Towards robustness guarantees for feedback-based optimization

Marcello Colombino

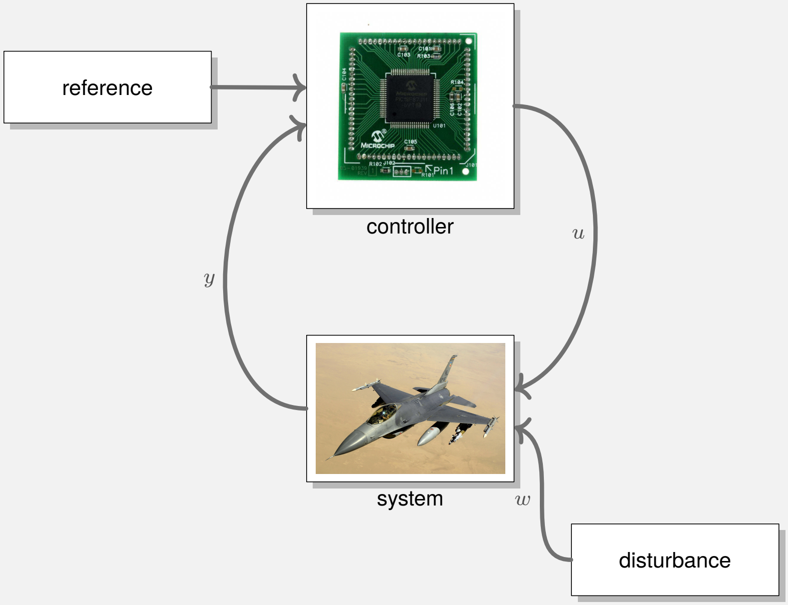
Joint work with John W. Simpson-Porco (U. Waterloo), and Andrey Bernstein (NREL)

April 11, 2019 / Autonomous Energy Systems Workshop

Towards robustness guarantees for feedback-based optimization

Feedback is traditionally associated with control and not with optimization

Example: autopilot



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Feedback

- simple: use linear model with a dozen states - undergrad exercise

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Feedforward

- complex: would require perfect model and incredible computation

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- fragile: small model mismatch generates large output deviations

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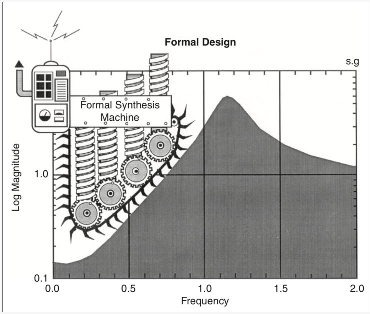
- robust: designed on a simple linear model - works on real plane
- the effect of the disturbance is observed through the output and corrected in real time

Feedforward

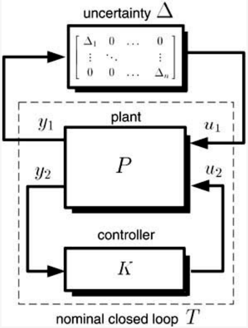
- complex: would require perfect model and incredible computation
- fragile: small model mismatch generates large output deviations
- since the input sequence is pre-computed all disturbances need to be known ahead of time

Established field



credit: G.Stein "Respect the unstable"

established machinery for control synthesis



established tools to study performance and robustness to model uncertainty

Feedback is becoming popular in optimization

Fifty-fourth Annual Allerton Conference
Allerton House, UIUC, Illinois, USA
September 27 - 30, 2016

Projected Gradient Descent on Riemannian Manifolds with Applications to Online Power System Optimization

Adrian Hauswirth[1,2], Saverio Bolognani[1], Gabriela Hug[2], and Florian Dörfler[1]

Abstract— Motivated by
in nonlinear power system
optimization over closed su
Compared to convention
explicitly consider inequali
sible set that is itself no
a continuous-
the feasible se
Under mild a

Real-Time Feedback-Based Optimization of Distribution Grids: A Unified Approach

Andrey Bernstein and Emiliano Dall'Anese*

$^{(t)}(\mathbf{p}, \mathbf{q})$

$\mathbf{p}_i, \mathbf{q}_i \in \mathcal{Y}_i^{(t)} \; \forall i$

$\mathbf{g}^{(t)}(\mathbf{p}, \mathbf{q}) \leq \mathbf{0}$

IEEE TRANSACTIONS ON SMART GRID, VOL. 8, NO. 6, NOVEMBER 2017 2963

Real-Time Optimal Power Flow

Yujie Tang, *Student Member, IEEE*, Krishnamurthy Dvijotham, and Steven Low, *Fellow, IEEE*

***Abstract*—Future power networks are expected to incorporate a large number of distributed energy resources, which introduce randomness and fluctuations as well as fast control capabilities. But traditional optimal power flow methods are only appropriate for applications that operate on a slow timescale. In this paper, we build on recent work to develop a real-time algorithm for AC optimal power flow, based on quasi-Newton methods. The algorithm uses second-order information to provide suboptimal solutions on a fast timescale, and can be shown to track the optimal power flow solution when the estimated second-order information is sufficiently accurate. We also give a specific implementation**

a continuous-time approach based on gradient dynamics for loss minimization; [2] proposed a distributed feedback algorithm for optimal reactive power flow that exploits real-time measurements, based on dual ascent method; [3] developed a fast VAR controller and analyzed its stability; [4] proposed an online OPF algorithm for distribution networks based on projected gradient descent and showed its convergence to the global optimum under certain conditions; [5] designed a composable framework for real-time control of distribu-

...and many more...

