A Computationally Efficient Algorithm for Computing Convex Hull Prices

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Introduction

Non-convex markets

- Operating markets with non-convexities poses challenges both theoretical and practical
- Theoretical: Typically no (good) convergence guarantees for associated optimization problems; uniform market-clearing prices may not exist
- Practical: Consequences of clearing the market with a suboptimal solution; which of many pricing mechanisms to use; how to enforce compliance with schedule for participants not correctly incentivized

Convex Hull Pricing (CHP)

- Minimizes specific side payments
  - lost opportunity costs are typically lower with CHP
- Not tied to a (suboptimal) primal solution
  - Can lead to counter-intuitive results; e.g., off-line units can be marginal, non-binding transmission constraints in primal solution can have non-zero shadow prices
- Can be difficult to compute in practice

Introduction

Non-convexities in Electricity Markets

- Two main sources: generator operating characteristics (minimum stable power level, minimum run/stop time, start-up costs) and AC power flow
- Market operators in the US typically linearize power flow constraints but not generator operating characteristics
- Medium- to long-term scheduling in US markets therefore involves solving a mixed-integer linear programming problem
- This work considers the convex hull pricing problem (CHPP) in this context

Contribution

Solving the Convex Hull Pricing Problem (CHPP)

- Hua and Baldick (2017) proved a primal linear programming problem could solve CHPP if explicit representations of the convex hull for all generating units is known
- Such linear programs can be very large – the best-known convex hull formulation for a generator grows cubically with the number of time periods
  - If some extra constraints, like daily maximum power are present, the best know convex hull formulation is exponential in the number of time steps
- Alvarez et al. (2019) and Yu et al. (2020) propose algorithms which are heuristics when generators have binding ramping limits
- This work proposed a Benders decomposition approach for solving CHPP, leveraging both recent generator convex hull developments and recent advancements in tight and compact formulations for UC (K. et al. 2020)

Primal Formulation & Decomposition for CHPP

Primal Formulation for CHPP

Unit Commitment

$$\min \sum_{g \in \mathcal{G}} c^g$$

$$\sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) + N(s) = D$$

$$(u^g, p^g, c^g) \in \Pi^g, \ \forall g \in \mathcal{G}$$

Primal CHPP

$$\min \sum_{g \in \mathcal{G}} c^g$$

$$\sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) + N(s) = D \qquad (\pi^{\mathrm{CH}})$$

$$(u^g, p^g, c^g) \in \mathrm{conv}(\Pi^g), \ \forall g \in \mathcal{G}.$$

Explicit formulations of the convex hull for every market participant, as required for the Primal CHPP, typically make the approach intractable for large systems. Recent research (Frangioni & Gentile 2015, K. et al. 2018, Guan et al. 2018) makes this less of an issue for the prototypical thermal generator.

Primal Formulation for CHPP

Unit Commitment

$$\min \sum_{g \in \mathcal{G}} c^g$$

$$\sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) + N(s) = D$$

$$(u^g, p^g, c^g) \in \Pi^g, \ \forall g \in \mathcal{G}$$

Primal aCHPP

$$\min \sum_{g \in \mathcal{G}} c^g$$

$$\sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) + N(s) = D \quad (\pi^{\mathrm{aCHP}(\mathcal{R})})$$

$$(u^g, p^g, c^g) \in \mathcal{R}(\Pi^g), \ \forall g \in \mathcal{G}.$$

One way around this issue is to instead compute “approximated” convex hull prices. Here $\mathcal{R}(\Pi^g)$ is some MILP relaxation for the set Π^g.

Issue: different MILP relaxations may lead to different prices (Zheng et al. 2018)

Primal Formulation for CHPP

Primal CHPP

$$\min \sum_{g \in \mathcal{G}} c^g$$

$$\sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) + N(s) = D \quad (\pi^{\mathrm{CH}})$$

$$(u^g, p^g, c^g) \in \mathrm{conv}(\Pi^g), \ \forall g \in \mathcal{G}.$$

Primal aCHPP

$$\min \sum_{g \in \mathcal{G}} c^g$$

$$\sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) + N(s) = D \quad (\pi^{\mathrm{aCHP}(\mathcal{R})})$$

$$(u^g, p^g, c^g) \in \mathcal{R}(\Pi^g), \ \forall g \in \mathcal{G}.$$

Because $\mathcal{R}(\Pi^g) \subseteq \mathrm{conv}(\Pi^g)$, the Primal aCHPP problem is a relaxation of the Primal CHPP problem. It follows that $\mathcal{R}(\Pi^g)$ can be added to the Primal CHPP problem and not change the feasible region.

