig. 1.)

#### State space model

The dynamics of the WEC in the pitch degree of freedom, assuming that the system's oscillations are modest, can be written in the time domain as follows [25]:

$$(J+J_{\infty})\ddot{\theta}(t) = -K_{hs}\,\theta(t) - b_{v}\,\dot{\theta}(t) - M_{rad}(t)$$

$$M_{exc}(t) + M_{pto}(t) \tag{4a}$$

$$\underline{\dot{r}}(t) = A_r \underline{r}(t) + B_r \dot{\theta}(t)$$
(4b)

$$M_{rad}(t) = C_r \underline{r}(t) + D_r \dot{\theta}(t)$$
(4c)

where:

•  $\theta$  represents the angular displacement of the arm with respect to the equilibrium position,  $\dot{\theta}$  and  $\ddot{\theta}$  represent the angular velocity and angular acceleration of the arm.



**FIGURE 1**. DIAGRAM OF THE WAVESTAR WEC SYS-TEM

- *J* is the total mass moment of inertia of the float and pivot arm.
- $J_{\infty}$  is the added mass moment of the inertia.
- $K_{hs}$  is the hydrostatic coefficient.
- $b_v$  is a linear damping coefficient.
- $M_{rad}$  is the radiation moment.
- $M_{exc}$  is the excitation moment due to the incident wave.
- $M_{pto}$  is the PTO moment (Input to the system).
- $A_r, B_r, C_r, D_r$ , are the state space matrices used to approximate the radiation moment  $M_{rad}$  avoiding the direct computation of the convolution integral in timedomain simulations. The state vector  $\underline{r}(t)$  have no physical meaning, but still contain information on the condition of the surrounding fluid [26].

For more details of the model development, the interested reader is referred to [3, 18].

*Remark:* Parameters and variables in Eq. (4) are specified with respect to the rotating point *A*.

#### Power take-off efficiency

PTO systems are not perfect in real-world applications, which means that the electrical power  $P_e$  is never equal to the absorbed mechanical power  $P_m$ , i.e.,  $0 \le P_e \le P_m$ . If a reactive control strategy is adopted, at certain times, the PTO system must return some electric power from the grid back into the ocean ( $P_m \le 0$ ). In those instants, and because of the losses in the conversion stages, the electrical power provided by the grid to the PTO system must be larger than  $|P_m|$ , i.e.,  $P_e \le P_m \le 0$ .

Given that the efficiency of the PTO system varies depending on the direction of the energy flow (float-to-grid or grid-to-float direction), the energy-maximising control strategy must consider the efficiency when solving for the optimal control input [27]. Other studies have discussed the impact of nonideal PTO efficiency on WEC control [11, 15–17, 28–32]. The main drawback of the cited studies is that none of them solves the optimal control problem related to the nonlinear output equation of the model in real time ( see Eq. (9) ), which is the main contribution of this paper.

The overall efficiency of a PTO system can be modelled using a modified-step function with two different values for the efficiency depending on whether the PTO system is working as a motor (grid-to-float) or as a generator (float-to-grid). Therefore, the instantaneous extracted power can be expressed as:

$$P_e(t) = \Gamma(t) P_m(t), \begin{cases} \Gamma(t) = \mu_{gen} \ if \quad P_m(t) \ge 0\\ \Gamma(t) = \mu_{mot} \ if \quad P_m(t) < 0 \end{cases}$$
(5)

where  $\mu_{gen}$  is the global efficiency of the PTO system when it delivers energy to the grid and  $\mu_{mot}$  is the global efficiency when the PTO system consumes power from the grid.

#### **WEC-Sim numerical model**

The work presented in this paper is based on a numerical simulation of the WEC device using the WEC-Sim code. WEC-Sim is a time-domain open-source code that solves the system dynamics of WECs consisting of rigid bodies, PTO systems, mooring systems, and control systems [3]. WEC-Sim calculates the dynamic response of the WEC device by solving the WEC's equation of motion for each rigid body about its centre of gravity  $C_g$  in the 6 degrees of freedom based on Cummins' equation [25]. The reader is referred to [3] for a detailed description of the code implementation and validation of the numerical model for the scaled Wavestar model against wave tank experiments.

