Stochastic Model Predictive Control for Demand Response in a Home Energy Management System

Preprint

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Stochastic Model Predictive Control for Demand Response in a Home Energy Management System

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Abstract—**This paper presents a chance constrained, model predictive control (MPC) algorithm for demand response (DR) in a home energy management system (HEMS). The HEMS optimally schedules controllable appliances given user preferences such as thermal comfort and energy cost sensitivity, and available residentially-owned power sources such as photovoltaic (PV) generation and home battery systems. The proposed control architecture ensures both the DR event and indoor thermal comfort are satisfied with a high probability given the uncertainty in available PV generation and the outdoor temperature forecast. The uncertainties are incorporated into the MPC formulation using probabilistic constraints instead of computationally limiting sampling-based approaches. Simulation results for various user preferences and probabilistic model parameters show the effectiveness of the HEMS algorithm response to DR requests.**

NOMENCLATURE

B_{util}	DR reduction request from utility (kW)
c_e	Cost of P_{grid} (\$/kWh)
d_{hvac}	HVAC control signal (duty cycle)
E	Battery state of charge (kWh)
$E_{\max}$	Maximum energy storage in battery (kWh)
$E_{\min}$	Minimum energy storage in battery (kWh)
η	Battery charging/discharging efficiency
P_{c}	Power consumed by HVAC when cooling (kW)
P_{ch}	Power injected into battery storage (kW)
P_{dis}	Power drawn from battery storage (kW)
P_{grid}	Power consumed from the grid (kW)
$P_{\max}$	Maximum charging/discharging power (kW)
P_{pred}	Predicted P_{grid} consumption with no DR (kW)
P_{PV}	Solar power consumed (kW)
P_{rad}	Solar Irradiance (W/m^2)
P_{sol}	Available solar power (kW)
P_{uc}	Residential load from uncontrollable devices (kW)
ρ_{DR}	Probability that DR request is satisfied
ρ_T	Probability that T_{in} bounds are satisfied
$t_{\text{DR,f}}$	End time of DR period
$t_{\text{DR,n}}$	Time of DR notice
$t_{\text{DR,s}}$	Start time of DR period
T_{in}	Indoor air temperature (°F)
$T_{\max}$	Maximum indoor air temperature (°F)
$T_{\min}$	Minimum indoor air temperature (°F)
T_{out}	Outdoor air temperature (°F)
U_{curt}	PV curtailment (percent)

I. INTRODUCTION

Renewable energy integration into the power grid has presented new challenges due to uncertainty in weather forecasts and renewable energy generation availability. While generation-side energy management solutions have been widely studied to account for intermittent renewable energy generation challenges, demand-side energy management solutions present another approach for integrating renewable energy sources given the current power grid architecture. In particular, residential demand-side energy management can be used to address stable renewable energy integration since the residential buildings account for 37.6% of total electricity consumption in the U.S., which is more than the commercial building, industrial building, or transportation sector [1]. Home energy management systems (HEMS) provide demand-side energy management by coordinating multiple residential appliances in real-time given user preferences and renewable energy resource forecasts [2], [3]. HEMS can increase the energy efficiency of a home by leveraging controllable residential devices, such as heating, ventilation, and air-conditioning (HVAC) systems, which account for over 50% of total residential load [1]. This work provides a stochastic model predictive control (MPC) algorithm for demand response (DR) in a HEMS that optimally coordinates home appliances and residentially-owned generation and storage given user comfort preferences, energy cost sensitivity, and uncertainty in available PV generation and outdoor temperature forecast.