Primal Projected/Benders Formulation for CHPP

Primal CHPP (redundant constraints/EF)

$$\min \sum_{g \in \mathcal{G}} c^g$$

$$\sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) + N(s) = D \quad (\pi^{\mathrm{CH}})$$

$$(u^g, p^g, c^g) \in \mathrm{conv}(\Pi^g), \ \forall g \in \mathcal{G}$$

$$(u^g, p^g, c^g) \in \mathcal{R}(\Pi^g), \ \forall g \in \mathcal{G}.$$

Primal Projected/Benders CHPP

$$\min \sum_{g \in \mathcal{G}} c^g$$

$$\sum_{g \in \mathcal{G}} (A^g p^g + B^g u^g) + N(s) = D \qquad (\pi^{\mathrm{CH}})$$

$$\alpha^T_{g,i} u^g + \beta^T_{g,i} p^g + \delta^T_{g,i} c^g \leq \eta_{g,i}, \ \forall i \in \mathcal{F}_g, \ \forall g \in \mathcal{G}$$

$$(u^g, p^g, c^g) \in \mathcal{R}(\Pi^g), \ \forall g \in \mathcal{G}.$$

Here the set $\mathcal{F}_g$ can be considered as the projection of $\mathrm{conv}(\Pi^g)$ onto the (u^g, p^g, c^g) variables, or in the Benders context, the subset of the projection (along with $\mathcal{R}(\Pi^g)$) sufficient to ensure $(u^g, p^g, c^g) \in \mathrm{conv}(\Pi^g)$.

Benders Decomposition for CHPP

Algorithm 1 (Benders CHP) Solves the CHPP using Benders decomposition.

$\mathcal{G}^R \leftarrow \{g \in \mathcal{G} \mid g$ has irredundant ramping constraints$\}$
$cuts \leftarrow True$
while $cuts$ **do**
  $cuts \leftarrow False$
  Solve master problem (3) with $\mathcal{R} = \mathcal{R}^*$
  for $g \in \mathcal{G}^R$ **do**
    Solve feasibility subproblem (7) for g with
      $(\hat{u}^g, \hat{p}^g, \hat{c}^g)$ fixed from solution of master
    if feasibility subproblem is infeasible **then**
      add cut to master problem
      $cuts \leftarrow True$
return dual values π of Constraint (3b)

Theorem 2 (Benders Procedure[13]). *If $conv(\Pi^g)$ is a polytope for every $g \in \mathcal{G}$, and the CGLP solver returns vertex solutions, then Algorithm 1 converges in a finite number of iterations.*

Theorem 3. *The price, π, returned by Algorithm 1 is a convex hull price. That is, there exists an optimal solution $(\pi^{conv}, \lambda^{conv})$ to the dual of the EF where $\pi^{conv} = \pi$.*

Computational Results

Test Instances & Platform

Test instances: pglib-uc (https://github.com/power-grid-lib/pglib-uc)

- 24 & 48 hourly time steps
- Three sets of instances, `ferc`, `ca`, `rts_gmlc`
- Number of generators
  - `ferc`: ~900, `ca`: ~600, `rts_gmlc`: ~100

Platform

- Gurobi used to solve all LPs
- 24-hour instances: MacBook Pro
- 48-hour instances: 64-core Linux workstation

Computational Results: 24-hour Instances

ferc

	Benders				EF
	Cut Time (s)	Cuts Added	Iterations	Total Time (s)	Time (s)
min	0.74	20	6	13.55	733.4
mean	3.11	115.79	14.20	20.55	1553.8
max	13.27	577	47	40.62	2932.2

ca

	Benders				EF
	Cut Time (s)	Cuts Added	Iterations	Total Time (s)	Time (s)
min	0.00	0	1	1.75	17.58
mean	0.05	0.15	1.15	2.70	24.23
max	0.19	1	2	3.87	35.08

rts_gmlc

	Benders				EF
	Cut Time (s)	Cuts Added	Iterations	Total Time (s)	Time (s)
min	0.01	0	1	0.37	17.13
mean	0.10	2.79	2.62	0.74	22.28
max	0.30	12	7	1.26	39.90

