



Evaluation of Topology Optimization to Achieve Energy Savings at the Urban District Level

Preprint

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Evaluation of Topology Optimization to Achieve Energy Savings at the Urban District Level

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ABSTRACT

Advanced district thermal energy systems have the potential to achieve significant energy savings and facilitate the integration of renewable thermal resources and waste heat, contributing to reductions in carbon emissions. Such systems, also known as fifth generation district heating and cooling (5GDHC) systems, circulate water at temperatures close to ambient, and leverage electrically driven water-source heat pumps located at connected buildings to further temper the water. However, barriers exist to the adoption of 5GDHC systems, including the factorial growth in potential network configurations as a function of the number of considered buildings. Topology optimization, which seeks to answer the questions, “Which is the best subset of buildings, if any, to connect to a district thermal energy system, and by what network should they be connected, to minimize life cycle cost?” can accelerate the adoption of 5GDHC systems. This study is part of an effort to develop a topology optimization framework for district thermal energy systems. In this study, a heuristic for one important aspect of the topology optimization problem—the use of the minimal spanning tree network to connect a given set of buildings at the least life cycle cost—is validated.

INTRODUCTION

Emerging technologies and design practices have enabled significant improvements in building energy performance (U.S. Department of Energy [DOE] 2015). Examining energy use intensity (EUI) beyond the level of an individual building, and at the level of an urban district, can unlock even greater reductions in energy use and associated carbon emissions (Polly 2016). Past studies have demonstrated that there is significant techno-economic potential for greater adoption of district energy systems (DES) in urban areas in the United States, and that advanced district thermal energy systems can meet targets for reductions in carbon emissions more cost-effectively than building-level systems alone (Gils et al. 2013; Connolly et al. 2014). Past work has characterized the evolution of district thermal energy systems into either four or five generations, with the later generations circulating water as opposed to steam and leveraging more moderate temperatures (Lund et al. 2018). As delineated in the work of Buffa

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et al. (2020), fifth generation district heating and cooling (5GDHC) systems are characterized by their use of water at temperatures close to ambient as a working fluid, and leveraging water-source heat pumps located at connected buildings to further temper the water as needed to heat and cool the buildings. A schematic representation of such a system is shown in Figure 1. This configuration facilitates synergistic exchange of heat among connected buildings and processes, through bi-directional thermal flow, offsetting the requirements for active heating and cooling of the district loop, as well as facilitating the integration of waste heat and renewable thermal sources. However, barriers exist to the adoption of 5GDHC systems, including the factorial growth in potential network configurations as a function of the number of considered buildings. Topology optimization seeks to answer the questions: “Which is the best subset of buildings to connect to a district thermal energy system, and by what network should they be connected to minimize life cycle cost?” Topology optimization has the potential to accelerate the adoption of 5GDHC systems.

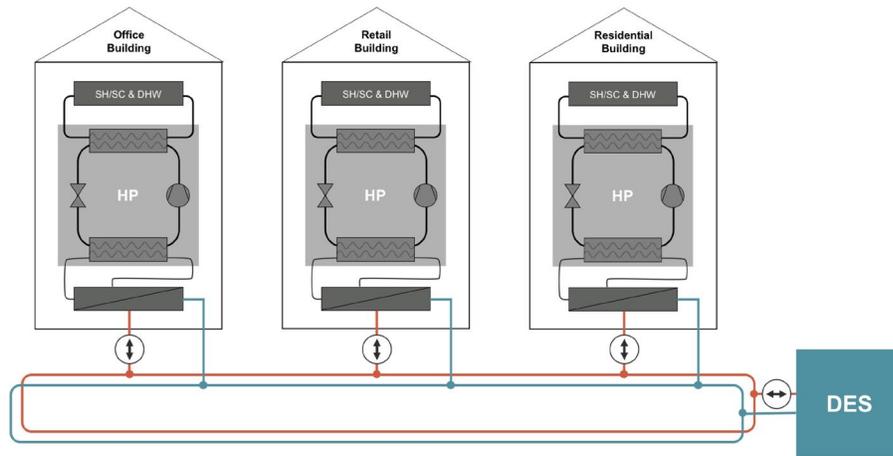


Figure 1 Schematic diagram of 5GDHC system (courtesy of von Rhein et al. 2019).