Various HEMS architectures have been proposed for increasing residential energy efficiency [3]. Many of the control methods studied for HEMS applications also include DR grid service capabilities such as real-time pricing and direct load control, which encourage consumers to shift the load of flexible devices away from peak demand periods [2], [3]. Typically, advanced HEMS algorithms use optimization techniques such as MPC [2], [4], mixed-integer linear programming (MILP) [5], or various artificial intelligence techniques [3], [6], [7]. One approach to using stochastic optimal control methods in HEMS algorithms is the use of Monte Carlo sampling for representing uncertainties in various parameters such as outdoor temperature and renewable energy source generation [8], which is computationally restrictive. Another approach for incorporating uncertainty in HEMS algorithms is using the Markov chain modeling framework [9]. To include probabilistic models in a HEMS algorithm, we propose a chance constrained MPC-based optimization formulation. Chance constrained optimization has been used to incorporate uncertainty in renewable energy generation into optimal energy storage sizing problems [10] and AC optimal power flow problems [11]. Chance constrained MPC has also been used to include uncertainty in weather forecasts for energy efficient HVAC system usage in buildings [12].

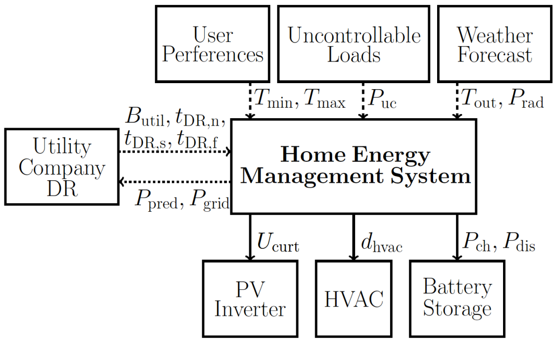

Fig. 1. Overall HEMS control and data schematic.

Chance constrained optimization for a HEMS has been used to incorporate uncertainty in dynamic pricing and system loads [13]. However, in this work we use a chance constrained MPC-based HEMS to incorporate uncertainty in the weather forecast and renewable energy generation while ensuring both the DR request and users' thermal preferences are satisfied with high probability.

In this work, we provide a chance constrained MPC algorithm for a HEMS capable of satisfying grid DR requests where usage of flexible (i.e. controllable) devices, such as HVAC systems, are shifted away from peak demand periods on the power grid. We assume the DR event is communicated to the HEMS before the DR period begins in the form of a request for some amount of grid power reduction B_{util} relative to amount of grid power the HEMS predicts the home requires during the DR period. We consider a HEMS that coordinates a residentially-owned PV array and battery storage system, an HVAC system, and uncontrollable devices to satisfy the DR event while ensuring user thermal comfort. Uncontrollable residential devices include lighting, television, and plug loads, which we assume cannot be controlled with a HEMS. The chance constraints in the HEMS algorithm are used to ensure that both the DR reduction request and the users' thermal comfort are satisfied with high probability given uncertainty in available PV power and outdoor air temperature. An overview of the considered HEMS system is shown in Fig. 1.

The rest of this paper is organized as follows: Section II provides the mathematical formulation of the HEMS device models and chance constraints for modeling uncertainty in PV generation and outdoor air temperature forecast. In Section III, we provide the overall stochastic MPC HEMS optimization problem. The simulation results of the proposed HEMS algorithm are provided in Section IV. In Section V, conclusions on the effectiveness of the proposed stochastic MPC-based HEMS algorithm are discussed, as well as areas of future work.

II. Residential DR Control Structure

In this section, the mathematical models for the indoor air temperature dynamics and residential devices are provided. In this work, we assume that the HEMS must coordinate the HVAC system, the residential PV generation, the battery storage system, uncontrollable loads, and any additional power needed from the grid to satisfy the load. The chance constraint formulations for incorporating the uncertainty in PV generation P_{PV} and outdoor air temperature T_{out} are also provided. The temporal index is denoted with the superscript (t).