Computational Results: 48-hour Instances

ferc

	Benders				EF
	Cut Time (s)	Cuts Added	Iterations	Total Time (s)	Time (s)
min	13.5	42	5	61.1	*
mean	93.4	184.6	20.2	165.0	*
max	480.5	1084	86	658.1	*

ca

	Benders				EF
	Cut Time (s)	Cuts Added	Iterations	Total Time (s)	Time (s)
min	0.00	0	1	5.91	317.3
mean	1.31	0.0	1.0	11.55	537.5
max	4.72	0	1	17.42	745.5

rts_gmlc

	Benders				EF
	Cut Time (s)	Cuts Added	Iterations	Total Time (s)	Time (s)
min	0.80	0	1	1.85	369.9
mean	4.27	5.29	4.0	6.59	473.7
max	18.49	24	16	21.34	645.5

Computational Results

LP Size

- Even though the convex hull formulations used are all representable with a polynomial number of variables and constraints, the associated Primal CHPP can be prohibitively large
- For the 48-hour `ferc` instances, the **EF** linear programs have 275 million non-zero constraint matrix elements (150 million after pre-solve)

Benders Decomposition

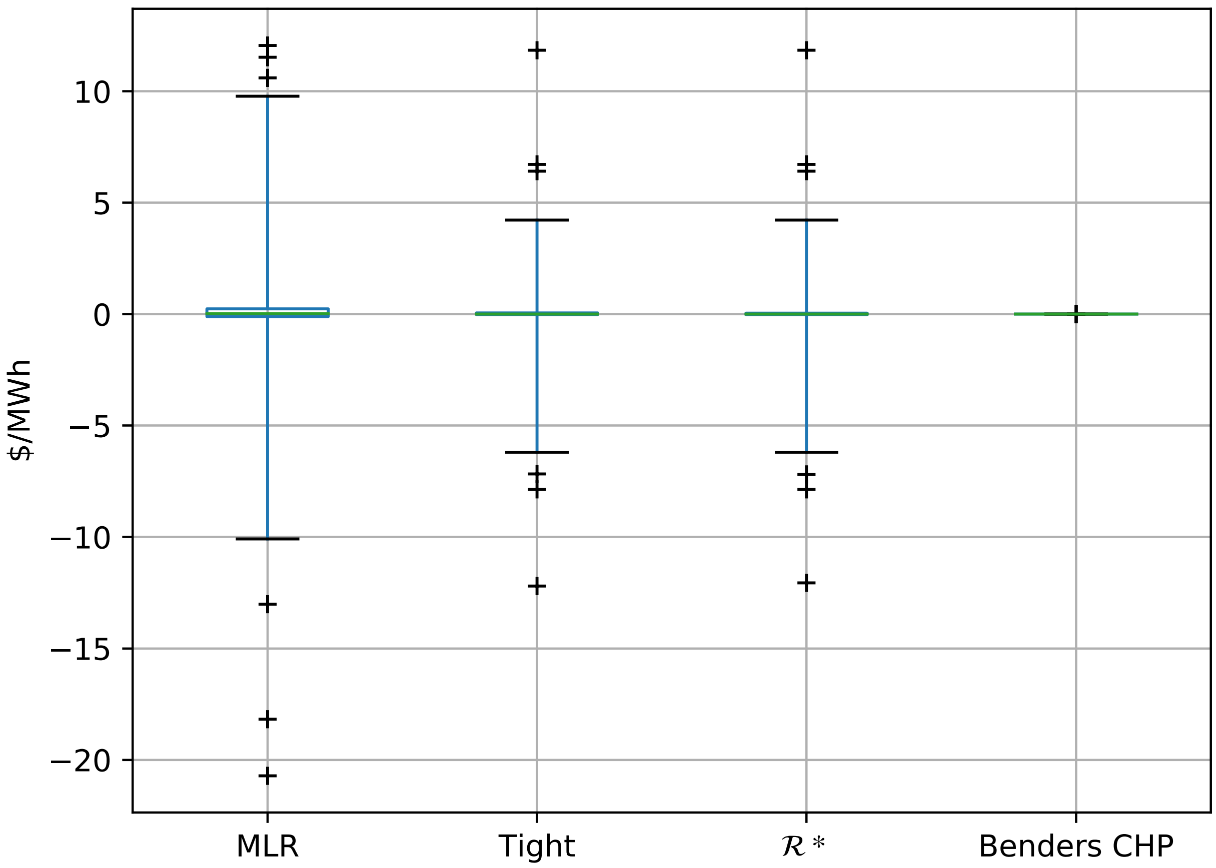
- Tight but compact generator formulations (a subject of much research in the past decade) can be useful for approximating convex hulls
  - Therefore only few Benders iterations are need in practice