Network topology optimization is particularly relevant in the context of 5GDHC systems relative to earlier generations of district heating and cooling systems. “Conventional” district thermal energy systems circulating steam or hot water and chilled water are typically configured with radial networks, or with ring networks if redundancy of supply is essential (von Rhein et al. 2019). The potential for synergistic thermal exchange among buildings tied to a 5GDHC system motivates consideration of ring or meshed networks (Jensen et al. 2017; Schluck et al. 2015). In the context of the network topology optimization problem, the number of potential thermal networks that could connect a subset of buildings in a given urban district grows factorially with the number of buildings considered and is a function of the number of subsets of buildings that can be selected from the district and the number of potential networks by which a given subset can be connected. Note that in this study, it is not assumed that every building in a given urban district will be served by a district thermal energy system. As an illustration of this concept, all potential networks that could connect buildings in a district consisting of three buildings and a central plant are shown in Figure 2.

This study is part of a larger effort to develop a topology optimization framework for district thermal energy systems. The goal of the framework is to assist urban planners and engineers in the design of advanced district thermal energy systems by determining the optimal subset of buildings (if any) in an urban district to connect to such a system, and the optimal network by which to connect them. In this work, optimality is defined with respect to life cycle cost, encompassing costs associated with infrastructure, operating energy, and carbon. The very large “search space” associated with this optimization problem results in a computationally intractable problem: the search space is

on the surprisingly high order of 10^{15} scenarios for a district consisting of 10 buildings, motivating consideration of heuristics to address this part of the problem.

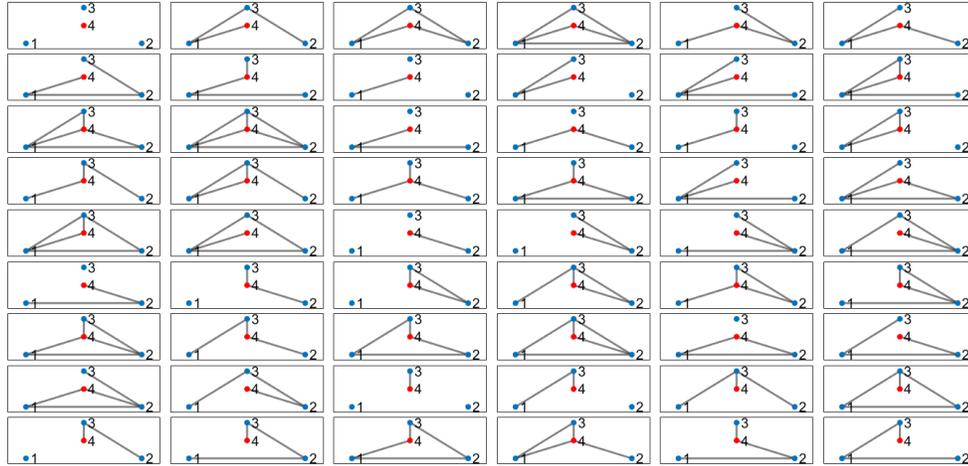


Figure 2 Illustration of all possible thermal networks to connect three buildings (shown with blue nodes) and a central plant (shown with a red node) (courtesy of Allen et al. 2020).

Concepts from graph theory can be leveraged to describe thermal networks. In the mathematical context, an “undirected graph” consists of a set of nodes and a set of edges, which can be represented as unordered pairs of nodes (Bullo et al. 2019). Such a graph can be represented by an “adjacency matrix,” a binary matrix with a number of rows and columns equal to the number of nodes of the graph, in which a given element of the matrix is equal to one if there exists an edge between the corresponding nodes, and zero if there does not (Bullo et al. 2019). An undirected graph provides a convenient structure for representing thermal networks with “nodes” corresponding to buildings or a central plant and “edges” corresponding to thermal connections. The “minimal spanning tree” (MST) is the graph connecting a given set of nodes with the least total edge length, while providing a path between any two nodes in the set (Bullo et al. 2019). In the context of DES, the MST represents the network that connects a given set of buildings with the least total length of pipes, thus minimizing the infrastructure cost. In this study, the use of the MST to identify the best (least life cycle cost) thermal network by which to connect a given set of buildings was evaluated as a heuristic for the topology optimization problem, substantially reducing the number of scenarios to be considered.

