

Computational Math Problems for a Clean Energy Future

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Introduction

- The mission of the Computational Science at NREL includes leading the lab's efforts to solve energy challenges using high-performance computing (HPC), computational science and applied mathematics.
- We provide a short overview of three areas of computational mathematics research at NREL: scenario generation for stochastic grid operations and planning, improved rational function approximations for electromagnetic transients codes, and wind farm yaw control using ADMM and reinforcement learning.

Rational Approximation for EMT Modelling

Rational approximations are essential for representing frequency-dependent phenomena in electromagnetic transients (EMT) simulations via the universal line model. The vector fitting (VF) algorithm for computing rational approximations is the current state of the art. We investigate using a multifunction variant of the AAA algorithm [1] to compute rational function approximations for EMT.

AAA approximation

Our variant of the AAA algorithm builds a rational approximation of a functions f_1, \dots, f_m in barycentric form from sample values $f(Z)$, $Z \subseteq \mathbb{C}$ by selecting support points $\{z_m\} \subset Z$ and subsequently solving

$$r_{k,m}(z) = \frac{\sum_{n=1}^m \frac{w_n f_k(z_n)}{z - z_n}}{\sum_{n=1}^m \frac{w_n}{z - z_n}}$$

for the $w = \{w_1, w_2, \dots, w_m\}$ over $Z \setminus \{z_m\}$ in a least squares sense. Support points are chosen greedily such that z_{m+1} maximizes $\sum_k |f_k - r_{k,m}|$ over $Z \setminus \{z_m\}$. When $|f_k - r_{k,m}|$ is less than some desired tolerance, $r_{k,m}(z)$ is output as rational approximations of $\{f_k\}$.

Example: Fitting entries of admittance matrix

We test our approach by approximating six entries in an admittance matrix as a function of frequency. This example is used as a test case in the Vector Fitting handbook [2].

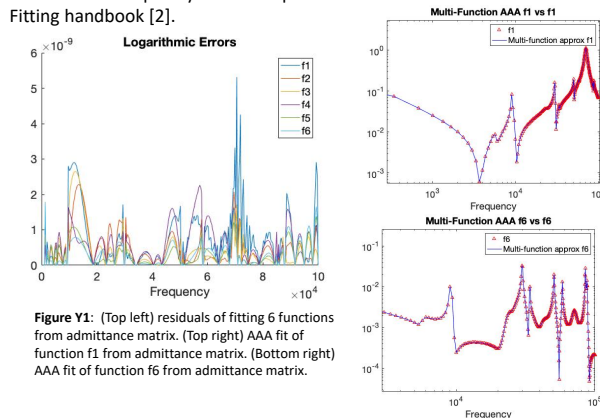


Figure Y1: (Top left) residuals of fitting 6 functions from admittance matrix. (Top right) AAA fit of function f1 from admittance matrix. (Bottom right) AAA fit of function f6 from admittance matrix.

Scenario Generation for Economic Dispatch

Increasing penetrations of renewable energy sources, e.g. wind, into power grids motivates investigating new approaches to computing 5-minute economic dispatch. We investigate using importance sampling with analog scenarios in two-stage stochastic economic dispatch.

Stochastic economic dispatch experiments on the RTS-GMLC network [2] were run for 200 unique timestamps in a simulated year. Six years of WIND Toolkit (WTK) [3] time series data were used as a source of analog scenarios, and 1 year of WTK data was used to simulate actuals.

Two-stage stochastic programming and SAA

Linear two-stage stochastic program:

$$\min_{\mathbf{x}} \mathbf{c}^T \mathbf{x} + \mathbb{E}_{\xi} [L(\mathbf{x}, \xi)] \text{ where } L(\mathbf{x}, \xi) = \min_{\mathbf{y}} \mathbf{c}_{\xi}^T \mathbf{y}$$

$$\text{s. t. } \mathbf{T}_{\xi} \mathbf{x} + \mathbf{W}_{\xi} \mathbf{y} = \mathbf{b}_{\xi}$$

$$\mathbf{y} \geq \mathbf{0}$$

\mathbf{x} – first stage variables (generator setpoints)

\mathbf{y} – second stage variables (e.g. amount of wind dispatched, slack)

ξ – uncertain variables (deviation from wind power persistence)

Sample average approximation

$$\mathbb{E}_{\xi} [L(\mathbf{x}, \xi)] \approx \frac{1}{N} \sum_{i=1}^N L(\mathbf{x}, \xi_i)$$