Energy shift

Energy shift

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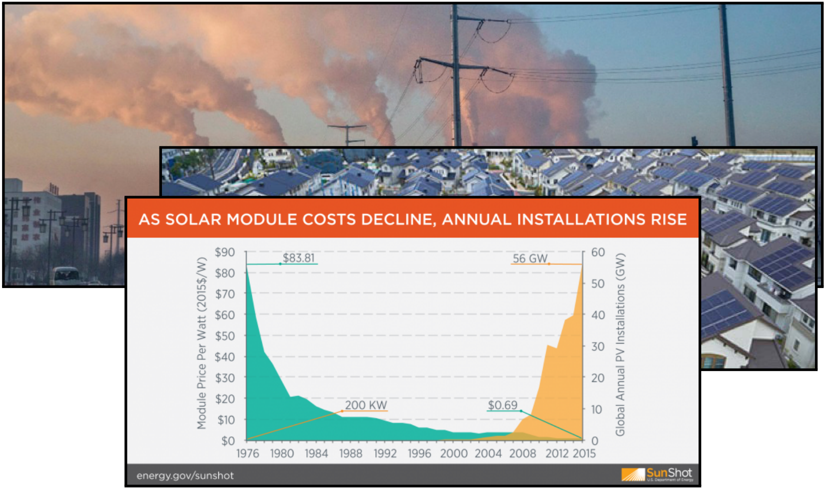

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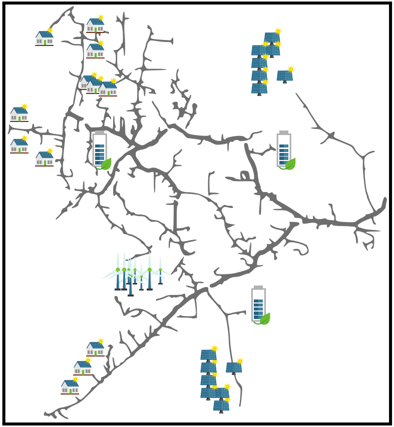

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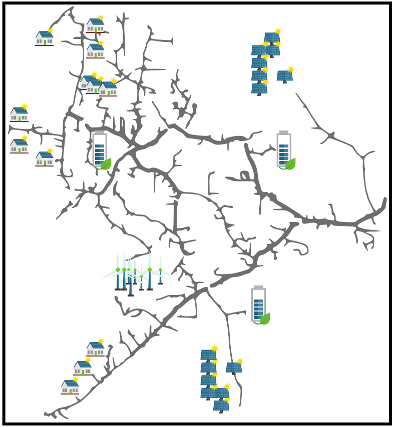
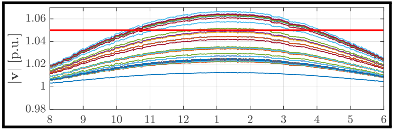
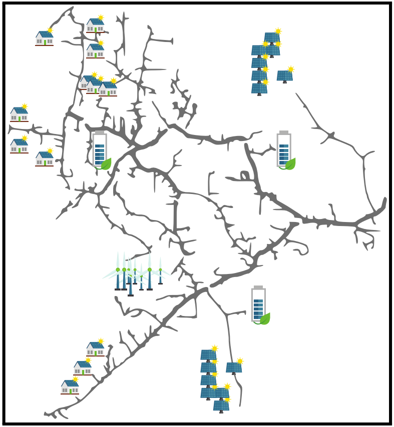


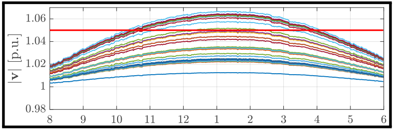


Much of the action will happen at the distribution level

- the distribution grid is designed for loads not generators

Energy shift





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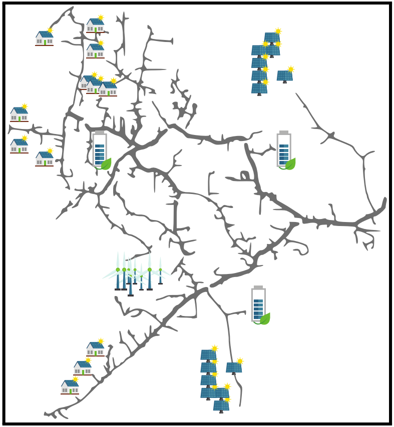
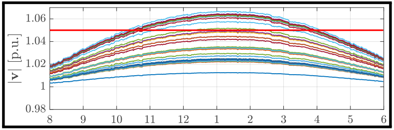
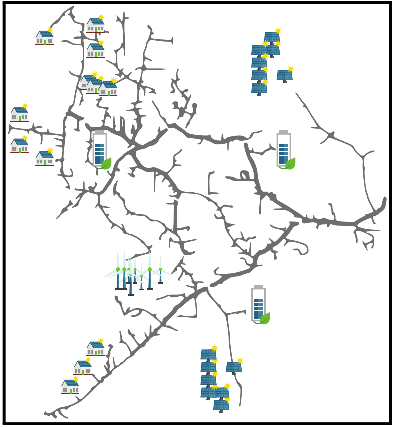




Much of the action will happen at the distribution level

- the distribution grid is designed for loads not generators
- renewables create uncertainty in effective load for transmission
- plenty of controllable devices offer an opportunity for corrective action

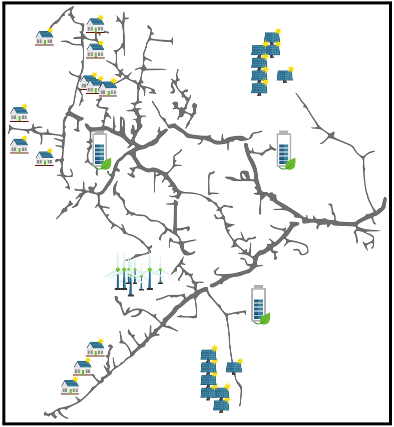


Model

- nonlinear physical system (power flow equations)

$$y = \pi(u, w)$$

often in implicit form



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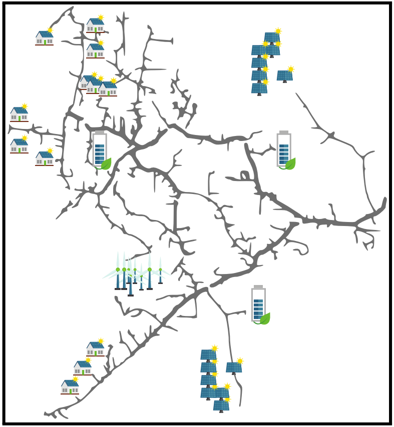