Past work in the area of topology optimization for district thermal energy systems, such as that of Li and Svendsen (2013), Marty et al. (2018), and Falke et al. (2016), has primarily focused on high-temperature heating-only networks, and has often addressed only radial networks, and treated the connection status of a particular building to a thermal network as a boundary condition. This study is distinct from past work because it addresses both the selection of the set of buildings to connect to a DES as well as the network by which they are connected, and considers 5GDHC systems, which introduce more complexities, and more interesting potential synergistic interactions among building loads, addressing a need identified by Best et al. (2020). This work builds on that of von Rhein et al. (2019), who developed a model of a 5GDHC system and evaluated the performance of particle swarm optimization for the network topology optimization problem for a district consisting of three prototypical buildings, with a small search space of 54 scenarios.

METHODS

In this study, the MST heuristic was evaluated by performing an exhaustive search of all potential network configurations for a prototypical urban district consisting of five buildings and a central plant and confirming that the MST indeed represented the network with the least life cycle cost to connect a given subset of buildings. The search space consisted of 30,770 possible scenarios, including the “null case” in which all buildings were served by independent systems, with 32 possible subsets of buildings in the district. For each subset, it was confirmed that the MST network had the least life cycle cost relative to other networks connecting the same subset of buildings, or a life cycle cost within a very small margin (0.1%) of the least life cycle cost.

For purposes of this study, the topology optimization problem was formulated based on minimizing the life cycle cost associated with the infrastructure for a district thermal energy system and operating energy costs for heating, ventilating, and air conditioning (HVAC), and associated carbon costs (assuming a carbon fee), for the urban district.¹ The infrastructure cost was defined as the cost associated with pipes for the district thermal energy system. It was assumed that any cost difference between district-level and building-level HVAC generation and distribution equipment would be insignificant. Note that the costs associated with operating energy encompassed all buildings in the considered district, whether or not they were served by the district thermal energy system in a particular scenario. The life cycle cost was calculated for a 30-year time period. The optimization problem was formulated as follows:

$$\min_{\mathbf{A}} C_{pipes} + C_{ele} UPV_{ele}(E_{de} + \sum_{i=1}^n E_{be,i}) + C_{gas} UPV_{gas} \sum_{j=1}^n E_{bg,j} \quad (1)$$

$$+ \sum_{t=1}^{30} m_{CO_2}(t) C_{CO_2}(t) UPV_{CO_2}$$

where \mathbf{A} is the adjacency matrix representing the connectivity of the thermal network; C_{pipes} is the cost associated with the infrastructure of the DES; C_{ele} and C_{gas} are the costs of electricity and gas per unit of consumption; UPV_{ele} , UPV_{gas} , and UPV_{CO_2} are the uniform present value factors used to convert an annual cost to the value over the lifetime of the system for electricity, gas, and carbon, respectively; E_{de} is the energy consumption associated with the centralized DES heat pump and distribution pumps; E_{be} is the electricity consumption for HVAC at a given building; E_{bg} is the gas consumption for HVAC at a given building; $m_{CO_2}(t)$ is the emissions of carbon dioxide associated with the district’s energy use for a given year; and $C_{CO_2}(t)$ is the unit cost of carbon dioxide emissions in a given year. Note that the gas consumption term exists because buildings with independent HVAC systems are served by gas heating systems. The uniform present value factors associated with electricity and natural gas account for expected escalation in the costs of these utilities projected by the National Institute for Standards and Technology, documented in the work of Lavappa and Kneifel (2019). The real annual electricity cost escalation rates (between -4% and 2%) and natural gas cost escalation rates (0% to 9%) varied from year to year over the time period considered. Escalation of the carbon price also varies from year to year, ranging from 0% to 33% annually over the time period considered (Lavappa and Kneifel 2019).

The economic inputs to the life cycle cost calculation considered in this study, and their sources, are summarized in Table 1. In this work, the energy consumption terms in the cost function were evaluated using an energy model developed in Modelica, an object-oriented, equation-based language that is often used for modeling physical systems (Modelica Association 2020). Note that for purposes of this study, the calculated energy consumption was taken to be the same for each year in the considered life span of the system (30 years). The model of a 5GDHC system was extended from one developed by von Rhein et al. (2019) as the 5GDHC Topology Analysis Tool. The 5GDHC system considered, similar to the one represented in Figure 1, was configured with a two-pipe network, circulating

¹ Note that the HVAC energy consumption does not include energy associated with humidification in the prototypical hospital building, which was not supplied through the district thermal energy system.

warm water at 26°C and cool water at 16°C, with the loop tempered by a ground-source heat pump, and water-source heat pumps tempering the water further at each connected building. The mass flow rate at each building was controlled to maintain a 10°C temperature differential between the “warm pipe” and “cool pipe.”

