SST $k - \omega$ Simulations of the Atmospheric Boundary Layer Including the Coriolis Effect

Christiane Adcock[1], Marc Henry de Frahan[2], Jeremy Melvin[3], Ganesh Vijayakumar[2], Shreyas Ananthan[4], Gianluca Iaccarino[1], Robert Moser[3], Michael Sprague[2]

[1]Stanford University
janiad@stanford.edu

[2]National Renewable Energy Laboratory

[3]University of Texas at Austin

[4]Siemens Gamesa Renewable Energy

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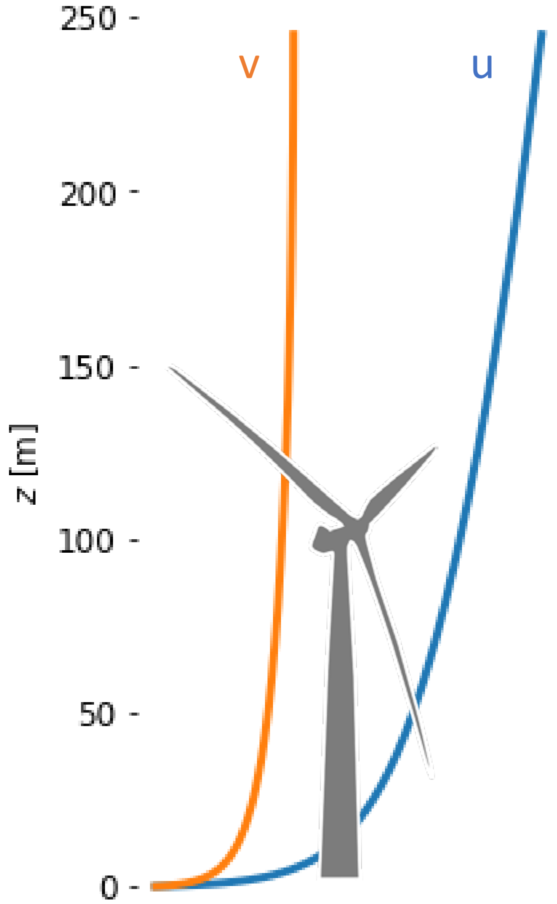
Blade-resolved simulations of wind turbines in the Atmospheric Boundary Layer (ABL) [1]

Most ABL simulations use LES or RANS $k-\varepsilon$

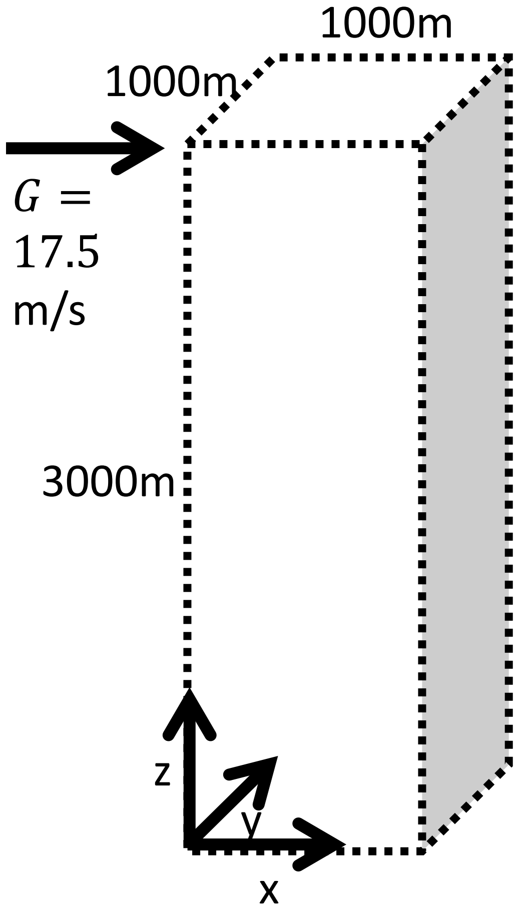
- LES prohibitively expensive for blade-resolved + multiple turbines [1]
- RANS $k-\varepsilon$ inaccurate for adverse pressure gradients near blades [2,3]
- RANS $k-\omega$ overly sensitive to freestream value of ω [2,3]