#### Wave excitation moment estimation

Many of the optimal control strategies for wave energy converters studied in the literature rely on the availability of measurements of the wave elevation and/or the exciting forces caused by the incoming waves [33–36]. This requirement is often difficult, if not impossible, to meet due to the limited number of sensors available and time. To that end, wave excitation force/moment has to be estimated via measuring other quantities, such as the position or velocity of the float.

The approach followed here, which was first proposed in [34] and implemented in [17], is based on a Kalman filter coupled with a random-walk model for the wave excitation moment. The main features of this solution are [17]: (1) only standard WEC measurements (position, velocity), (2) there is no significant lag compared to "true" values, and (3) no (implicit) unrealistic assumption about the timeinvariant nature of the sea state is made; therefore it can be implemented in any sea state. The algorithm is fully described in [34].

#### Wave excitation moment prediction

This study used a linear AR model to predict the wave excitation moment. The AR model, which was first introduced in [37], implies that the wave excitation moment  $M_{exc, k}$  at any given time  $t_k$  is linearly dependent on its past values via the parameters  $a_i$  (in [37] a concept was proposed for sea surface elevation, but the analogy to excitation moment is immediate):

$$M_{exc,k} = \sum_{i=1}^{N} a_i \cdot M_{exc,k-i} + \zeta_k \tag{6}$$

where *N* is the AR model order and  $\zeta$  is a disturbance term considered in the prediction.

If an estimate of the parameters  $\hat{a}_{i,k}$  at time instant  $t_k$  is computed and the noise is assumed to be Gaussian and white, the best prediction for the wave excitation moment  $M_{exc,k+p|k}$  at instant  $t_k$  can be derived from Eq. (6) as:

$$\hat{M}_{exc,k+p|k} = \sum_{i=1}^{N} \hat{a}_{i,k} \cdot \hat{M}_{exc,k+p-i|k}$$
(7)

where,  $\hat{M}_{exc,k+p-i|k} \equiv \hat{M}_{exc,k}$  if  $k+p-i \leq k$  ( i.e., information already acquired, no need of prediction).

For brevity, the reader is invited to extract in-depth descriptions of AR models from [37–39].

#### 4 NONLINEAR MODEL PREDICTIVE CONTROL

NMPC is becoming increasingly popular for real-time optimal control solutions due to its ability to explicitly handle constraints and nonlinear dynamics that define the system of interest [40]. The following subsections are meant to provide a quick overview of each of the steps involved in RTI-NMPC.

Let us first define some notations used in the following sections. The upper-bar (-) represents a nominally guessed point that is considered a "desirable-optimal" trajectory that the NMPC framework will use to optimise and improve the solution iteratively. Similarly, the hat  $(^)$ represents the predicted trajectory value, whereas the variable with no additional notation will be reserved for the real/simulated value. In addition, for readability and to simplify the notation of the following equations, the underbar notation for vectors  $(_)$  will be dropped.

#### Modelling

In this paper, a discrete-time nonlinear dynamic model describing the dynamics of a generic wave energy converter of the following form is considered:

$$x_{k+1} = f(x_k, u_k, w_k)$$
 (8a)

$$y_k = g(x_k, u_k, w_k) \tag{8b}$$

where  $x_{k+1}$ ,  $u_k$  and  $y_k$  are vectors containing the  $n_x$  states,  $n_u$  inputs and  $n_y$  the outputs of the system, respectively.