A. HEMS Device Models

The residential indoor temperature model, coupled with the HVAC model and thermal comfort preferences, is given by the following dynamic equations:

$$T_{\mathrm{in}}^{(t+1)} = T_{\mathrm{in}}^{(t)} + \beta_1 (T_{\mathrm{out}}^{(t)} - T_{\mathrm{in}}^{(t)}) - \beta_2 d_{\mathrm{hvac}}^{(t)} + \beta_3 P_{\mathrm{rad}}^{(t)}, \quad (1)$$

$$T_{\min} \leq T_{\mathrm{in}}^{(t+1)} \leq T_{\max}, \quad (2)$$

where $d_{\mathrm{hvac}}^{(t)} \in [0, 1]$ is the HVAC duty cycle, β_1 represents the building envelope, β_2 is the cooling gain, and β_3 is the solar gain. The residential PV system model is given by

$$P_{\mathrm{PV}}^{(t)} = (1 - U_{\mathrm{curt}}^{(t)}) P_{\mathrm{sol}}^{(t)}, \quad (3)$$

where $U_{\mathrm{curt}} \in [0, 1]$ is the fraction of available PV generation that is curtailed, and $P_{\mathrm{sol}}^{(t)}$ is a function of the solar irradiance, array size, and array efficiency.

The residential battery storage system state of charge (SOC) and power charged/discharged are modeled by the following:

$$E^{(t+1)} = E^{(t)} + \eta P_{\mathrm{ch}}^{(t)} \Delta t - \tfrac{1}{\eta} P_{\mathrm{dis}}^{(t)} \Delta t, \quad (4)$$

$$E_{\min} \leq E^{(t+1)} \leq E_{\max}, \quad (5)$$

$$0 \leq P_{\mathrm{dis}}^{(t)}, P_{\mathrm{ch}}^{(t)} \leq P_{\max}, \quad (6)$$

where $E_{\min}$ and $E_{\max}$ are 15% and 85% of the rated energy storage capacity, respectively, which limits battery degradation when operating in this region [14]. From the structure of this problem, it is assumed the optimization will not find it optimal for the battery to charge and discharge simultaneously. This simplification can be addressed with MILP or including the constraint $P_{\mathrm{ch}}^{(t)} P_{\mathrm{dis}}^{(t)} = 0$. While the constraint is not included in this work, we have confirmed that for all simulation results provided in Section IV, the battery does not simultaneously charge and discharge.

The overall power balance in a home is given by the following equation:

$$0 = -P_{\mathrm{grid}}^{(t)} + P_{\mathrm{load}}^{(t)} - P_{\mathrm{PV}}^{(t)} + P_{\mathrm{ch}}^{(t)} - P_{\mathrm{dis}}^{(t)}, \quad (7)$$

where the total power demand for a home is given by $P_{\mathrm{load}}^{(t)} = P_{\mathrm{uc}}^{(t)} + P_{\mathrm{c}} d_{\mathrm{hvac}}^{(t)}$. The total HVAC load is given by $P_{\mathrm{hvac}}^{(t)} = P_{c} d_{\mathrm{hvac}}^{(t)}$.

B. Inclusion of Uncertainty

Chance constraints are introduced into the problem to ensure both the DR request is satisfied and the user thermal comfort is maintained with high probability given uncertainty in forecasting errors in weather parameters that dictate available PV power and the indoor air temperature. First, we will focus on the constraint that ensures the DR reduction request is satisfied with probability ρ_{DR}, which is given by the following:

$$\Pr\left(\sum_{t=t_{\mathrm{DR,s}}}^{t_{\mathrm{DR,f}}} P_{\mathrm{pred}}^{(t)} - \sum_{t=t_{\mathrm{DR,s}}}^{t_{\mathrm{DR,f}}} P_{\mathrm{grid}}^{(t)} \geq B_{\mathrm{util}} \right) \geq \rho_{\mathrm{DR}}, \quad (8)$$

where the actual grid power consumed $P_{\text{grid}}^{(t)}$ is dependent on the power balance equation in (7). The available PV power can be represented as $P_{\text{PV}}^{(t)} = P_{\text{f}}^{(t)} + P_{\text{err}}^{(t)}$, where $P_{\text{f}}^{(t)}$ and $P_{\text{err}}^{(t)}$ are the solar forecast and solar forecast error at time t, respectively. We assume the solar forecast error $P_{\text{err}}^{(t)}$ is Normally distributed $P_{\text{err}}^{(t)} \sim \mathcal{N}(\mu_{P_{\text{err}}}^{(t)}, \sigma_{P_{\text{err}}}^{2(t)})$ and $P_{\text{err}}^{(t)}$ are independent. Then, we obtain the following for $P_{\text{grid}}^{(t)}$ in (8):