Current goal: RANS SST $k-\omega$ to simulate ABL w/ Coriolis, w/out buoyancy, w/out turbines



[1] 2020 Sprague et. al, NAWEA WindTech
[2] 1992 Menter, NASA Technical Memorandum
[3] 2006 Wilcox, Turbulence modeling for CFD

Test Case Set-Up



BCs: Top=symmetry. Sides=periodic. Bottom:

- $u = 0.\ k = \frac{u_\tau^2}{\sqrt{\beta^*}};\ \omega = \frac{u_\tau}{\sqrt{\beta^*}\kappa z_0}$ [4]
- Derive: $\varepsilon = \frac{u_\tau k \sqrt{\beta^*}}{\kappa z_0}$ by $\omega = \frac{\varepsilon}{\beta^* k}$

Model constants: SST $k - \omega$ [4]; $k - \varepsilon$ [5]

Implemented in **Nalu-Wind**: open-source, incompressible, massively parallel flow solver for simulations of turbines in wind farms. https://github.com/Exawind/nalu-wind

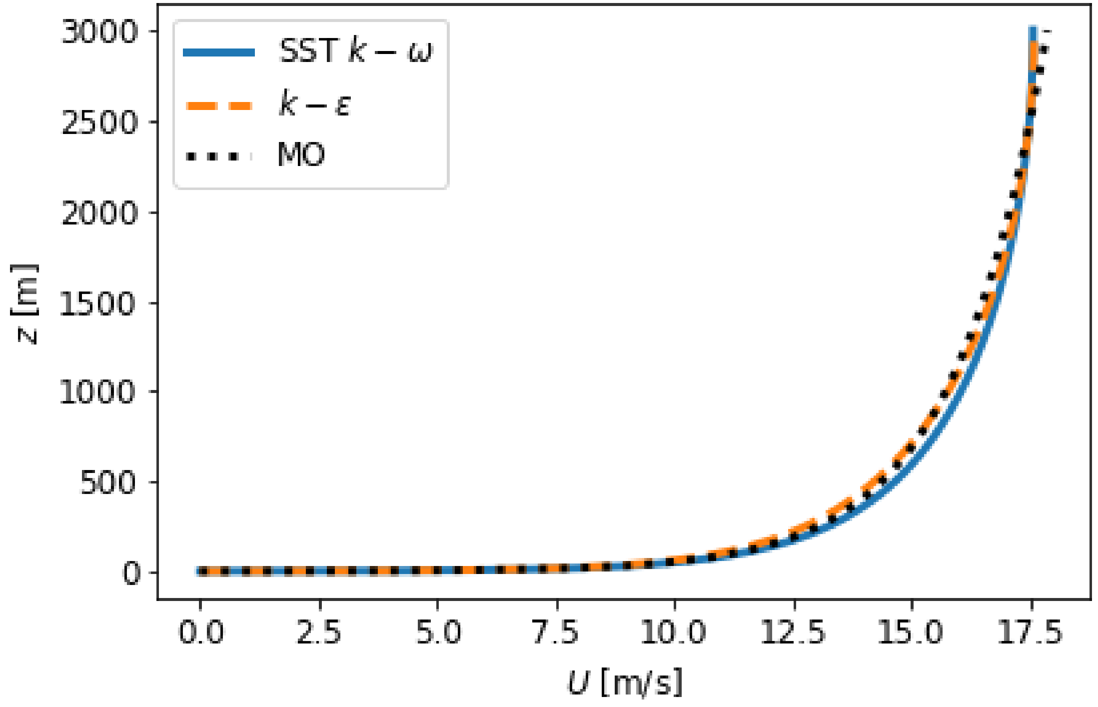
Test cases:

(1) W/out Coriolis. (2) W/ Coriolis:

$$\frac{\partial U_i}{\partial t} + \frac{\partial U_i U_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial(2\nu S_{ij})}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j} + F_i^C$$

