A Computationally Efficient Algorithm for Computing Convex Hull Prices

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Introduction

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Analysis of Approximate Convex Hull Prices

Conclusion
Introduction

Non-convex markets

– Operating markets with non-convexities poses challenges both theoretical and practical
– Theoretical: Typically no (good) convergence guarantees for associated optimization problems; uniform market-clearing prices may not exist
– Practical: Consequences of clearing the market with a suboptimal solution; which of many pricing mechanisms to use; how to enforce compliance with schedule for participants not correctly incentivized

Convex Hull Pricing (CHP)

– Minimizes specific side payments
  • lost opportunity costs are typically lower with CHP
– Not tied to a (suboptimal) primal solution
  • Can lead to counter-intuitive results; e.g., off-line units can be marginal, non-binding transmission constraints in primal solution can have non-zero shadow prices
– Can be difficult to compute in practice
Introduction

Non-convexities in Electricity Markets

- Two main sources: generator operating characteristics (minimum stable power level, minimum run/stop time, start-up costs) and AC power flow
- Market operators in the US typically linearize power flow constraints but not generator operating characteristics
- Medium- to long-term scheduling in US markets therefore involves solving a mixed-integer linear programming problem
- This work considers the convex hull pricing problem (CHPP) in this context
Contribution

Solving the Convex Hull Pricing Problem (CHPP)

- Hua and Baldick (2017) proved a primal linear programming problem could solve CHPP if explicit representations of the convex hull for all generating units is known.
- Such linear programs can be very large – the best-known convex hull formulation for a generator grows cubically with the number of time periods.
  - If some extra constraints, like daily maximum power are present, the best known convex hull formulation is exponential in the number of time steps.
- Alvarez et al. (2019) and Yu et al. (2020) propose algorithms which are heuristics when generators have binding ramping limits.
- This work proposed a Benders decomposition approach for solving CHPP, leveraging both recent generator convex hull developments and recent advancements in tight and compact formulations for UC (K. et al. 2020).
Primal Formulation & Decomposition for CHPP
Explicit formulations of the convex hull for every market participant, as required for the Primal CHPP, typically make the approach intractable for large systems. Recent research (Frangioni & Gentile 2015, K. et al. 2018, Guan et al. 2018) makes this less of an issue for the prototypical thermal generator.
Primal Formulation for CHPP

Unit Commitment

\[ \min \sum_{g \in \mathcal{G}} c^g \]

\[ \sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) + N(s) = D \]

\[(u^g, p^g, c^g) \in \Pi^g, \ \forall g \in \mathcal{G} \]

Primal aCHPP

\[ \min \sum_{g \in \mathcal{G}} c^g \]

\[ \sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) + N(s) = D \] \hspace{1cm} (\pi^{a\text{CHP}(\mathcal{R})})

\[(u^g, p^g, c^g) \in \mathcal{R}(\Pi^g), \ \forall g \in \mathcal{G}. \]

One way around this issue is to instead compute “approximated” convex hull prices. Here \( \mathcal{R}(\Pi^g) \) is some MILP relaxation for the set \( \Pi^g \).

Issue: different MILP relaxations may lead to different prices (Zheng et al. 2018)
Because $\mathcal{R}(\Pi^g) \subseteq \text{conv}(\Pi^g)$, the Primal aCHPP problem is a relaxation of the Primal CHPP problem. It follows that $\mathcal{R}(\Pi^g)$ can be added to the Primal CHPP problem and not change the feasible region.
Here the set $\mathcal{F}_g$ can be considered as the projection of $\text{conv}(\Pi^g)$ onto the $(u^g, p^g, c^g)$ variables, or in the Benders context, the subset of the projection (along with $\mathcal{R}(\Pi^g)$) sufficient to ensure $(u^g, p^g, c^g) \in \text{conv}(\Pi^g)$. 

Primal CHPP (redundant constraints/EF)

$$\min \sum_{g \in \mathcal{G}} c^g$$

$$\sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) + N(s) = D \quad (\pi_{\text{CH}})$$

$$(u^g, p^g, c^g) \in \text{conv}(\Pi^g), \ \forall g \in \mathcal{G}.$$ 

$$(u^g, p^g, c^g) \in \mathcal{R}(\Pi^g), \ \forall g \in \mathcal{G}.$$ 

Primal Projected/Benders CHPP

$$\min \sum_{g \in \mathcal{G}} c^g$$

$$\sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) + N(s) = D \quad (\pi_{\text{CH}})$$

$$\alpha_{g,i}^T u^g + \beta_{g,i}^T p^g + \delta_{g,i}^T c^g \leq \eta_{g,i}, \ \forall i \in \mathcal{F}_g, \ \forall g \in \mathcal{G}$$

$$(u^g, p^g, c^g) \in \mathcal{R}(\Pi^g), \ \forall g \in \mathcal{G}.$$
Benders Decomposition for CHPP

Algorithm 1 (BENDERS CHP) Solves the CHPP using Benders decomposition.

\[ \mathcal{G}^R \leftarrow \{g \in \mathcal{G} \mid g \text{ has irredundant ramping constraints}\} \]
\[ \text{cuts} \leftarrow \text{True} \]
\[ \text{while cuts do} \]
\[ \quad \text{cuts} \leftarrow \text{False} \]
\[ \quad \text{Solve master problem (3) with } \mathcal{R} = \mathcal{R}^* \]
\[ \quad \text{for } g \in \mathcal{G}^R \text{ do} \]
\[ \quad \quad \text{Solve feasibility subproblem (7) for } g \text{ with} \]
\[ \quad \quad \quad (\hat{u}^g, \hat{p}^g, \hat{c}^g) \text{ fixed from solution of master} \]
\[ \quad \quad \text{if feasibility subproblem is infeasible then} \]
\[ \quad \quad \quad \text{add cut to master problem} \]
\[ \quad \quad \quad \text{cuts} \leftarrow \text{True} \]
\[ \quad \text{return dual values } \pi \text{ of Constraint (3b)} \]

Theorem 2 (Benders Procedure[13]). If \( \text{conv}(\Pi^g) \) is a polytope for every \( g \in \mathcal{G} \), and the CGLP solver returns vertex solutions, then Algorithm 1 converges in a finite number of iterations.