The output function  $g(x_k, u_k, w_k)$  is selected as follows:

$$y_k = g_k = \begin{bmatrix} \dot{\theta}_k \\ \gamma_k u_{k-1} \end{bmatrix}$$
(9)

Equation (5), on the other hand, which models PTO efficiency, has a discontinuity between the two cases:  $P_m(t) \ge 0$  and  $P_m(t) < 0$ . Such a discontinuous function is undesirable in gradient-based optimisation approaches. A

smoothed approximation to Eq. (5), at  $P_m(t) = 0$ , must be implemented to avoid problems with the efficient implementation of the optimisation algorithm.

In [32] and [16], a modified-hyperbolic tangent function is used to approximate Eq. (5). The approximation using *tanh* is preserved in this study and is given by:

$$\Gamma_{approx}(t) = \alpha + \beta \tanh(\varphi P_m(t))$$
(10)

where  $\alpha$  is an offset,  $\beta$  is a scaling factor, and  $\varphi$  is a real positive parameter that determines the accuracy of the approximation.

#### Prediction

The prediction model discussed in this section is derived similarly to that presented in [41] and is presented here to allow the contents of this paper to be self-contained. Using a first order multivariable Taylor series expansion, the linearised model for Eq. (8a) at a given time step  $t_k$  is given by:

$$\hat{x}_{k+1} = \bar{x}_{k+1} + A_k \,\delta \hat{x}_k + B_k \,\delta \hat{u}_k + B_{w,k} \,\delta \hat{w}_k \tag{11}$$

where  $\delta \hat{x}_k = \hat{x}_k - \bar{x}_k$ ,  $\delta \hat{u}_k = \hat{u}_k - \bar{u}_k$ , and  $\delta \hat{w}_k = \hat{w}_k - \bar{w}_k$  are the deviations of the state, control input and wave excitation moment from their nominal points  $(\bar{x}_k, \bar{u}_k, \bar{w}_k)$  at time step  $t_k$  respectively, and  $A_k$ ,  $B_k$ , and  $B_{w,k}$  are the partial derivatives with respect to the states, control input, and wave excitation input moment, which will be defined shortly.

The wave excitation moment deviation  $\delta \hat{w}_k$  requires special consideration at this stage. Because the approach described in this study is based on the prediction of future wave excitation moment, the nominal and predicted trajectory for the wave excitation moment is considered to be the same at each time step, i.e.,  $\delta \hat{w}_k = 0$  for all k, and thus the following derivation ignores this term.

$$A_{k} = \frac{\partial f(x,u,w)}{\partial x} \bigg|_{\substack{\bar{x}_{k} \\ \bar{w}_{k}}} B_{k} = \frac{\partial f(x,u,w)}{\partial u} \bigg|_{\substack{\bar{x}_{k} \\ \bar{w}_{k}}}$$
$$\bar{x}_{k+1} = f(\bar{x}_{k}, \bar{u}_{k}, \bar{w}_{k})$$

The deviation  $\delta \hat{x}_{k+1} = \hat{x}_{k+1} - \bar{x}_{k+1}$  at time step  $t_{k+1}$ 

can be approximated by:

$$\delta \hat{x}_{k+1} = A_k \,\delta \hat{x}_k + B_k \,\delta \hat{u}_k \tag{12}$$

Given that the nominal point  $\bar{x}_{k+1}$  and the linearisation matrices  $A_k, B_k$  are parametrically dependent on  $\bar{x}_k, \bar{u}_k, \bar{w}_k$ , and that the value for  $x_k$  is already known at a given sampling time  $t_k$  (either by measurements or by state estimation), the value for  $\bar{x}_{k+1}$  can only be derived by guessing (or estimating) an optimal-nominal value for  $\bar{u}_k$  around which the trajectory will be linearised.