Analysis of Approximate CHPs

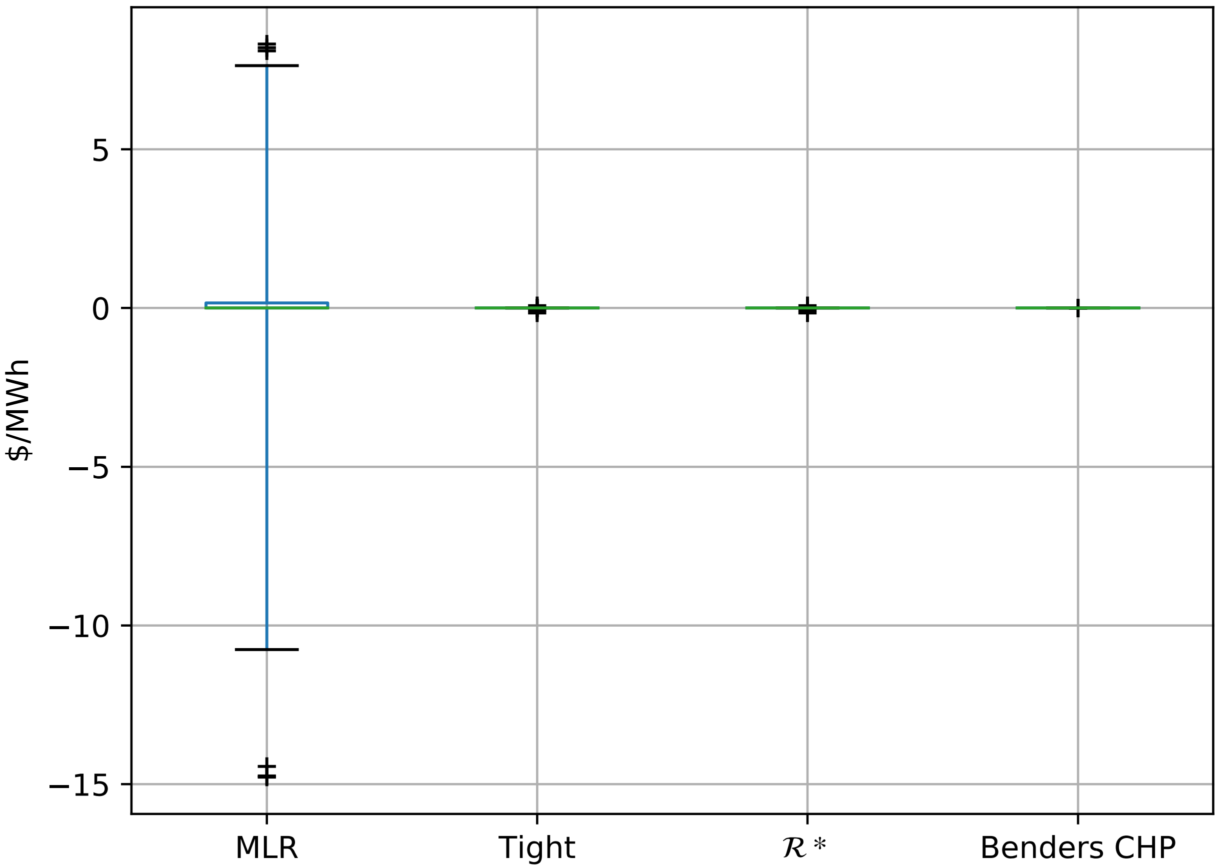
`ferc` energy price differences

Difference in energy prices in every hour against the EF approach across all 24-hour `ferc` instances.