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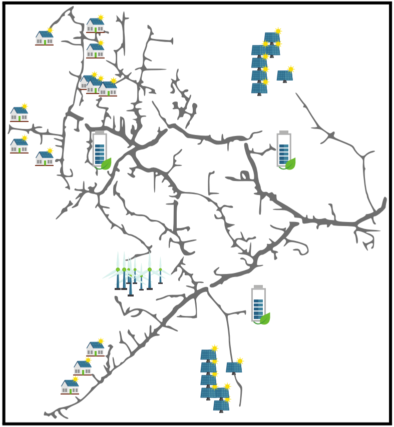

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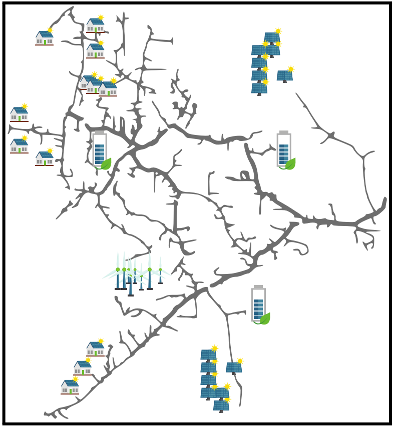
- y = regulated variables (e.g voltages, line powers, overall power output at PCC)



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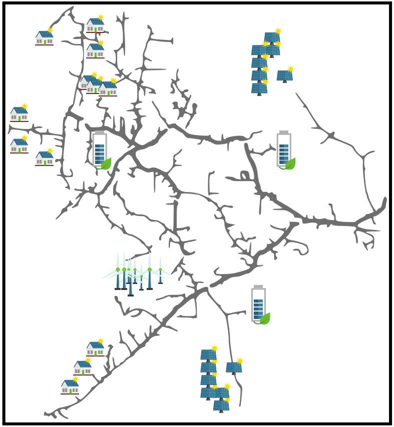

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  $\pi(u, w) \approx \Pi u + \Xi w$

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(e.g. DC or linearized optimal power flow)

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- use a simple model of the system to come up with a control law

$$u_{k+1} = k(y_k)$$

to steer the system to $u^\star(w)$

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to steer the system to $u^\star(w)$
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- there are chances for distributed computation
- warning! we are closing a loop, we need to ensure stability

Online approximate gradient

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Gradient descent

$$\overbrace{y_k = \pi(u_k, w)}^{\text{measured}}$$

$$u_{k+1} = \mathrm{Proj}_{\mathcal{U}}\left\{ u_k \quad \tau \left(\nabla f(u_k) + \partial\pi(u_k, w)^{\mathsf{T}} \nabla g(y_k) \right) \right\}$$

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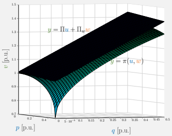
Pick a power-flow linearization



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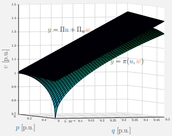

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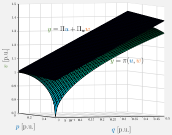
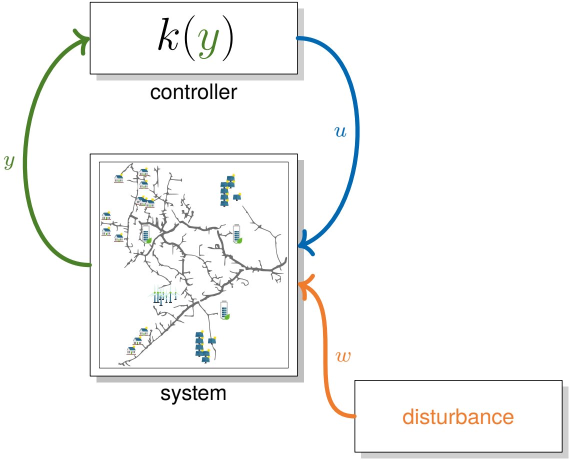
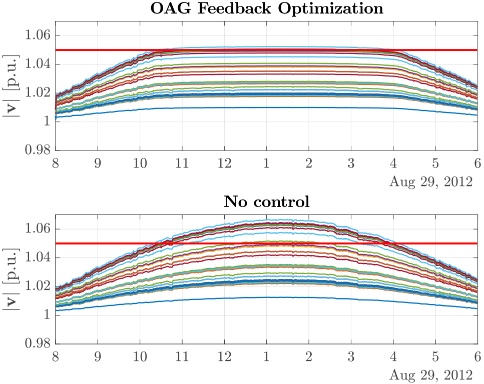


$$y \approx \Pi u + \Xi w$$

Closed loop



It works really well



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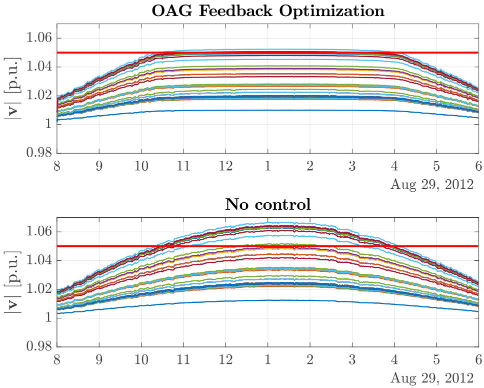

- radical cost improvement over volt-var (state of the art)

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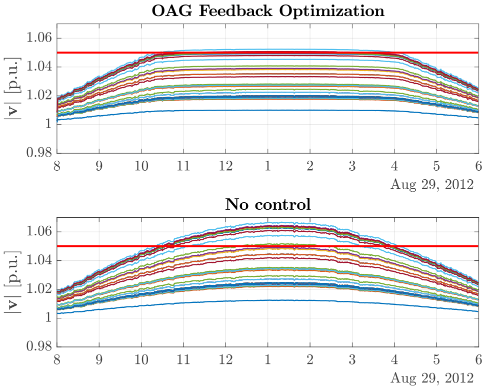