If values for the future optimal-nominal input trajectory  $\overline{U} = [\overline{u}_k^T, \overline{u}_{k+1}^T, \cdots, \overline{u}_{k+N_p-1}^T]^T$  are guessed, the projected nominal state trajectory  $\overline{X} = [\overline{x}_{k+1}^T, \overline{x}_{k+2}^T, \cdots, \overline{x}_{k+N_p}^T]^T$  and the linearisation matrices  $A_k, B_k$  can be computed for future time steps  $t = k + 1, k + 2, \dots, k + N_p$ , where  $N_p$  is known as the prediction horizon. This technique is often referred as *single-shooting*. Other techniques such as *multiple shooting* and *collocation points* can also be used with the proposed approach [40].

After obtaining  $\bar{X}$  with  $\bar{U}$ , Eq. (12) can be shifted forward:

$$\delta \hat{x}_{k+2} = A_{k+1} \,\delta \hat{x}_{k+1} + B_{k+1} \,\delta \hat{u}_{k+1} \tag{13}$$

Substituting Eq. (12) into Eq. (13) yields:

$$\delta \hat{x}_{k+2} = A_{k+1} \left( A_k \,\delta \hat{x}_k + B_k \,\delta \hat{u}_k \right) + B_{k+1} \,\delta \hat{u}_{k+1} \qquad (14)$$

By recursively repeating the preceding procedure for  $N_p$  steps and considering just the system output (Eq. (9)), the predicted deviations from the nominal output trajectory may be expressed in a matrix form by:

$$\delta \hat{Y} = G_{y} \, \delta x_{k} + H_{y} \, \delta \hat{U} \tag{15}$$

where  $\delta \hat{Y} = \hat{Y} - \bar{Y} = [\delta y_{k+1}^T, \delta y_{k+2}^T, \cdots, \delta y_{k+N_p}^T]^T$ are the output deviations,  $\delta \hat{U} = \hat{U} - \bar{U} = [\delta \hat{u}_k^T, \delta \hat{u}_{k+1}^T, \cdots, \delta \hat{u}_{k+N_p-1}^T]^T$  are the control input deviations. The matrices  $G_y$  and  $H_y$  are given by:

$$G_{y} = \begin{bmatrix} C_{1}A_{0} \\ C_{2}A_{1}A_{0} \\ \vdots \\ C_{N_{p}}A_{N_{p}-1}\cdots A_{1}A_{0} \end{bmatrix}$$
(16)

$$H_{y} = \begin{bmatrix} C_{1}B_{0} & \mathbf{0} & \dots & \mathbf{0} \\ C_{2}A_{1}B_{0} & C_{2}B_{1} & \dots & \vdots \\ C_{3}A_{2}A_{1}B_{0} & C_{3}A_{2}B_{1} & \ddots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ C_{N_{p}}A_{N_{p}-1}\cdots A_{1}B_{0} & C_{N_{p}}A_{N_{p}-1}\cdots A_{2}B_{1} & \cdots & C_{N_{p}}B_{N_{p}-1} \end{bmatrix}$$
(17)

The dimensions for these matrices are  $G_y = [N_p n_y \times n_x]$ ,  $H_y = [N_p n_y \times N_p n_u]$ , and  $C_k$  is the partial derivative of Eq. (9) with respect to nominal state, evaluated at the specific time step t = k, and is given by:

$$C_k = \frac{\partial g(x, u, w)}{\partial x} \bigg|_{\substack{\bar{x}_k \\ \bar{u}_k \\ \bar{w}_k}}$$

In addition, the matrix **0** represents a matrix of zeros with the same dimensions as the matrix  $C_k B_k$ .

#### Optimisation

Following the definition of the prediction models, the cost function described in Eq. (3a) can be recast as follows:

$$J = \frac{1}{2}\hat{Y}^{T}Q\hat{Y} + \frac{1}{2}\delta\hat{U}^{T}R\delta\hat{U}$$
(18)

where the matrix *R* is a positive definite matrix with dimensions  $[N_p n_u \times N_p n_u]$  and constant elements over its diagonal. *Q* is selected as a block diagonal matrix with dimensions  $[N_p n_u \times N_p n_u]$  and inner matrices  $q_i$  used to compute the product  $\dot{\theta}_k \times \gamma_k u_{k-1}$  as defined in [42].