Whiskers are drawn to cover 99% of observations.



ca energy price differences



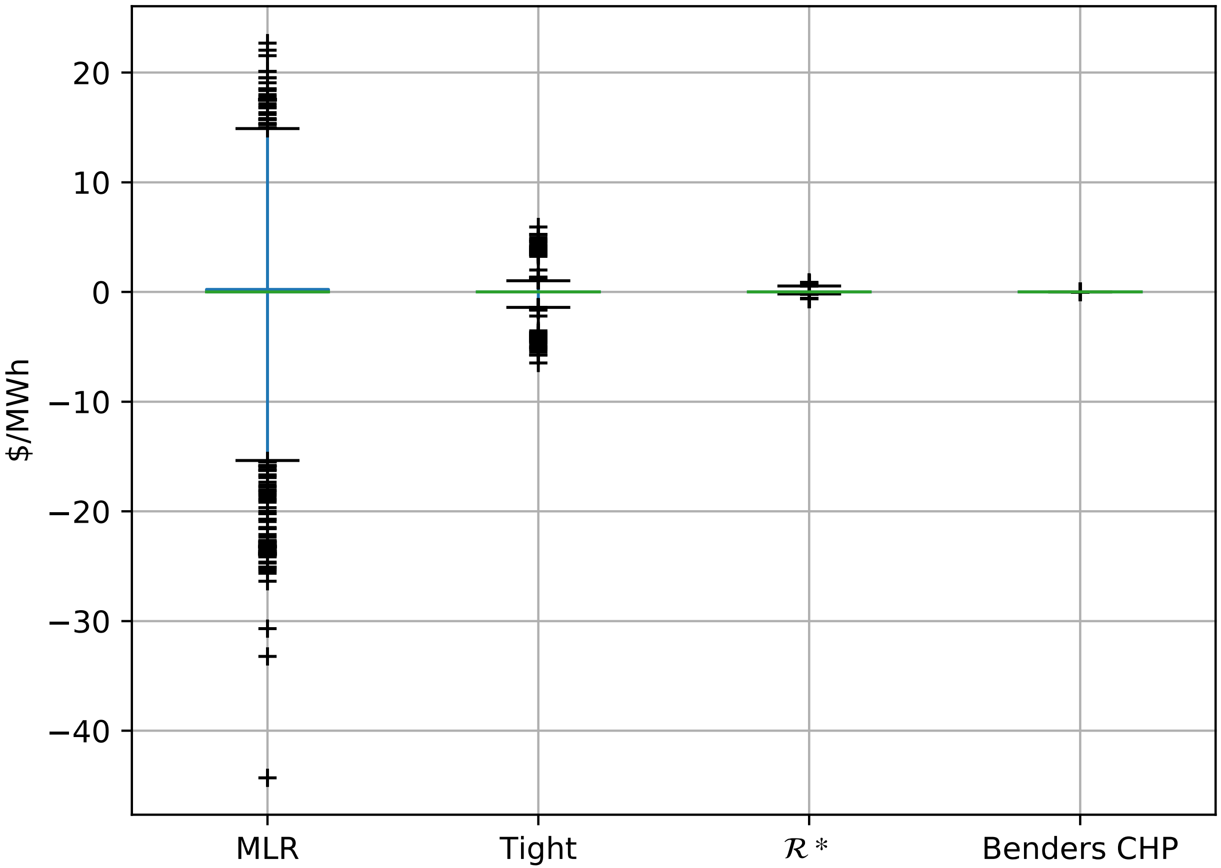
Difference in energy prices in every hour against the EF approach across all 24-hour ca instances.

Whiskers are drawn to cover 99% of observations.

rts_gmlc energy price differences

Difference in energy prices in every hour against the EF approach across all 24-hour rts_gmlc instances.

Whiskers are drawn to cover 99% of observations.



Conclusion

Conclusion

- Convex hull prices for energy for large systems can be efficiently computed leveraging the proposed Benders decomposition approach
- “Approximated” convex hull prices can have both small and large errors – the tightness of the approximation is empirically related to the tightness of the generator formulation
- The proposed Benders algorithm can be used as an approximation scheme (terminated after any iteration) if runtime limits are binding
- More work is needed on convex hull representations of generators with complex ancillary service offers and for market participants whose best-known convex hull representation has an exponential number of variables and constraints (e.g., units with maximum daily power, storage).

Q&A

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