- radical cost improvement over volt-var (state of the art)
- very robust to model uncertainty (still stable and near-optimal for 40% variation in line impedances)

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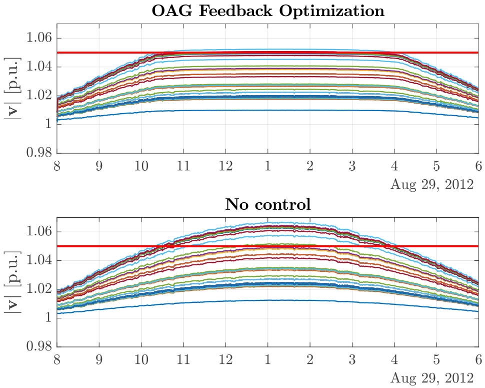

- radical cost improvement over volt-var (state of the art)
- very robust to model uncertainty (still stable and near-optimal for 40% variation in line impedances)
- no need for measuring or knowing a-priori the disturbance w.

Approximate online gradient descent

$$\overbrace{y_k}^{\text{measured}} = \pi(u_k, w)$$

$$u_{k+1} = \underbrace{\mathrm{Proj}_{\mathcal{U}}\left\{u_k \quad \tau\left(\nabla f(u_k) + \Pi^{\mathsf{T}}\nabla g(y_k)\right)\right\}}_{k(y_k)}$$

comments

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- feedback interconnection of two nonlinear systems - stability?
- there is plenty of freedom in choosing the linearization and the cost functions
- diagonal Π plus right cost function $\implies$ Volt Var control (state of the art)
- intuitively the quality of the approximation

$$\Pi \approx \partial\pi(u_k, w)$$

should play an important role in stability - performance - robustness

Basic assumptions

$$\begin{array}{ll} \min\limits_{u \in \mathcal{U}} & f(u) + g(y) \\ \text{subject to:} & y = \pi(u, w) \end{array}$$

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- u, w live in compact sets $\mathbb{U}$, $\mathbb{W}$

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- $f(\cdot), g(\cdot)$ convex and differentiable

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- $\pi(\cdot, \cdot)$ continously differentiable w.r.t u.

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In the next 20-30 mins

- tractable test for (robust) stability of the feedback interconnection

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Assumptions

- u, w live in compact sets $\mathbb{U}, \mathbb{W}$
- $f(\cdot), g(\cdot)$ convex and differentiable
- $\pi(\cdot, \cdot)$ continously differentiable w.r.t u.

In the next 20-30 mins

- tractable test for (robust) stability of the feedback interconnection
- error bounds for distance to kkt points

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In the next 20-30 mins

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- demonstration on small feeder examples & interesting observations

Stability

Approximate gradient

$$u_{k+1} = \mathrm{Proj}_{\mathcal{U}} \left\{ u_k \quad \tau \left(\underbrace{\nabla f(u_k) + \Pi^{\mathsf{T}} \nabla g(\pi(u_k, w))}_{F_w(u_k)} \right) \right\}$$

$$\partial F_w(u_k) = \partial^2 f(u_k) + \Pi^{\mathsf{T}} \partial^2 g(y_k) \partial \pi(u_k, w)$$

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Lipschitz-continuity

Given $P \quad 0$, the map $F_w(\cdot)$ is L Lipschitz continuous w.r.t $\langle \cdot, \cdot \rangle_P$ if

$$\partial F_w(u)^{\mathsf{T}} P \partial F_w(u) \quad L^2 P, \qquad \forall\, u \in \mathcal{U}$$

always true with our assumptions..

Stability

Proposition [e.g Facchinei]

If F_w is ρ strongly monotone and L Lipschitz continuous, the iteration

$$u_{k+1} = \mathrm{Proj}_{\mathcal{U}} \{u_k \quad \tau F_w(u_k)\}$$

converges to a unique fixed point for $\tau < \frac{\rho}{L^2}$

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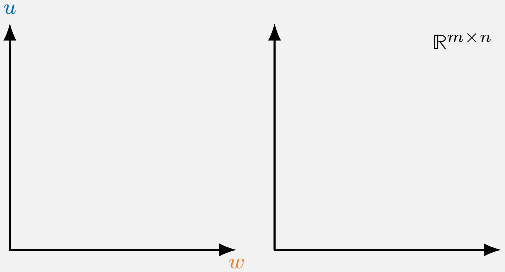
- these properties depend on the Jacobian

$$\partial F_w(u) = \partial^2 f(u) + \Pi^\intercal \partial^2 g(y) \partial \pi(u, w)$$

- we need a test that guarantees that these properties are satisfied for all operating points in $\mathbb{U} \times \mathbb{W}$.

Building a stability test

Over-approximate the set of possible Jacobians as a “nice” set



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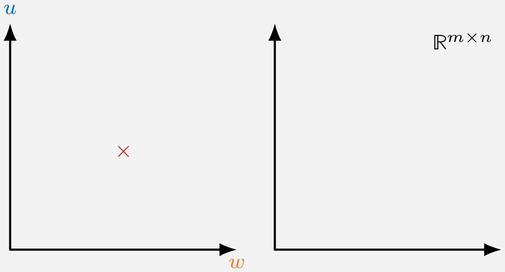

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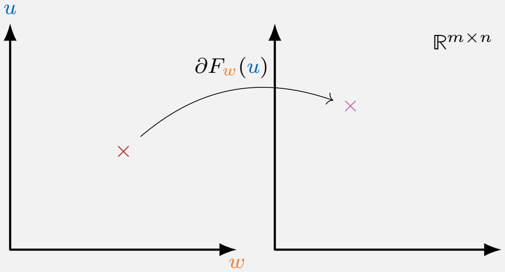

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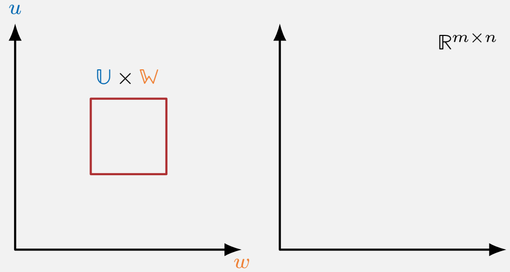