Theorem 3. The price, \( \pi \), returned by Algorithm 1 is a convex hull price. That is, there exists an optimal solution \((\pi_{\text{conv}}, \lambda_{\text{conv}})\) to the dual of the EF where \( \pi_{\text{conv}} = \pi \).
Computational Results
Test Instances & Platform

Test instances: pglib-uc (https://github.com/power-grid-lib/pglib-uc)

- 24 & 48 hourly time steps
- Three sets of instances, ferc, ca, rts_gmlc
- Number of generators
  - ferc: ~900, ca: ~600, rts_gmlc: ~100

Platform

- Gurobi used to solve all LPs
- 24-hour instances: MacBook Pro
- 48-hour instances: 64-core Linux workstation
## Computational Results: 24-hour Instances

<table>
<thead>
<tr>
<th></th>
<th>Cut Time (s)</th>
<th>Cuts Added</th>
<th>Iterations</th>
<th>Total Time (s)</th>
<th>EF Time (s)</th>
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<tr>
<td><strong>ferc</strong></td>
<td></td>
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<tr>
<td>min</td>
<td>0.74</td>
<td>20</td>
<td>6</td>
<td>13.55</td>
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<tr>
<td>mean</td>
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<td>115.79</td>
<td>14.20</td>
<td>20.55</td>
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<tr>
<td>max</td>
<td>13.27</td>
<td>577</td>
<td>47</td>
<td>40.62</td>
<td>2932.2</td>
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</table>

| **ca** |              |            |            |                |             |
| min    | 0.00         | 0          | 1          | 1.75           | 17.58       |
| mean   | 0.05         | 0.15       | 1.15       | 2.70           | 24.23       |
| max    | 0.19         | 1          | 2          | 3.87           | 35.08       |

| **rts_gmlc** |              |            |            |                |             |
| min    | 0.01         | 0          | 1          | 0.37           | 17.13       |
| mean   | 0.10         | 2.79       | 2.62       | 0.74           | 22.28       |
| max    | 0.30         | 12         | 7          | 1.26           | 39.90       |
## Computational Results: 48-hour Instances

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<tr>
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<th></th>
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<td>Time (s)</td>
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<tr>
<td><strong>min</strong></td>
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<td>42</td>
<td>5</td>
<td>61.1</td>
<td></td>
<td>*</td>
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<tr>
<td><strong>mean</strong></td>
<td>93.4</td>
<td>184.6</td>
<td>20.2</td>
<td>165.0</td>
<td></td>
<td>*</td>
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<tr>
<td><strong>max</strong></td>
<td>480.5</td>
<td>1084</td>
<td>86</td>
<td>658.1</td>
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### ferc

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<td>Cuts Added</td>
<td>Iterations</td>
<td>Total Time (s)</td>
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<td>Time (s)</td>
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<td><strong>mean</strong></td>
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<td>1.0</td>
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<td>537.5</td>
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<td><strong>max</strong></td>
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<td>0</td>
<td>1</td>
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<td>745.5</td>
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### ca

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<td>Cut Time (s)</td>
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<td>Time (s)</td>
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<tr>
<td><strong>max</strong></td>
<td>18.49</td>
<td>24</td>
<td>16</td>
<td>21.34</td>
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<td>645.5</td>
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### rts_gmlc
Computational Results

LP Size

– Even though the convex hull formulations used are all representable with a polynomial number of variables and constraints, the associated Primal CHPP can be prohibitively large
– For the 48-hour \texttt{ferc} instances, the EF linear programs have 275 million non-zero constraint matrix elements (150 million after pre-solve)

Benders Decomposition

– Tight but compact generator formulations (a subject of much research in the past decade) can be useful for approximating convex hulls
  • Therefore only few Benders iterations are need in practice
Analysis of Approximate CHPs
Difference in energy prices in every hour against the EF approach across all 24-hour FERC instances.

Whiskers are drawn to cover 99% of observations.
Energy price differences

Difference in energy prices in every hour against the EF approach across all 24-hour instances.

Whiskers are drawn to cover 99% of observations.
Difference in energy prices in every hour against the EF approach across all 24-hour rts_gmlc instances.

Whiskers are drawn to cover 99% of observations.
Conclusion
Conclusion

– Convex hull prices for energy for large systems can be efficiently computed leveraging the proposed Benders decomposition approach.

– “Approximated” convex hull prices can have both small and large errors – the tightness of the approximation is empirically related to the tightness of the generator formulation.

– The proposed Benders algorithm can be used as an approximation scheme (terminated after any iteration) if runtime limits are binding.

– More work is needed on convex hull representations of generators with complex ancillary service offers and for market participants whose best-known convex hull representation has an exponential number of variables and constraints (e.g., units with maximum daily power, storage).