$$P_{\text{grid}}^{(t)} = P_{\text{c}} d_{\text{hvac}}^{(t)} + P_{\text{uc}}^{(t)} + P_{\text{ch}}^{(t)} - P_{\text{dis}}^{(t)} - P_{\text{f}}^{(t)} - P_{\text{err}}^{(t)}. \quad (9)$$

Notice that the sum of Normally distributed forecast errors will result in another Normally distributed random variable denoted $P_{\text{err}} \sim \mathcal{N}(\mu_P, \sigma_P)$ where $\mu_P = \sum_{t=t_{\text{DR,s}}}^{t_{\text{DR,f}}} \mu_{P_{\text{err}}}^{(t)}$ and $\sigma_P^2 = \sum_{t=t_{\text{DR,s}}}^{t_{\text{DR,f}}} \sigma_{P_{\text{err}}}^{2(t)}$. Then we can write the constraint in (8) that ensures DR is met with probability ρ_{DR} as the following:

$$\Pr(\mathcal{X}_P \leq 0) \geq \rho_{\text{DR}}. \quad (10)$$

where $P_{\text{grid}}^{(t)}$ is given in (9) and $\mathcal{X}_P$ is given by:

$$\mathcal{X}_P = \sum_{t=t_{\text{DR,s}}}^{t_{\text{DR,f}}} P_{\text{grid}}^{(t)} + B_{\text{util}} - \sum_{t=t_{\text{DR,s}}}^{t_{\text{DR,f}}} P_{\text{pred}}^{(t)}.$$

Since P_{err} is Normally distributed, $\mathcal{X}_P$ is also Normally distributed with the following mean μ and variance σ:

$$\begin{aligned}\mu = \; & \sum_{t=t_{\text{DR,s}}}^{t_{\text{DR,f}}} (P_{\text{c}} d_{\text{hvac}}^{(t)} + P_{\text{uc}}^{(t)} + P_{\text{ch}}^{(t)} - P_{\text{dis}}^{(t)} - P_{\text{f}}^{(t)}) - \mu_P \\ & + B_{\text{util}} - \sum_{t=t_{\text{DR,s}}}^{t_{\text{DR,f}}} P_{\text{pred}}^{(t)}, \\ \sigma = \; & \sigma_P.\end{aligned}$$

The chance constraint in (10) can be written as:

$$\Pr(\mathcal{X}_P \leq 0) = \Phi\left(\tfrac{0-\mu}{\sigma}\right) \geq \rho_{\text{DR}}, \quad (11)$$

where $\Phi(\cdot)$ is the CDF of the Normal distribution $\mathcal{N}(0,1)$. The chance constraint that ensures the DR request is satisfied with a high probability is obtained by taking the inverse CDF of both sides of (11), which is given by:

$$\begin{aligned}\sum_{t=t_{\text{DR,s}}}^{t_{\text{DR,f}}} P_{\text{pred}}^{(t)} - \sum_{t=t_{\text{DR,s}}}^{t_{\text{DR,f}}} (P_{\text{c}} d_{\text{hvac}}^{(t)} + P_{\text{uc}}^{(t)} + P_{\text{ch}}^{(t)} - P_{\text{dis}}^{(t)} - P_{\text{f}}^{(t)}) \\ + \mu_P - B_{\text{util}} \geq \Phi^{-1}(\rho_{\text{DR}}) \sigma_P. \quad (12)\end{aligned}$$