The prototypical district considered consisted of three multi-family buildings, a hospital, and a retail building. The building types were selected based on a thermal load diversity metric developed by Zarin Pass et al. (2018). The load diversity metric reflects the extent to which simultaneous heating and cooling loads exist in the district, which improves the exergetic efficiency of a 5GDHC system, by increasing the extent to which buildings can reject heat to or draw heat from the thermal network in a synergistic manner. In the work of Zarin Pass et al. (2018), a threshold for the diversity metric was established (a value of 0.60), above which 5GDHC systems are likely to exceed distributed building-level HVAC systems in exergetic efficiency. The combination of buildings selected for this study, three multi-family buildings and one hospital, yield a value of the thermal load diversity metric of 0.58, close to this threshold. For purposes of calculating pipe lengths, the locations of the buildings were superimposed on an existing block in Golden, Colorado. The analysis was performed using the Typical Meteorological Year 3 weather file for Golden-NREL (724666).

Table 1. Summary of Economic Parameters in Life Cycle Cost Calculation

Parameter	Value		Units		Source
Discount rate	3%		N/A		von Rhein et al. (2019)
Electricity cost, base year	0.10	27.8	\$/kWh	\$/GJ	U.S. Energy Information Administration (EIA) (2019)
Natural gas cost, base year	0.68	6.48	\$/therm	\$/GJ	EIA (2019)
Carbon cost, base year	18.1	20.0	\$/st	\$/mt	Lavappa and Kneifel (2019)
Pipe cost	167	548	\$/ft	\$/m	Best et al. (2020) and Rafferty (1996)

Building heating and cooling loads were represented using data-driven metamodels, leveraging the Metamodeling Framework developed by Long (2018). The Metamodeling Framework has been demonstrated to represent building loads in an accurate manner and reduces the computational complexity of the district energy model relative to the use of full-order models to represent building loads (von Rhein et al. 2019). The metamodels were trained based on data generated from the DOE prototype building models in EnergyPlus® (DOE 2017). Separate metamodels were trained to represent building thermal loads for the case in which the building was connected to the district thermal energy system, and the case in which the building was served by independent systems. For the metamodels representing the case in which the buildings were served by independent systems, the HVAC system types existing in the prototype building models were preserved. For the case in which the buildings were served by a district thermal energy system, the underlying EnergyPlus models were altered to represent a consistent HVAC system type, with water-source heat pumps tied to a heat source and sink at temperatures corresponding to that of the district loop. It was confirmed that the modified HVAC systems continued to meet thermal loads in the modeled buildings. Details of the prototypical buildings are summarized in Table 2.

Table 2. Characteristics of Prototypical Buildings Considered in Study

Building	Floor Area (m ² /ft ²)	Individual. HVAC System
Hospital	22,436/241,500	Variable-air volume reheat system served by water-cooled chillers and hot water boilers
Multifamily	3,120/33,585	Zone-level direct-expansion cooling and gas heating
Retail	2,295/24,705	Roof-top units with DX cooling and gas heating

Due to the computational burden of the energy model simulations, hierarchical clustering was performed on the

building heating and cooling load profiles to select a smaller set of days to represent an entire year, following an approach similar to that outlined in the work of Nahmmacher et al. (2016). It was found that a set of 30 days could represent the annual heating and cooling load profiles with a coefficient of variation of the root-mean squared error CV(RMSE) of 10.8% and 5.9%, respectively, which was deemed to be acceptable per criteria in ASHRAE Guideline 14 (ASHRAE 2014).