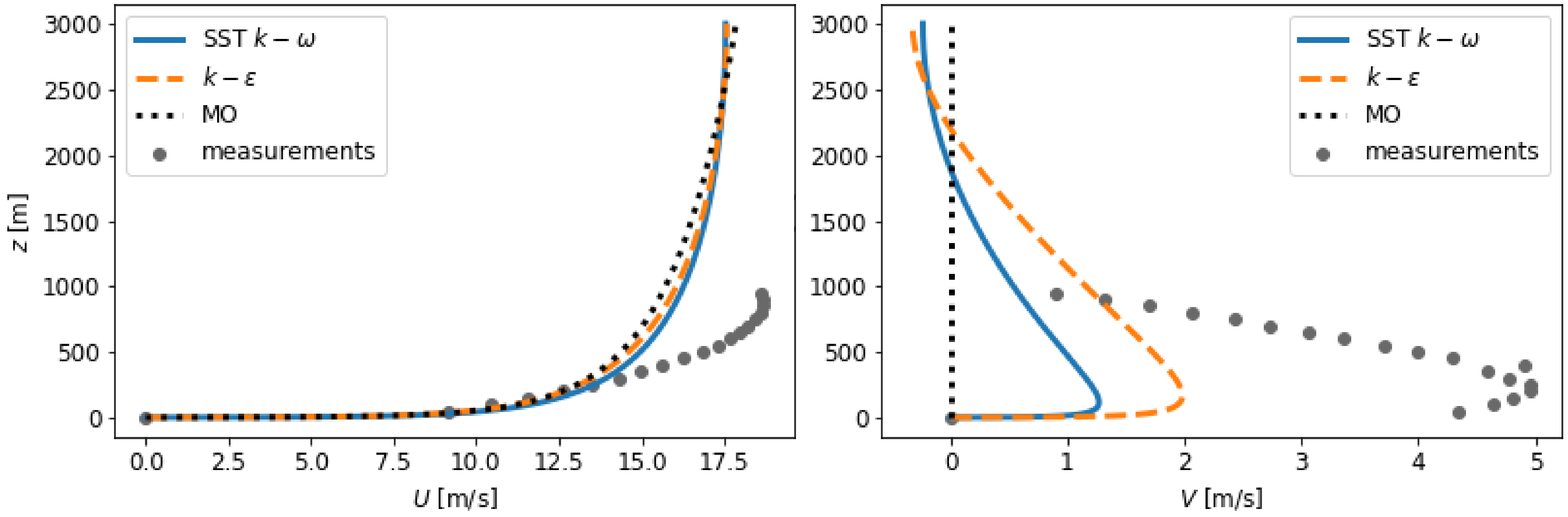
[1] 2020 Sprague e al, NAWEA WindTech
[4] 2015 Bautista, Dufresne, Masson, E3S Web of Conferences
[5] 2013 Koblitz, Ph.D. thesis, DTU

Baseline SST $k-\omega$ and $k-\varepsilon$ w/out Coriolis match Monin-Obukhov similarity theory



- Monin-Obukhov (MO) log wind profile:
- $u = \frac{u_\tau}{\kappa} \ln\left(\frac{z-z_0}{z_0}\right)$