Next, we provide the mathematical formulation for the chance constraints that ensure the users' thermal comfort preferences are satisfied with a high probability during the DR period given uncertainty in the outdoor temperature. Given the thermal comfort bounds in (2), the constraints requiring that the thermal comfort bounds are satisfied with at least probability ρ_T during the DR period are as follows:

$$\Pr(T_{\min} \leq T_{\text{in}}^{(t+1)}) \geq \rho_T, \quad (13)$$

$$\Pr(T_{\text{in}}^{(t+1)} \leq T_{\max}) \geq \rho_T. \quad (14)$$

For the probabilistic equation in (13), let the outdoor temperature be written in terms of the forecasting error: $T_{\text{out}}^{(t)} = T_{\text{f}}^{(t)} + T_{\text{err}}^{(t)}$, where $T_{\text{f}}^{(t)}$ is the forecast outdoor temperature and $T_{\text{err}}^{(t)}$ is the outdoor temperature forecast error. The outdoor temperature forecast error is assumed to be Normally distributed $T_{\text{err}}^{(t)} \sim \mathcal{N}(\mu_T^{(t)}, \sigma_T^{2(t)})$ [15]. With similar analysis, we obtain the following constraint that ensures the users' thermal comfort lower bound in (13) is satisfied with probability ϵ_T:

$$\begin{aligned}(1-\beta_1) T_{\text{in}}^{(t)} + \beta_1 (T_{\text{f}}^{(t)} + \mu_T^{(t)}) - \beta_2 d_{\text{hvac}}^{(t)} + \beta_3 P_{\text{rad}}^{(t)} - T_{\min} \\ \geq -\Phi^{-1}(\rho_T) \beta_1 \sigma_T^{(t)}. \quad (15)\end{aligned}$$

Similarly, the chance constraint for the thermal comfort upper bound in (14) is obtained

$$\begin{aligned}T_{\max} - (1-\beta_1) T_{\text{in}}^{(t)} - \beta_1 (T_{\text{f}}^{(t)} + \mu_T^{(t)}) + \beta_2 d_{\text{hvac}}^{(t)} - \beta_3 P_{\text{rad}}^{(t)} \\ \geq \Phi^{-1}(\rho_T) \beta_1 \sigma_T^{(t)}. \quad (16)\end{aligned}$$

III. Stochastic MPC Formulation for HEMS

In this section, we provide the overall chance constrained MPC optimization problem for the HEMS. The following objective function is minimized at each step of the optimization:

$$f_{\text{cost}}(t, \{\mathbf{x}^{(t)}, \mathbf{u}^{(t)}\}_{t=1}^{N_h}) = \sum_{t=1}^{N_h} (c_e P_{\text{grid}}^{(t)} + P_{\text{sol}}^{(t)} U_{\text{curt}}^{(t)}),$$

where N_h denotes the prediction horizon, the optimization decision variables at each time t are collected in the vector $\mathbf{u}^{(t)} = [P_{\text{grid}}^{(t)}, U_{\text{curt}}^{(t)}, d_{\text{hvac}}^{(t)}, P_{\text{ch}}^{(t)}, P_{\text{dis}}^{(t)}]$, and the state variables at each time t are collected in the vector $\mathbf{x}^{(t)} = [T_{\text{in}}^{(t)}, E^{(t)}, P_{\text{PV}}^{(t)}]$. The overall chance constrained convex optimization problem is the following:

$$\min_{\{\mathbf{x}^{(t)}, \mathbf{u}^{(t)}\}_{t=1}^{N_h}} \quad f_{\text{cost}}(t, \{\mathbf{x}^{(t)}, \mathbf{u}^{(t)}\}_{t=1}^{N_h}) \quad (17)$$

subject to

$$0 \leq P_{\text{grid}}^{(t)} \quad \forall t, \quad (18)$$

$$0 \leq U_{\text{curt}}^{(t)} \leq 1 \quad \forall t, \quad (19)$$

$$P_{\text{PV}}^{(t)} = (1 - U_{\text{curt}}^{(t)}) P_{\text{sol}}^{(t)} \quad \forall t, \quad (20)$$