$$Q = \begin{bmatrix} q_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & q_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & q_{N_p} \end{bmatrix} \qquad q_i = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (19)$$
$$\forall i = [1, N_p]$$

Equation (18) includes an additional term that penalises the input deviation. This term is included for two reasons: first, it smooths out the control signal, making the requirement for the actuator's response limit less stringent; and second, according to [10, 22, 32, 43], a reactive control strategy with a cost function solely based on maximising

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the extracted energy can result in overall negative energy absorbed, implying that the system is losing energy rather than absorbing energy from the waves.

Finally, by inserting the linearised output prediction Eq. (15) in Eq. (18), grouping comparable terms with respect to the decision variable  $\delta \hat{U}$ , and excluding any constant terms in the cost function, the standard quadratic programming (QP) formulation is obtained:

$$J = \frac{1}{2} \,\delta \hat{U}^T E \,\delta \hat{U} + \delta \hat{U}^T f \qquad s.t. \quad M \,\delta \hat{U} \le \rho \quad (20a)$$

$$E = H_y^T Q H_y + R \tag{20b}$$

$$f = H_y^T Q \left[ \bar{Y} + G_y \, \delta x_k \right] \tag{20c}$$

where  $E \in \mathbb{R}^{N_p n_u \times N_p n_u}$  is a symmetric matrix known as the Hessian and  $f \in \mathbb{R}^{N_p n_u}$  is a column vector usually referred as the linear term;  $M \in \mathbb{R}^{2N_p n_u \times N_p n_u}$  is the constraints matrix and  $\rho \in \mathbb{R}^{2N_p n_u}$  is the constraints vector, defined as:

$$M = \begin{bmatrix} I \\ -I \end{bmatrix} \qquad \rho = \begin{bmatrix} U_{max} - \bar{U} \\ -(U_{min} - \bar{U}) \end{bmatrix}$$
(21)

In the present work, only constraints in the control input are considered. If any states' constraints are required, Mand  $\rho$  must be slightly reformulated. It is worth noting that  $G_y$  and  $H_y$ , and hence E and f, are time-dependent, which is one of the main reasons why NMPC is computationally expensive.

Having defined *E*, *f*, *M*, and  $\rho$ , the OCP can be solved using any QP solver, such as Matlab's quadprog function and qpOASES [44], to mention two. In this paper, quadprog was used. The new control input sequence is computed once the QP problem is solved, recalling that  $\hat{U} = \bar{U} + \delta U$ . From the new control input sequences, only the first input is applied to the system, and the procedure is repeated at the next time step, which is known as the *receding horizon* scheme [45].

#### **Real-time iterations scheme**

The RTI scheme was first introduced in [19] for nonlinear optimisation in optimal feedback control. A fully converged NMPC should ideally re-linearise the predictions and thus cost function Eq. (20) until no deviations are necessary, i.e.,  $\delta \hat{U} = 0$  [45]. This is not computationally tractable in real-time applications since one must provide a solution at each time step under strict time constraints and avoid solving a problem that is just "getting older" [46].

The RTI scheme is briefly commented on in the following subsections.

**Initial value embedding.** Choosing an appropriate initial estimate for  $\hat{U}$  optimal, denoted as  $\hat{U}^*$ , is critical for fast and reliable convergence of the SQP iteration. To facilitate the estimation, the previous optimal input trajectory is employed in a shifted version to *hot-start* the solution at the following sampling time, generally by duplicating the last value [46].

**Single SQP Iteration.** The computing burden can be further decreased by executing only a single SQP iteration at each time step, i.e., only linearising the OCP once instead of re-linearising it until convergence.

**Computation separation.** The separation of the computation is perhaps the essential aspect of the RTI scheme. It divides the calculations into preparation and feedback phases. A timing diagram that illustrates this can be seen in [46].