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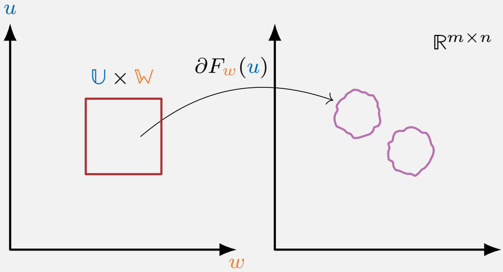

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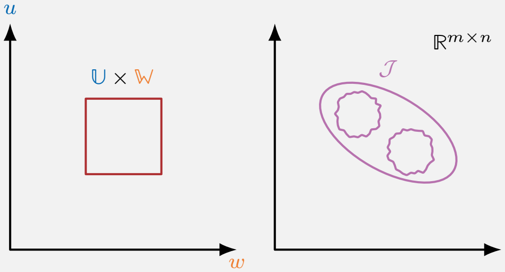

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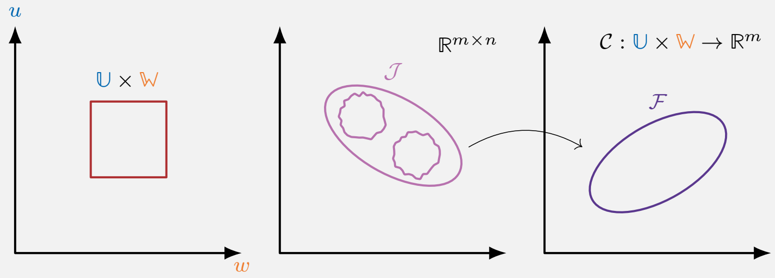

$\mathcal{F}$ is the set of all continuous functions whose jacobian lies in $\mathcal{J}$ for all operating points in $\mathbb{U} \times \mathbb{W}$. $\mathcal{J}$ over-approximation $\implies F_w(u) \in \mathcal{F}$.

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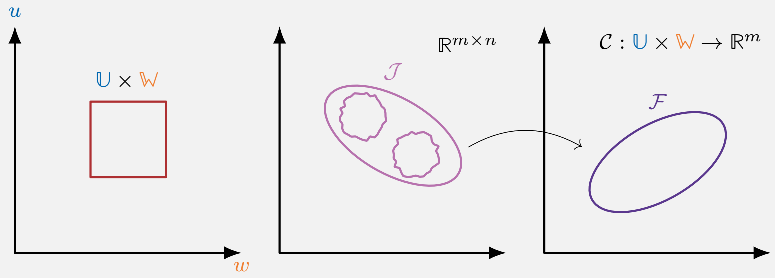

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We choose the set $\mathcal{J}$ such that:

- it is an over-approximation

Building a stability test

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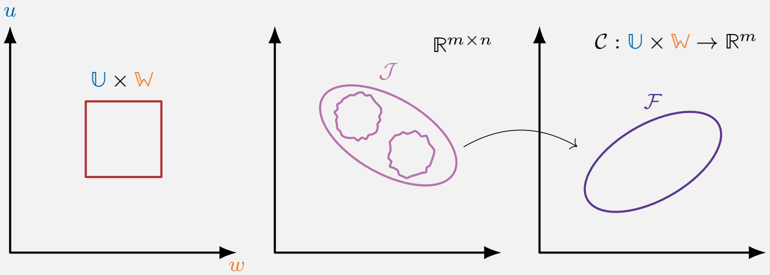

$\mathcal{F}$ is the set of all continuous functions whose jacobian lies in $\mathcal{J}$ for all operating points in $\mathbb{U} \times \mathbb{W}$. $\mathcal{J}$ over-approximation $\implies F_w(u) \in \mathcal{F}$.

We choose the set $\mathcal{J}$ such that:

- it is an over-approximation
- we can easily guarantee monotonicity for all "approximate gradient" maps $F_w(u) \in \mathcal{F}$

Building a stability test

Monotonicity

Given $P \quad 0$, the map $F_w(\cdot)$ is ρ strongly monotone w.r.t $\langle \cdot, \cdot \rangle_P$ if and only if

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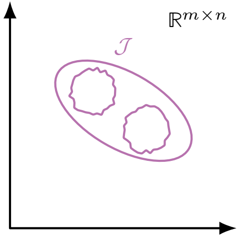

Robustly ρ-strongly-monotone iff for all $J \in \mathcal{J}$

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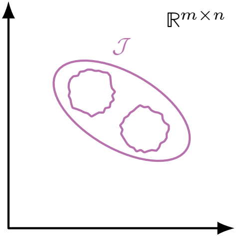

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how easy this test is depends on the structure of $\mathcal{J}$...

Building a stability test

Example 1: polytopic uncertainty

$$\mathcal{J}^{\text{poly}} := \operatorname{co}\{J_i,\ i = 1, \ldots, \nu\}.$$

$$\mathcal{F}^{\text{poly}} := \{F_w \mid \partial F_w(u) \subseteq \mathcal{J}^{\text{poly}}, \forall\, u \in \mathbb{U}, \forall\, w \in \mathbb{W}\}.$$

Building a stability test

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Numerical test for (robust) stability

Given $P \quad 0$ and a constant $\rho > 0$, the following two statements are equivalent:

(i) the set $\mathcal{F}^{\text{poly}}$, is ρ strongly monotone w.r.t $\langle \cdot, \cdot \rangle_P$ on the set $\mathcal{U}$;

(ii) the following Matrix Inequality holds true

$$\frac{1}{2}\left[J_i^\mathsf{T} P + P J_i\right] \} \quad \rho P, \quad i = 1, \ldots, \nu.$$

Building a stability test

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Can easily blow up in size...

Building a stability test

Example 2: LFT uncertainty

$$\mathcal{J}^{\mathsf{lft}} := \quad A + B \quad (I \quad D \quad)^{1}C : \quad \in \leftarrow \quad .$$

$\mathcal{J}^{\mathsf{lft}}$ is a Linear Fractional Transformation of a known set .