$$0 \leq d_{\text{hvac}}^{(t)} \leq 1 \quad \forall t, \quad (21)$$

$$\begin{aligned}T_{\text{in}}^{(t+1)} = T_{\text{in}}^{(t)} + \beta_1 (T_{\text{out}}^{(t)} - T_{\text{in}}^{(t)}) - \beta_2 d_{\text{hvac}}^{(t)} + \beta_3 P_{\text{rad}}^{(t)} \\ \forall t \in [1, \ldots, N_h - 1], \quad (22)\end{aligned}$$

$$T_{\min} \leq T_{\text{in}}^{(t+1)} \leq T_{\max} \quad \forall t \notin [t_{\text{DR,s}}, t_{\text{DR,f}}], \quad (23)$$

$$\begin{aligned}T_{\max} - (1-\beta_1) T_{\text{in}}^{(t)} - \beta_1 (T_{\text{f}}^{(t)} + \mu_T^{(t)}) + \beta_2 d_{\text{hvac}}^{(t)} \\ - \beta_3 P_{\text{rad}}^{(t)} \geq \Phi^{-1}(\rho_T) \beta_1 \sigma_T^{(t)} \quad \forall t \in [t_{\text{DR,s}}, t_{\text{DR,f}}], \quad (24)\end{aligned}$$

$$\begin{aligned}(1-\beta_1) T_{\text{in}}^{(t)} + \beta_1 (T_{\text{f}}^{(t)} + \mu_T^{(t)}) - \beta_2 d_{\text{hvac}}^{(t)} + \beta_3 P_{\text{rad}}^{(t)} \\ - T_{\min} \geq -\Phi^{-1}(\rho_T) \beta_1 \sigma_T^{(t)} \quad \forall t \in [t_{\text{DR,s}}, t_{\text{DR,f}}], \quad (25)\end{aligned}$$

$$P_{\text{grid}}^{(t)} + P_{\text{PV}}^{(t)} + P_{\text{dis}}^{(t)} = P_{\text{uc}}^{(t)} + P_{\text{c}} d_{\text{hvac}}^{(t)} + P_{\text{ch}}^{(t)} \quad \forall t, \quad (26)$$

$$E^{(t+1)} = E^{(t)} + \eta P_{\text{ch}}^{(t)} \Delta t - \tfrac{1}{\eta} \Delta t P_{\text{dis}}^{(t)} \; \forall t \in [1, \ldots, N_h - 1], \quad (27)$$

$$E_{\min} \leq E^{(t+1)} \leq E_{\max} \quad \forall t \in [1, \ldots, N_h - 1], \quad (28)$$

$$0 \leq P_{\text{dis}}^{(t)} \leq P_{\max} \quad \forall t, \quad (29)$$

$$0 \leq P_{\text{ch}}^{(t)} \leq P_{\max} \quad \forall t, \quad (30)$$

$$\begin{aligned}\sum_{t=t_{\text{DR,s}}}^{t_{\text{DR,f}}} P_{\text{pred}}^{(t)} - B_{\text{util}} - \sum_{t=t_{\text{DR,s}}}^{t_{\text{DR,f}}} P_{\text{grid}}^{(t)} + \mu_P \\ \geq \Phi^{-1}(\rho_{\text{DR}}) \sigma_P \quad \forall t \in [t_{\text{DR,n}}, t_{\text{DR,f}}]. \quad (31)\end{aligned}$$

IV. SIMULATIONS: LOAD SHEDDING IN SUMMER

To demonstrate the effectiveness of the proposed HEMS algorithm in (17)-(31), we provide simulation results for DR during a summer afternoon for two different houses. The MATLAB-based modeling system for solving disciplined convex programs, CVX, is used in this work. For all simulations, we assume the DR notice is received by the HEMS 2 hours before the start of the summer load reduction DR event from 4pm to 6pm, and the DR reduction request needs to be satisfied with probability $\rho_{\text{DR}} = 95\%$ while maintaining the thermal comfort bounds with probability $\rho_{\text{T}} = 95\%$. The preferred indoor thermal comfort band is 68°F $\leq T_{\text{in}}^{(t)} \leq$ 72°F from 9am to 5pm and restricted to 69°F $\leq T_{\text{in}}^{(t)} \leq$ 71°F otherwise when the residence is assumed to be occupied. The simulation has a 24 hour prediction horizon with 1 hour time intervals.