RESULTS

In this study case, the nature of the set of buildings connected to the network was highly influential on the overall life cycle cost. As shown in Figure 3a, the inclusion of the prototypical hospital building in the set of connected buildings was necessary in order for the life cycle cost of a particular scenario to be less than that associated with the “null case,” in which all buildings are served by independent heating and cooling systems. The hospital building is the only prototypical building for which a connection to the DES results in lower annual building-level energy costs relative to independent systems. For all of the prototypical buildings, a connection to the DES results in lower source- and site-EUI (albeit a very slight reduction in the case of source EUI for the multi-family building), but the low costs of natural gas result in lower annual energy costs when independent systems are employed for the multi-family and retail buildings. Note that the DES considered uses only electricity for both heating and cooling. The prototypical hospital building benefits from a substantial reduction in EUI with a connection to the DES relative to building-level systems due to its year-round base cooling load, which, when the building is connected to the DES, can offset a simultaneous heating load through the shared water loop serving the building’s heat pumps and through bi-directional heat and mass flow in the energy transfer station. Note that the zone-level HVAC systems implemented in the “connected” case for the hospital building also eliminate the use of reheat for temperature control. Figure 3b shows heating and cooling EUIs, on both a site and source basis, for the prototypical buildings considered, in both their “connected” and “independent” states. This benefit from simultaneous heating and cooling loads at the building level is consistent with results observed in other studies, including that of Wirtz et al. (2020).

District-wide source EUI for HVAC follows a trend similar to the one observed for life cycle cost, as shown in Figure 4a. Note that this EUI value encompasses energy consumption for generation and rejection of heat at the building level and at the centralized heat pump serving the DES, and distribution pumping energy for the DES. Site-source multipliers for electricity and natural gas were obtained from the U.S. EPA (2019). The scenarios in which the hospital building is not served by the DES have a similar source HVAC EUI to the null case, as the prototypical hospital building has the largest thermal load. The scenarios in which the hospital building is served by the DES have an associated source EUI that is approximately 25% lower than that of the null case. This result highlights the potential reductions in source energy and associated carbon emissions from 5GDHC systems, as well as the benefits of optimization in the selection of the subset of buildings to connect to the network.

Analysis of the results confirms that the MST network indeed represents the least-cost network to connect any of the thirty-two subsets of buildings associated with the prototypical urban district considered, or results in a life cycle cost within 0.1% of that of the least-cost network. These results are summarized in Figure 4b with a plot of the “MST ratio” as a function of network length. The “MST ratio” is defined as the ratio of the life cycle cost of a given scenario to the life cycle cost associated with the MST connecting the same subset of buildings. The fact that the MST ratio is always greater than or equal to unity affirms the validity of the heuristic. Note that the network with the greatest life cycle cost to connect a given subset of buildings never exceeds the cost associated with the MST by more than 20%. In this case study, the selection of the subset of buildings to connect to the DES has more influence on life cycle cost than the network by which they are connected. For the set of possible networks connecting a given subset of buildings, the life cycle cost generally increases with the length of the network, as the infrastructure cost scales with the network length, and little variation in energy and carbon costs is observed among networks connecting the same subset of buildings. Thus, in Figure 4b, a linear trend is generally observed in the MST ratio as a function of the total network length.

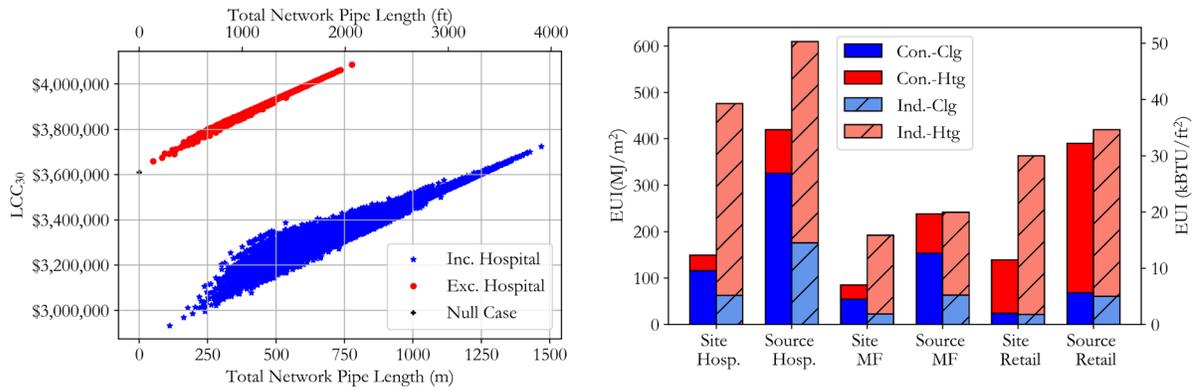


Figure 3 (a) (Left): life cycle cost (LCC) as a function of total network pipe length for all scenarios, sub-divided by whether they include or exclude the hospital building; (b) (right): building-level HVAC EUI for prototypical buildings considered.