#### 5 RESULTS

The numerical results of the proposed control strategy applied on the Wavestar benchmark scale model simulated on WEC-Sim are reported in this section.

#### Model parameters

Table 1 summarises the parameters used in the equation of motion for the dynamics of the WEC, Eq. (4).

#### Wave conditions

The performance of the proposed controller is evaluated in a series of three unidirectional sea states generated by the JONSWAP spectrum. In general, wave climate is characterised by the significant wave height  $H_{m0}$ , the peak wave period  $T_p$ , and wave direction. The spectrum parameters are shown in Table 2, based on the sea states utilised in the WECCCOMP [24].

TABLE	1.	MODEL	PARAMETERS	FOR	THE	SCALE
MODEL	OF 7	THE WAV	ESTAR DEVICE	[17,47	].	

J	$1.04 \mathrm{kg}\mathrm{m}^2$				
$J_{\infty}$	$0.4805kgm^2$				
K <sub>hs</sub>	$92.33Nmrad^{-1}$				
$b_v$	$1.80{ m N}{ m m}{ m rad}^{-1}{ m s}^{-1}$				
	$\frac{\text{arame}}{J}$ $J_{\infty}$ $K_{hs}$ $b_{v}$				

$A_r =$	-13.59 -	-13.35	$B_r$ 8.0
,	8.00	0.00	
$C_r =$	4.739	0.5	$D_r = -0.1586$

**TABLE 2**. PARAMETERS FOR WAVE GENERATION US-ING JONSWAP SPECTRUM. SIGNIFICANT WAVE HEIGHT  $H_{m0}$ , PEAK PERIOD  $T_p$  AND PEAK ENHANCEMENT FAC-TOR  $\gamma$ .

Name‡	$H_{m0}$	$T_p$	γ	Duration
Ivanie	[m]	[s]	[-]	[s]
SS4	0.0208	0.988		98.8
SS5	0.0625	1.412	3.3	141.2
SS6	0.1042	1.836		183.6

<sup>‡</sup> Names are given to have consistency with the names given in the WECCCOMP [24].

#### Simulation and control parameters

Regarding the prediction horizon, research on wave excitation force prediction suggests that prediction strategies can predict wave excitation force for swell waves extremely accurately up to two peak wave periods in the future [37]. However, the prediction horizon chosen in this study for each sea state is more conservative, i.e., one peak wave period  $(N_p = 1 \times T_p/dt)$ .

After studying the literature on wave prediction for WEC, specifically AR models, we found that there are

**TABLE 3.** SIMULATION AND CONTROL TUNING PA-<br/>RAMETERS.

Parameter	Value
Simulation time [s]	$100  imes T_p$
Control sampling time [ms]	50
AR Order [lags]	18
AR training set [-]	$10 \times AR$ order
Prediction horizon [samples]	$\operatorname{Ceil}(1 \times Tp/dt, 5)$
$\mu_{gen}$	0.7
$\mu_{mot}$	$0.7^{-1}$
$oldsymbol{arphi}^{st}$	1000
Control limit [N m]	±12

<sup>®</sup> Used in PTO efficiency function approximation.

widely disparate claims regarding the model order required to predict wave excitation, ranging from 12 lags to 32 lags in [37] to 10 lags to 200 lags in [38].

Therefore, the following procedure was followed to determine the model order: first, the WEC system was simulated without a control law extracting the wave excitation moment from the simulation. Second, using this information, partial autocorrelation on the wave excitation moment signal was performed, and it was found that a model with 18 lags would be sufficient to predict the wave excitation moment. The AR model is updated every second during simulations independently of the sea state selected.

Other relevant control tuning parameters are summarised in Table 3.

The simulation for each sea state lasts at least 100 times the peak period, with the first 25 s used as a wave ramp and hence omitted in the power and energy computation. Also, the prediction algorithm started working after 10 s of simulation, whereas the controller started working after 15 s.