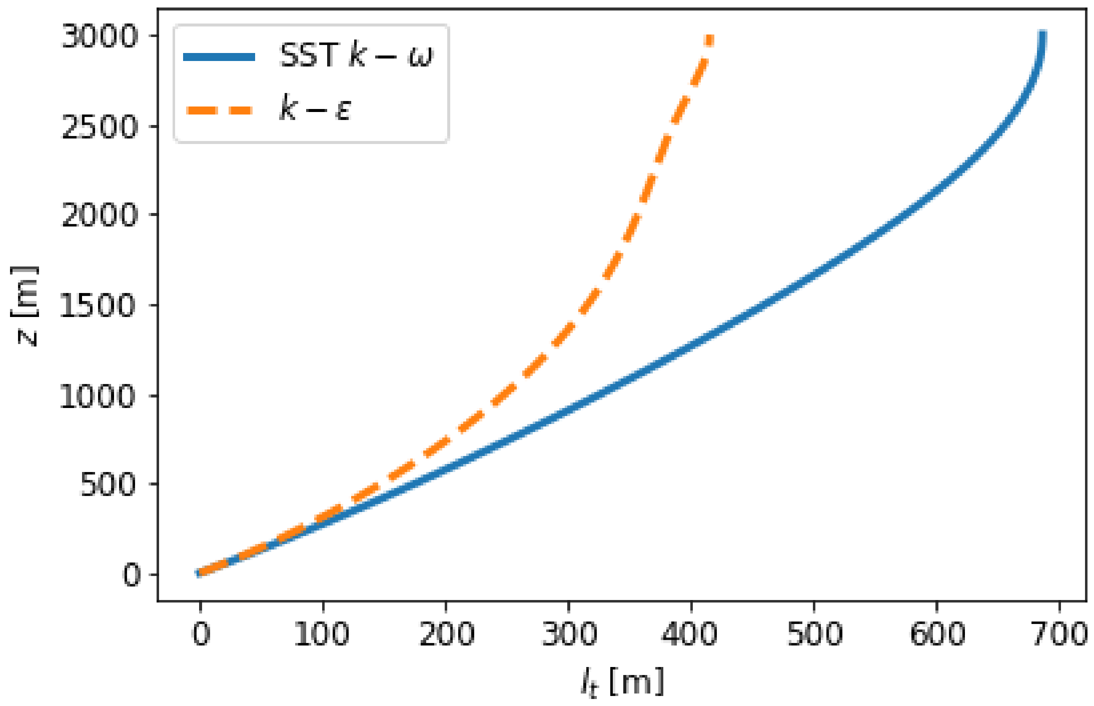
Baseline SST $k - \omega$ and $k - \varepsilon$ w/ Coriolis don't fully capture Coriolis effect



Measurements from Leipzig field test [6]

[6] 1950 Lettau, Tellus

Baseline SST $k-\omega$ and $k-\varepsilon$ don't capture Coriolis effect b/c of large mixing length



SST $k-\omega$ blending function, F_1:

- 0 away from wall ($k-\varepsilon$)
- 1 near wall ($k-\omega$)

Here, $F_1 \approx 1$ always b/c in F_1:

$$\tanh\left(\left(\max\left(\frac{\boldsymbol{l_t}}{\boldsymbol{d}}, \frac{\cdot}{d^2}\right)\right)^4\right)$$

$$= \tanh(large) \approx 1$$

$k - \varepsilon$ mixing length limiter [5]

$k - \varepsilon$ [7]: RANS, eddy viscosity model, $\nu_t = C_\mu \frac{k^2}{\varepsilon}$, 2 eqn.—$k, \varepsilon$

- ε eqn.: $\frac{\partial \varepsilon}{\partial t} + U_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\varepsilon}{k}(C_{\varepsilon 1} P - C_{\varepsilon 2} \varepsilon) + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]$

[5] 2013 Koblitz, Ph.D. thesis, DTU
[7] 2003 Menter et. Al., Turbulence, Heat and Mass Transfer

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$\boldsymbol{C_{\varepsilon 1}} \rightarrow \boldsymbol{C^*_{\varepsilon 1}} = C_{\varepsilon 1} + (C_{\varepsilon 2} - C_{\varepsilon 1}) l_t / l_e$

- Mixing length: $l_t = \beta^{*3/4} k^{3/2} / \varepsilon$
- Maximum mixing length: $l_e = 0.00027 G / f_c$
- Coriolis force: $f_c = 2\Omega sin\lambda$

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As $\frac{l_t}{l_e} \rightarrow 1$, $C^*_{\varepsilon 1} \rightarrow C_{\varepsilon 2}$ so $\frac{\varepsilon}{k}(C^*_{\varepsilon 1} P - C_{\varepsilon 2}\varepsilon) \rightarrow C_{\varepsilon 2}\frac{\varepsilon}{k}(P - \epsilon)$ so limit l_t