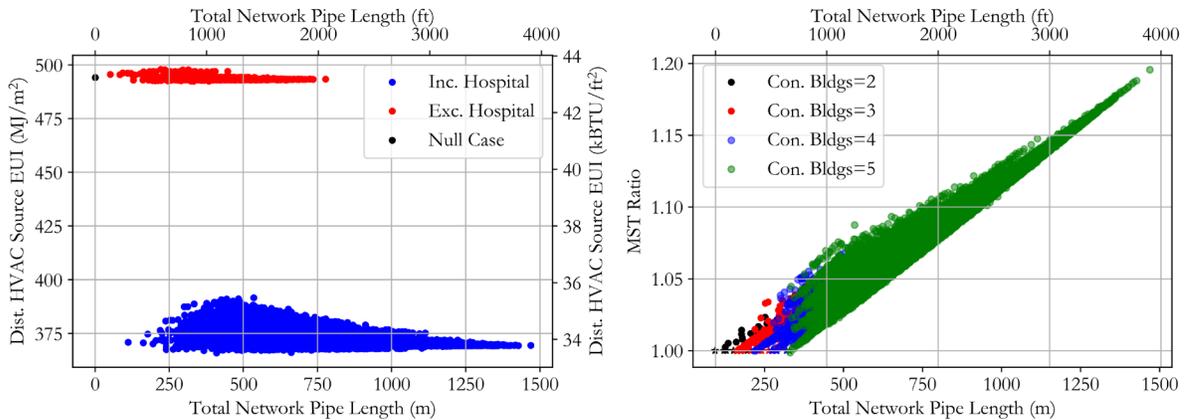


Figure 4 (a) (Left): District-level HVAC source EUI as a function of total network pipe length for all scenarios; (b) (right): MST ratio as a function of total network pipe length for all scenarios considered in this case study.

Figure 5 shows a disaggregation of life cycle cost for a random subset of the scenarios considered in this study. Among all scenarios considered, the fraction of life cycle cost attributable to energy costs ranged from 68% to 83%, carbon from 10% to 21%, and infrastructure, 0% to 22% (The null case, representing a scenario in which all buildings are served by independent systems, has no costs for DES infrastructure). The fact that infrastructure-related costs account for less than 25% of the overall life cycle cost in all scenarios in this case study is consistent with the fact that the selection of the subset of buildings has a greater influence on life cycle cost than the network by which they are connected.

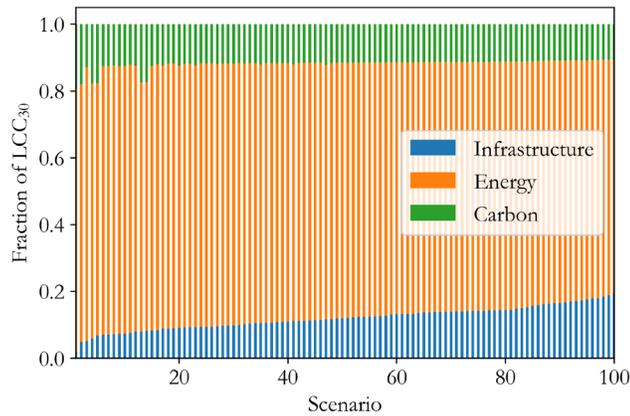


Figure 5 Disaggregation of life cycle cost for selected set of representative scenarios.

CONCLUSION

This study has validated the use of the MST as a heuristic for identifying the thermal network with the least life cycle cost for connecting a given set of buildings in an urban district. This study also demonstrated a potential for significant reduction in HVAC-source EUI (on the order of 25%) from the use of a 5GDHC system with an optimally selected subset of buildings in an urban district, relative to independent, building-level HVAC systems. Future work will quantify the benefits of integration of available low-temperature waste heat into a similar 5GDHC network, which is expected to improve further the energy and economic performance of the 5GDHC network relative to independent systems. Future work will also investigate the use of an algorithm, such as particle swarm optimization, for selection of the “best” subset of buildings (if any) in an urban district to connect to a 5GDHC network. An approach for selecting the best subset of buildings will be combined with the MST heuristic to create a topology optimization framework for district thermal energy systems. It is anticipated that this framework will help facilitate the adoption of 5GDHC systems in appropriate applications, and contribute to energy and cost savings as well as reduced carbon emissions.

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