Let us now turn our attention to the extracted energy and power. Figure 2 shows the control input and absorbed power over the simulation time for sea state SS6 running under the RTI-NMPC controller. For this simulation, the absorbed energy was 127.43 J with a mean power



**FIGURE 2**. CONTROL INPUT AND ABSORBED POWER BY THE WAVESTAR SCALED MODEL FOR SEA STATE SS6 WITH REAL-TIME ITERATION NONLINEAR MODEL PREDICTIVE CONTROL. THE RED DASHED LINE REPRE-SENTS THE MEAN ABSORBED POWER.

of 0.6204W. Results obtained for sea states SS4 and SS5 are summarised in Table 4.

From the data depicted in Fig. 2 we can make the following remarks. First, it is clear that the proposed control strategy successfully absorbs a net positive power from the ocean waves; second, RTI-NMPC strives to avoid consuming energy from the grid.

To compare the control strategy proposed around RTI-NMPC, one additional set of simulations was performed using a proportional controller, proportional to the arm angular velocity of the WEC, also known as resistive control in the ocean wave energy community [8, 11–13].

For comparative purposes, Fig. 3 shows the control input and absorbed power by the Wavestar model for sea state SS6 running a resistive controller. For this case, the absorbed energy was 73.09 J with a mean power of 0.3538 W. Results obtained for sea states SS4 and SS5 are summarised in Table 4.

Finally, Fig. 4 shows the energy absorbed by the WEC for each control strategy. From there, we can observe that RTI-NMPC can absorb roughly 1.75 times more than the resistive controller.



**FIGURE 3**. CONTROL INPUT AND ABSORBED POWER BY THE WAVESTAR SCALED MODEL FOR SEA STATE SS6 WITH RESISTIVE CONTROL. THE RED DASHED LINE REPRESENTS THE MEAN ABSORBED POWER.

**TABLE 4.** ENERGY ABSORBED AND MEAN POWERFOR RESISTIVE CONTROL AND RTI-NMPC FOR EACHSEA STATE.

San Stata	Resi	stive	RTI - NMPC		
Sea State	Absorbed	Mean	Absorbed	Mean	
	Energy [J]	Power [W]	Energy [J]	Power [W]	
SS4	2.121	0.01968	3.103	0.02867	
SS5	23.055	0.14807	42.855	0.27624	
SS6	73.092	0.35385	127.434	0.62041	

#### 6 CONCLUSIONS

This work describes an NMPC approach based on the RTI scheme for including the PTO efficiency system when solving the OCP at each time step in a control policy that maximises the energy harvested from ocean waves.

WEC-Sim simulations of the Wavestar-scaled model wave energy converter demonstrate that RTI-NMPC is able to solve in real time a nonlinear optimal control problem



**FIGURE 4**. ENERGY ABSORBED BY THE WAVESTAR MODEL FOR SEA STATE SS6 WITH DIFFERENT CON-TROL STRATEGIES. IN BLUE RTI-NMPC, IN BLACK RE-SISTIVE CONTROL.

that includes the nonideal efficiency of the PTO system. At the same time, the proposed RTI-NMPC approach can significantly improve wave energy converter performance.

Figures from Section 5 show the performance of the proposed RTI-NMPC approach for sea state SS6. Results show that RTI-NMPC clearly outperforms the resistive controller, harvesting roughly 1.75 times the amount of energy extracted by a resistive controller while keeping the amount of power "borrowed" from the grid to a bare minimum.

Future work on the proposed strategy will focus on the controller's robustness in the face of unmodeled system dynamics, the incorporation of nonlinear hydrodynamics, and the controller's performance with alternative lengths for the prediction horizon in the wave excitation moment algorithm.

To summarise, it appears that nonlinear model predictive control based on the real-time iteration method could be utilised to considerably enhance the absorbed energy from ocean waves, hence reducing the levelised cost of electricity.

Finally [48], to improve peer cooperation and openness, the findings presented in this paper and the code used in the simulations are available through a GitHub repository available at [49].

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