As $\frac{l_t}{l_e} \rightarrow 0$, $C^*_{\varepsilon 1} \rightarrow C_{\varepsilon 1}$ so $\frac{\varepsilon}{k}(C^*_{\varepsilon 1} P - C_{\varepsilon 2}\varepsilon) \rightarrow \frac{\varepsilon}{k}(C_{\varepsilon 1} P - C_{\varepsilon 2}\varepsilon)$ so reverts

[5] 2013 Koblitz, Ph.D. thesis, DTU
[7] 2003 Menter et. Al., Turbulence, Heat and Mass Transfer

Derivation of SST $k-\omega$ length scale limiter

SST $k-\omega$ [8]: RANS, eddy viscosity model, $\nu_t \approx \frac{k}{\omega}$, 2 eqn.—$k, \omega$

- ω eqn.: $\frac{\partial(\rho\omega)}{\partial t} + \frac{\partial(\rho u_j \omega)}{\partial x_j} = \frac{\gamma}{\nu_t} P - \beta\rho\omega^2 + \frac{\partial}{\partial x_j}\left[(\mu + \sigma_\omega \mu_t)\frac{\partial \omega}{\partial x_j}\right] + 2(1-F_1)\frac{\rho\sigma_{\omega 2}}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}$

[8] 1974 Launder and Spalding, Computer Methods in Applied Mechanics and Engineering

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Start from $k-\varepsilon$ eqns. w/ $C^*_{\varepsilon 1}$

Transform into $k-\omega$ eqns. w/ $C^*_{\varepsilon 1}$; find

- $\boldsymbol{\gamma} \rightarrow \boldsymbol{\gamma}^*$

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Start from $k-\varepsilon$ eqns. w/ $C^*_{\varepsilon 1}$

Transform into $k-\omega$ eqns. w/ $C^*_{\varepsilon 1}$; find

- $\boldsymbol{\gamma \rightarrow \gamma^*}$

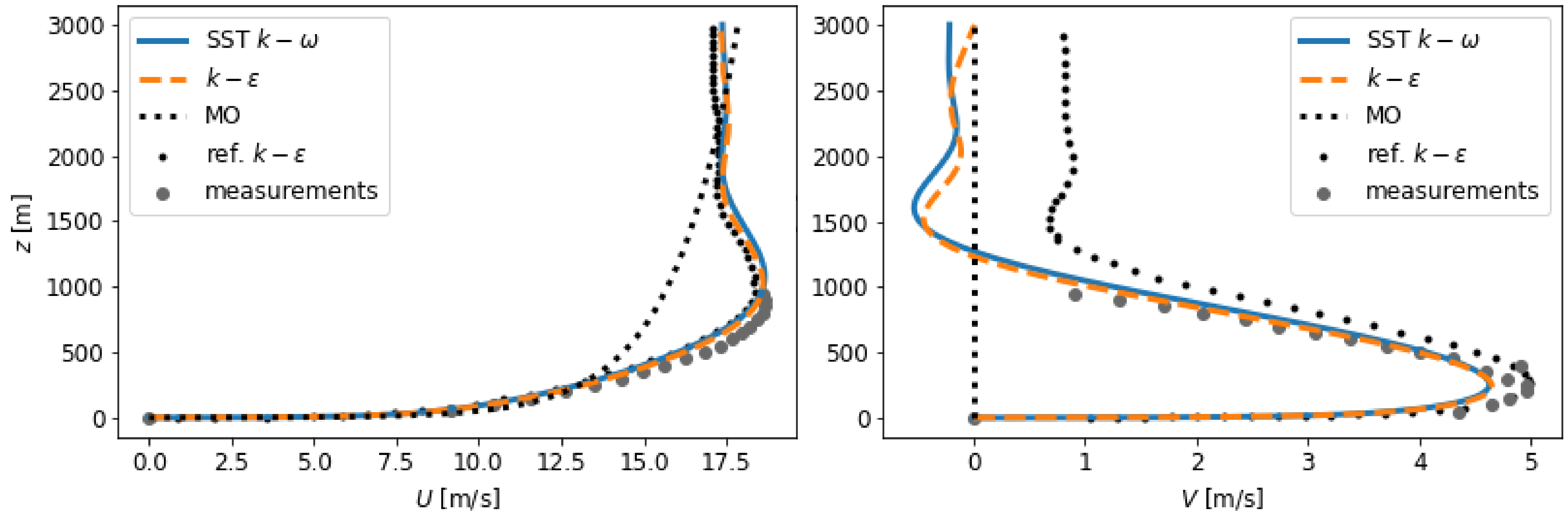
Apply to $k-\varepsilon$ and $k-\omega$ contributions to SST $k-\omega$:

- $\gamma_1^* = C^*_{\varepsilon 1} - 1$
- $\gamma_2^* = C^*_{\varepsilon 2} - 1$

Blend: $\gamma^* = F\gamma_1^* + (1-F)\gamma_2^*$

[8] 1974 Launder and Spalding, Computer Methods in Applied Mechanics and Engineering

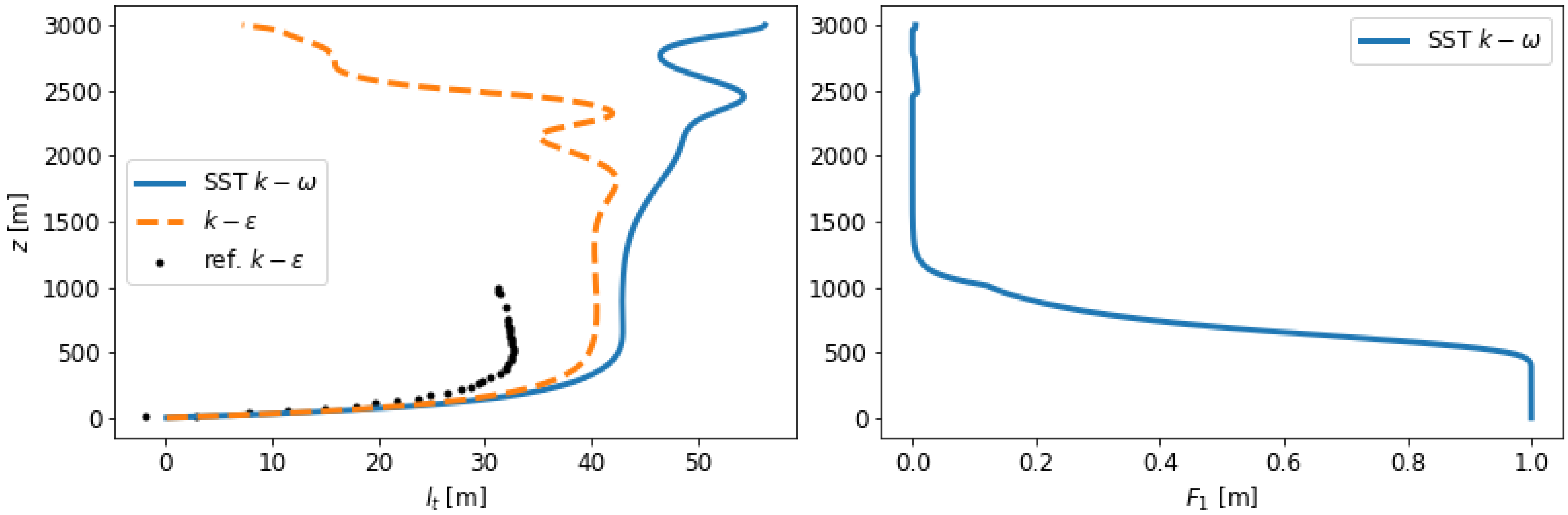
SST $k-\omega$ and $k-\varepsilon$ w/ limiter capture Coriolis effect



Reference $k-\varepsilon$ w/ limiter from [5] (also w/ wall function, extra diffusion)