Building a stability test

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Building a stability test

Convex numerical test for robust stability

The set $\mathcal{F}^{\text{lft}}$, is ρ strongly monotone w.r.t $\langle\cdot,\cdot\rangle_P$ if there exist $\Theta \in \mathbf{\Theta}$ such that

$$\begin{bmatrix} A_\rho^\top P + PA_\rho & PB \\ B^\top P & \mathbb{0} \end{bmatrix} \begin{bmatrix} C & D \\ \mathbb{0} & I_s \end{bmatrix}^\top \Theta \begin{bmatrix} C & D \\ \mathbb{0} & I_s \end{bmatrix} \succcurlyeq 0,$$

where $A_\rho := A + \rho I$

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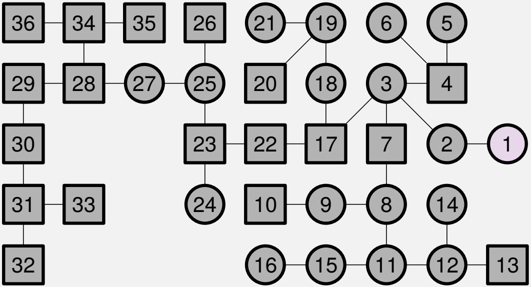
- convex in P, Θ - easy to verify up to moderate size networks

Example

Power-curtailment / Voltage regulation (E. Dall'Anese - OPF Pursuit)



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- PVs at every ***square*** bus
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Goal: minimize curtailment and reduce voltage violations

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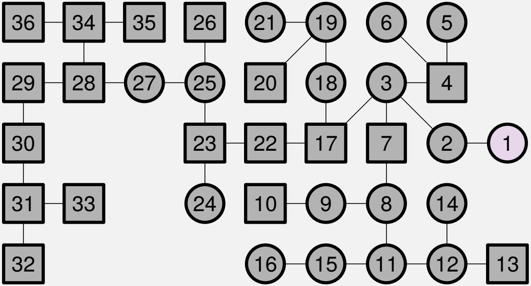

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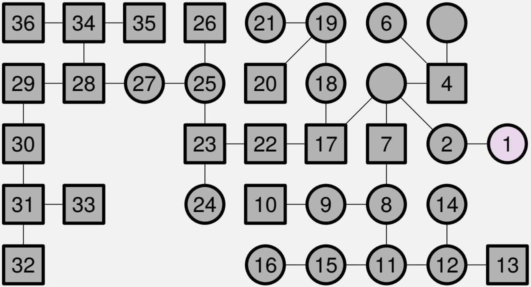

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$y =$ voltages, $u =$ controllable p, q inj., $w =$ uncontrollable p, q inj.

$$\min_{u_i \in \mathcal{U}_i} \quad u^\mathsf{T} H u + h^\mathsf{T} u + \eta \sum_{i=1}^{m} \} \max(0, \underline{y_i} \quad y_i, y_i \quad \overline{y_i})^2$$

$$\text{subject to :} \quad y = \pi(u, w)$$ power flow equations

Example

Gradient descent

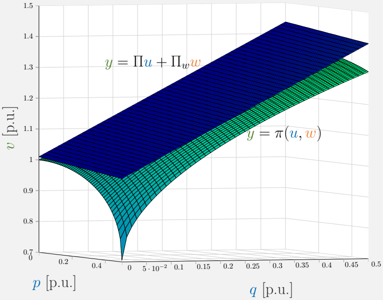
$$u_{k+1} = \mathrm{Proj}_{\mathcal{U}} \left\{ u_k \quad \tau \left(H u^k + \eta\, \partial\pi(u_k, w)^\top \underbrace{s_{\underline{y},\bar{y}}(y^k)}_{\text{measured}} \right) \right\}$$

Example

Gradient descent

$$u_{k+1} = \mathrm{Proj}_{\mathcal{U}} \left\{ u_k - \tau \left(H u^k + \eta\, \partial\pi(u_k, w)^\mathsf{T} \underbrace{s_{\underline{y},\bar{y}}(y^k)}_{\text{measured}} \right) \right\}$$

Pick a power-flow linearization



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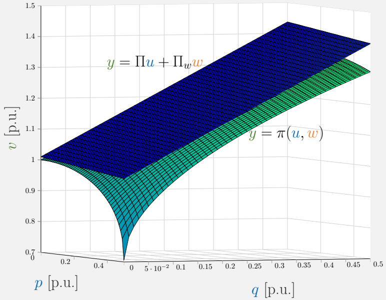

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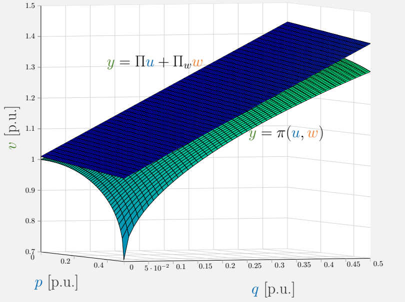
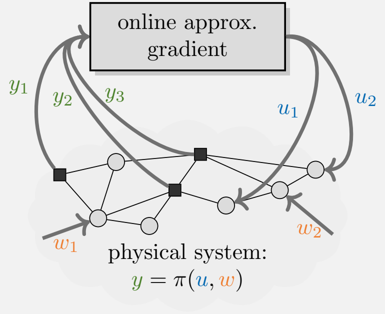
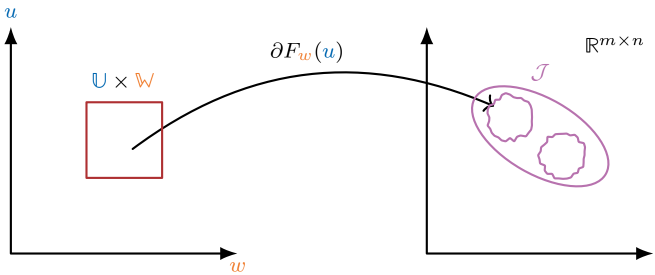