Continuous importance sampling

$$\mathbb{E}_p [L(\mathbf{x}, \xi)] = \int_{\Omega} L(\mathbf{x}, \xi) p(\xi) d\xi$$

$$= \int_{\Omega} \frac{L(\mathbf{x}, \xi) p(\xi)}{q(\xi)} q(\xi) d\xi = \mathbb{E}_q \left[\frac{L(\mathbf{x}, \xi) p(\xi)}{q(\xi)} \right]$$

Sampling scenarios from WTK:

1. Compute deterministic costs \tilde{c}_j of scenarios.
2. Set probability of selecting scenario j by $\tilde{c}_j / \sum_j \tilde{c}_j$.
3. Draw scenarios

Two-stage stochastic economic dispatch over varying dates and times

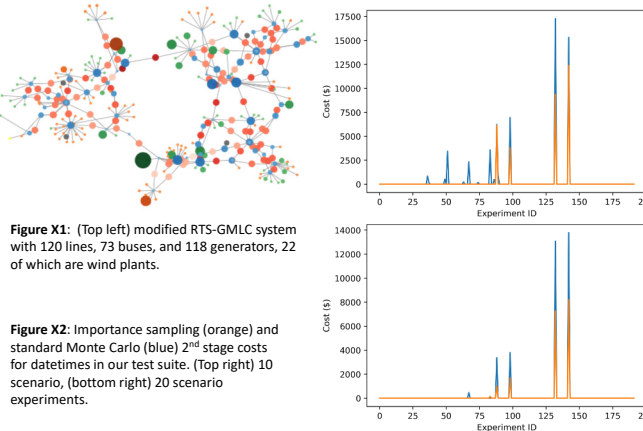


Figure X1: (Top left) modified RTS-GMLC system with 120 lines, 73 buses, and 118 generators, 22 of which are wind plants.

Figure X2: Importance sampling (orange) and standard Monte Carlo (blue) 2nd stage costs for datetimes in our test suite. (Top right) 10 scenario, (bottom right) 20 scenario experiments.

Sampling method	# of scenarios	First stage costs (\$)	Second stage costs (\$)
MC	10	620418	58556
MC	20	620937	34512
IS	10	621374	31700
IS	20	622311	18317

Table X1: Sums of 1st and 2nd stage costs from economic dispatch experiments in Fig. x2.

Wind Farm Yaw Control via ADMM-RL

In this work[5], an innovative distributed control algorithm is proposed by combining Alternating Direction Method of Multipliers (ADMM) [6] and reinforcement learning. This algorithm replaces one or more of the subproblems in ADMM with several steps of RL. When the nested iterations converge, there gives a pretrained sub-solver that can potentially increase the efficiency of the deployed distributed controllers by orders of magnitude.

Wind farm yaw control as a "consensus problem"

Wind Farm Yaw Control Objective:

Objective for wind farm yaw control is to maximize the total power generation of N turbines over control horizon T , under the stochastic wind behavior ϵ^t . x^t is the control variable, yaw angle.

$$\max_x P_{tot}(x) = \sum_t \sum_i P_i(x^t, \epsilon^t)$$

This problem has the form of ADMM consensus problem, using RL to solve distributed subproblems every iteration.

$$x_i^{k+1} = \arg \min_{x_p(\epsilon^t)} \text{RL} f_i(x_i) + y_k^{k,T} (x_i - \bar{x}^k) + \frac{\rho}{2} \|x_i - \bar{x}^k\|^2$$

$$y^{k+1} = y^k + \rho(x_i^{k+1} - \bar{x}^k)$$

Example: Six-turbine wind farm yaw control

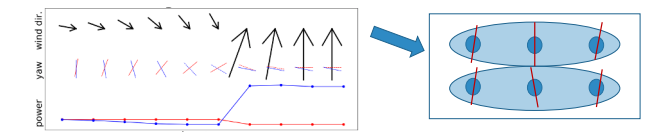


Figure Z1: (Top Left) Single turbine multi-step control demo using Floris. Black arrows indicate the wind speed and direction over time. RL (blue) controller shows the ability to "plan ahead" and generate more power than the baseline controller (red).

Figure Z2: (Top Right) Test case of 6 turbines.

Figure Z3: (Right) Result shows turbines are able to steer their wakes away from downwind turbines, in order to achieve higher power production.

Table Z1: Episodic power production for 6 turbines, ADMM-RL learnt controller achieves comparable result but can be operated in real time.

	floris $\Delta\gamma \infty$	floris $\Delta\gamma 10$	ADMM-RL
Power(MW)	81.6	79.0	77.5

References

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