The device models for both houses are assumed to have the same parameters. The residential PV array size is 20m^2 with a tilt of 30° and an efficiency of 16%. The 5-kWh residential battery system is restricted to 15% to 85% of the maximum SOC to preserve the battery lifetime [14]. The battery inverter power limit is 3 kW with an inverter efficiency of 95%. The battery charging/discharging efficiency η is 95%. The power consumption of the HVAC when cooling P_{c} is 3 kW. The cost of energy from the grid is assumed to be a flat rate of \$0.11/kWh. The outdoor air temperature and solar irradiance forecasts were obtained from NOAA USCRN data [16].

A. Simulation Results

The proposed HEMS algorithm is simulated for two homes, denoted House 1 and House 2, which have different house model parameters and are given in Table I. The model parameters are designed such that House 2 is a less-insulated version of House 1 and has a less efficient HVAC cooling system. The initial battery SOC is 1.5 kWh and initial indoor temperature is 73°F for both homes. The simulation results for House 1 are shown in Figs. 2 and 3 for a DR reduction request $B_{\text{util}} = 0.75$ kW with uncertainty in available PV $\sigma_P = 0.5$ kW and uncertainty in the outdoor forecast $\sigma_T = 6$°F. From Fig. 2, we can see that the battery only charges when the available PV power exceeds the base load of the home, and is discharging (supplying power to the home) otherwise. During the DR notice period, the P_{grid} usage is greater than the predicted grid usage without DR P_{pred} to account for pre-cooling before the DR event. During the DR period, the HEMS reduces grid power usage of House 1 by leveraging the stored energy in the battery system. The indoor air temperature $T_{\text{in}}^{(t)}$ stays within the preferred thermal comfort band throughout the simulation, as shown in Fig. 3 (top). The battery SOC is shown in Fig. 3 (middle), which highlights the usage of energy stored in the battery during the DR period. The battery behavior is included in Fig. 3 (bottom) to show there is no simultaneous battery charging and discharging.

Additional simulation results for both houses with varying DR requests B_{util} and uncertainties σ_P and σ_T are given in Table II. Let B_{H1} and B_{H2} denote the actual reduction in grid

TABLE I
HOUSE MODEL PARAMETER VALUES FOR HOUSE 1 AND HOUSE 2.

Parameter	House 1	House 2
β_1	0.03	0.035
β_2	4	3
β_3	0.000163	0.000326

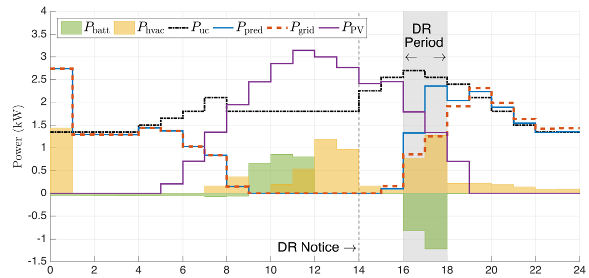

Fig. 2. Power profiles for House 1 with proposed HEMS algorithm responding to a summer load reduction DR event of $B_{\text{util}} = 0.75$ kW with uncertainty σ_P in available PV forecast and σ_T in outdoor air temperature forecast. The battery is discharging when $P_{\text{batt}} < 0$ and charging when $P_{\text{batt}} > 0$. The HEMS predicted grid power usage is given by P_{pred}. The actual grid power usage in the case of a DR event is denoted P_{grid}.