Feedback implementation



How to assess stability

$$\partial F_w(u) = H + \eta \Pi^\top Q(y) \partial \pi(u, w)$$



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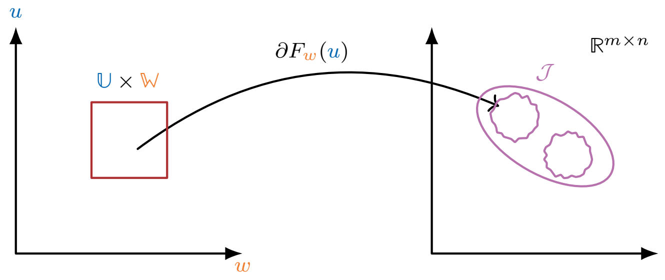

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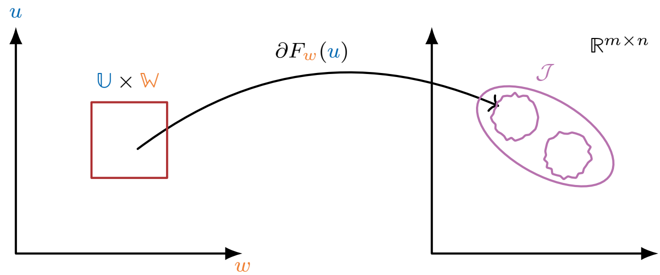

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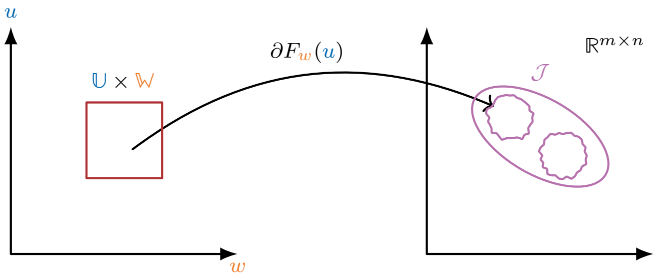

- $Q(y) \in \leftarrow_{Q} := \{$ diagonal $: 0$ $I\}$ LFT representable
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- the “product” of LFT representable sets is LFT representable

Example (cont'd)

LFT representation for the power flow Jacobians

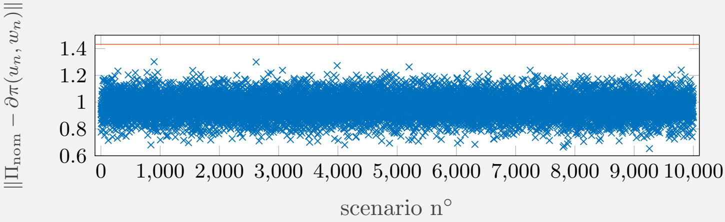
$$\partial\pi(u, w) = \Pi_{\text{nom}} + \quad {}_{\pi}(u, w), \quad \| \quad {}_{\pi}(u, w)\| \leq$$

Example (cont'd)

LFT representation for the power flow Jacobians

$$\partial\pi(u, w) = \Pi_{\text{nom}} + \Delta_\pi(u, w), \quad \|\Delta_\pi(u, w)\| \le \gamma$$

Choose γ by sampling



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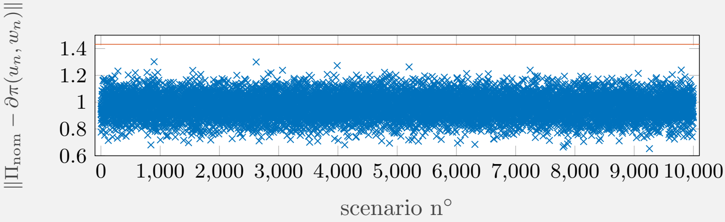
Choose γ by sampling



this step has the potential to be made more rigorous by exploiting the structure of the power-flow Jacobian (e.g. relaxation methods [Misra, Molzhan, Krishnamurthy 18])

How to assess stability

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How to assess stability

Stability test

$$\begin{bmatrix} A_\rho^\mathsf{T} P + P A_\rho & P B \\ B^\mathsf{T} P & \mathbb{0} \end{bmatrix} - \begin{bmatrix} C & D \\ \mathbb{0} & I \end{bmatrix}^\mathsf{T} \Theta \begin{bmatrix} C & D \\ \mathbb{0} & I \end{bmatrix} \succcurlyeq 0$$

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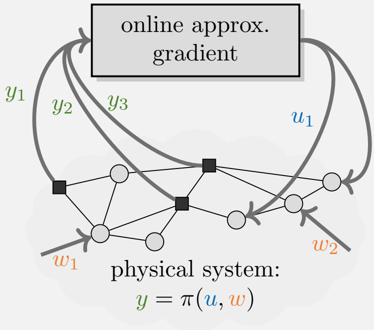
if the test succeeds, the online algorithm is provably stable



Observations

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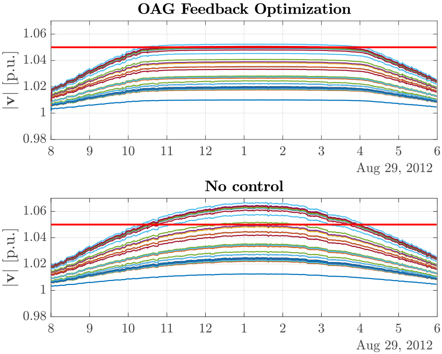

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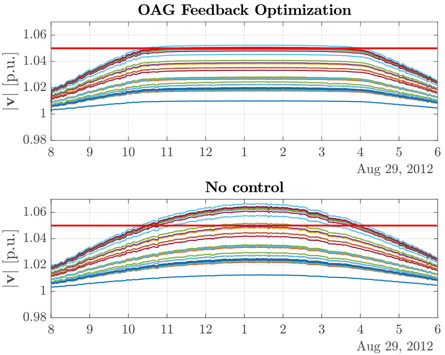

- test case with ieee 37 test feeder with high solar penetration
- real data (load, irradiance) from Anatolia CA
- $\sim 40\%$ less curtailment than Volt-Var