[5] 2013 Koblitz, Ph.D. thesis, DTU

Limiter stops growth of mixing length; restores desired SST $k-\omega$ blending



Conclusions

- Showed baseline SST $k - \omega$ and $k - \varepsilon$ match MO similarity theory but don't fully capture Coriolis effect
- Derived mixing length limiter for SST $k - \omega$ using ideas from [5]
- Showed SST $k - \omega$ and $k - \varepsilon$ w/ limiter match measured ABL profile
- Showed $k - \varepsilon$ w/ limiter models this ABL case well w/out
  - Wall function, e.g. that in [5]
  - Diffusion term analogous to that in SST $k - \omega$, e.g. as in [5]

[5] 2013 Koblitz, Ph.D. thesis, DTU

Future Work

- Hybrid RANS-LES (HRL) of ABL [9], specifically Active Model Split (AMS) [10], w/ SST $k - \omega$ w/ limiter
- SST $k - \omega$ and AMS of ABL including buoyancy
- AMS blade-resolved simulations of O(10) wind turbines in an ABL [1]

[9] 2021 Adcock et al. AIAA SciTech (accepted)
[10] 2020, Haering et al., 2020, arXiv (preprint)
[1] 2020 Sprague et al, NAWEA WindTech

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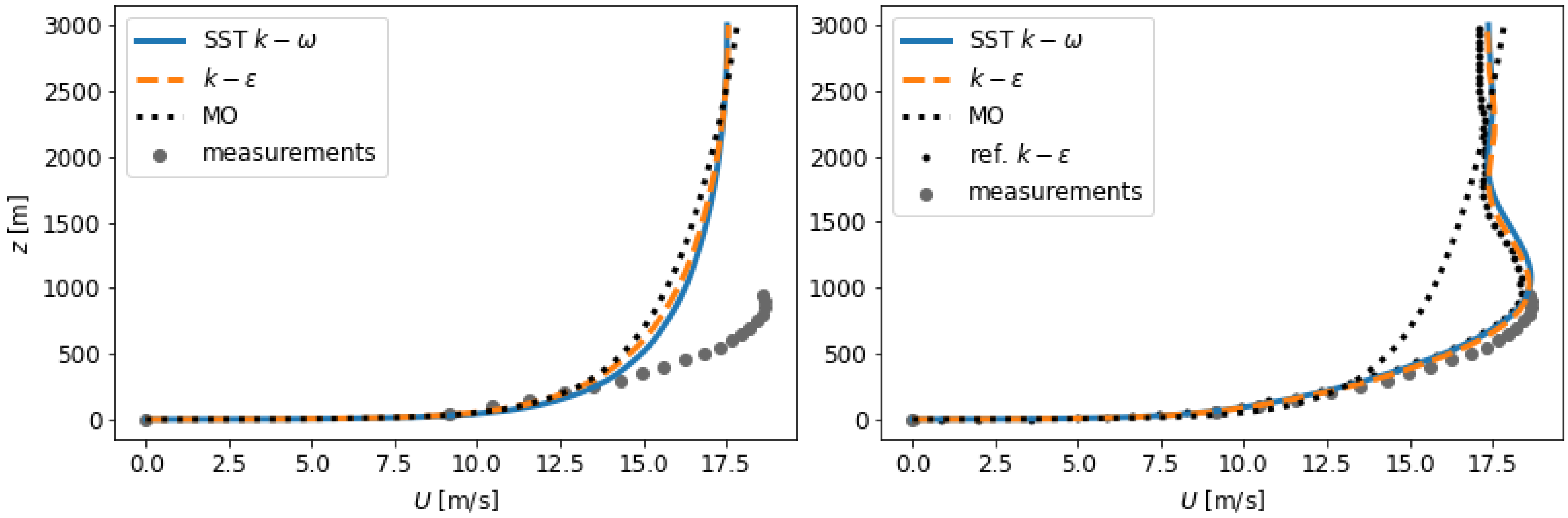
Questions


SST $k-\omega$
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