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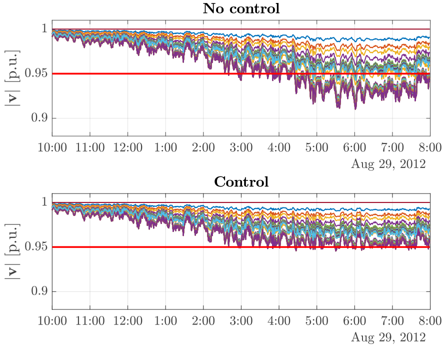


- test case with ieee 123 test feeder with high solar penetration
- real data (load, irradiance) from Anatolia CA

Performance

Proposition

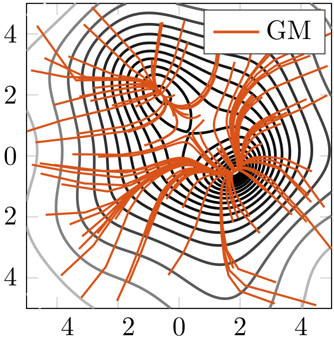
If $F_w(u)$ is ρ strongly monotone then the OAG algorithm converges to the unique point $\bar{u}(w)$

$$\|\bar{u}(w) \quad u^\star(w)\|_P \leq \frac{1}{\rho}\|(\Pi \quad \partial\pi(u^\star, w))^\top \nabla g(\pi(u^\star, w))\|_P$$

where $u^\star$ a KKT point of the original (non-convex) optimization problem

Inexact gradient can be helpful

$f(u_k) + \partial\pi(u_k, w)^\mathsf{T} \quad g(y_k)$



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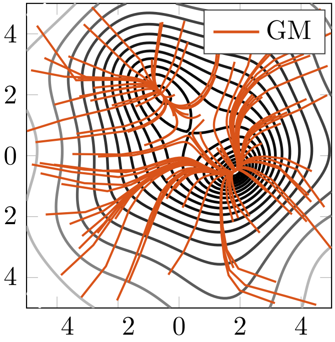

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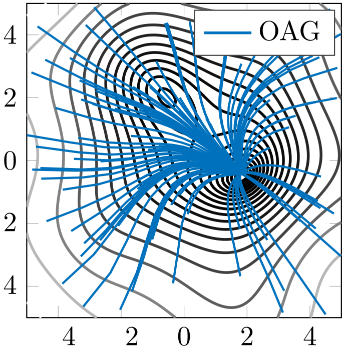
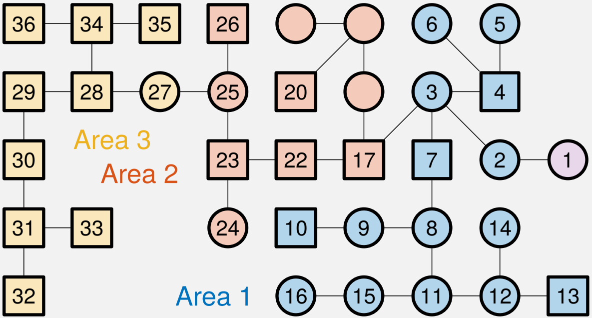


What about design?

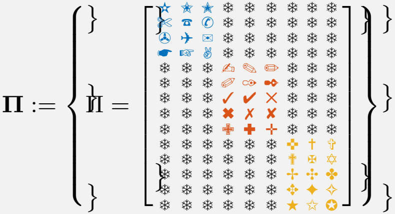
What about design?

Design a provably robustly stable **distributed** algorithm



What about design? (cont'd)

Structured Π = distributed algorithm



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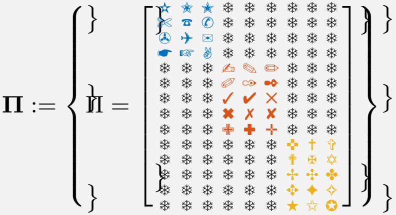

We can use the LMI to **design** Π & guarantee robust stability

$$\min_{\Pi,\Theta} \quad \|\Pi \quad \hat{\Pi}\| \leftarrow$$

subject to $\Pi \in \mathbf{\Pi}$

$$\begin{bmatrix} A_\rho^\top P + PA_\rho & PB(\Pi) \\ B(\Pi)^\top P & \mathbb{0} \end{bmatrix} \quad \begin{bmatrix} C & D \\ \mathbb{0} & I_s \end{bmatrix}^\top \Theta \begin{bmatrix} C & D \\ \mathbb{0} & I_s \end{bmatrix} \succcurlyeq 0$$

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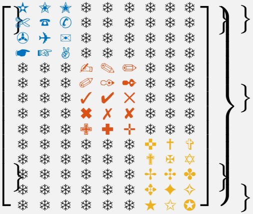

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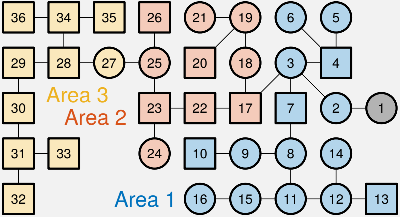
Synthesis of a linearization of the power flow equations with desired properties.

What about design? (cont'd)

Provably robustly stable distributed optimization algorithm



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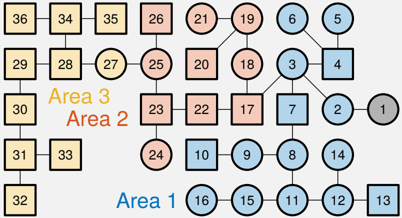

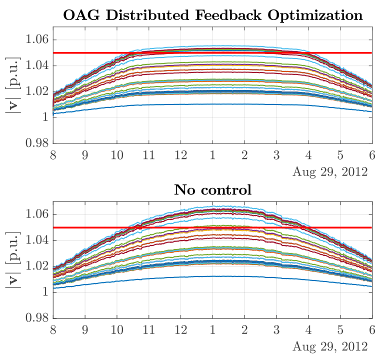

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Thanks

Questions?

Thank you

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