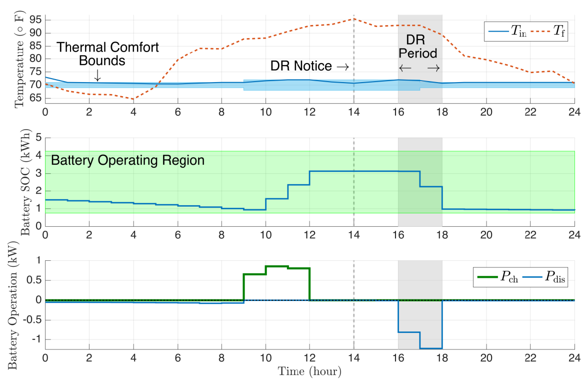

Fig. 3. Simulation of House 1 with proposed HEMS algorithm responding to a summer load reduction DR event of $B_{\text{util}} = 0.75$ kW with uncertainty σ_P in available PV forecast and σ_T in outdoor air temperature forecast. Indoor air temperature T_{in} (top). The outdoor air temperature forecast T_{f} is shown for reference. Residential battery system SOC (middle). Battery charging and discharging behavior (bottom).

power usage that House 1 and House 2 were able to achieve, respectively. From Table II, we can see that the B_{H1} and B_{H2} increase as the uncertainty in the solar forecast increases. For House 1 with constant σ_P, as the uncertainty in the outdoor air temperature forecast increases, B_{H1} decreases since more energy is required to maintain the indoor house temperature. For the simulation of House 2 with constant σ_P, in some cases the HEMS does not need to compensate further as σ_T varies since the uncertainty in the available PV is more restrictive. House 1 is able to achieve grid power usage reduction greater than or equal to the reduction achieved by House 2 due to the increased cooling efficiency and lower building envelope coefficient of House 1 relative to House 2.

B. Chance Constraint Validation

Monte Carlo simulations were conducted to validate that the chance-constrained optimization solutions actually satisfy

TABLE II
ADDITIONAL HEMS ALGORITHM SIMULATION RESULTS.

σ_P (kW)	σ_T (°F)	B_{util} (kW)	B_{H1} (kW)	B_{H2} (kW)
0.25	3	0.5	0.9443	0.9172
0.25	6	0.5	0.9392	0.9139
0.5	3	0.5	1.3379	1.3224
0.5	6	0.5	1.3348	1.3224
0.25	3	0.75	1.1836	1.1626
0.25	6	0.75	1.1787	1.1612
0.5	3	0.75	1.5802	1.5724
0.5	6	0.75	1.5771	1.5724

TABLE III
MONTE CARLO SIMULATIONS FOR CHANCE CONSTRAINT VALIDATION.

Constraint	House 1	House 2
$B_{\mathrm{H}} \geq B_{\mathrm{util}}$	95.28%	95.16%
$T_{\min} \leq T_{\mathrm{in}}^{(17)} \leq T_{\max}$	95.16%	95.2%
$T_{\min} \leq T_{\mathrm{in}}^{(18)} \leq T_{\max}$	95.24%	95.4%

the DR request with probability $\rho_{\mathrm{DR}} = 95\%$ and the thermal comfort bounds are satisfied with probability $\rho_T = 95\%$. Monte Carlo simulations use the chance-constrained optimal solution and solar forecast error and outdoor temperature forecast error sampled randomly from their respective distributions to check that constraints in (2) and (8) are satisfied. Using 5000 Monte Carlo simulations for validating the HEMS solutions for both House 1 (shown in Figs. 2 and 3) and House 2 with the same simulation parameters, it is confirmed that the chance constrained optimal solution satisfies the constraints in (2) and (8), which is shown in Table III. The percentages in Table III represent the empirical distributions of the Monte Carlo simulations that satisfy the DR request in (8) and temperature bounds in (2) during the DR period. Thus, in view of Table III, the Monte Carlo simulations validate that the solution obtained with the chance constrained MPC optimization problem ensures the DR request and temperature bounds are satisfied with probability ρ_{DR} and ρ_T, respectively.

V. CONCLUSION

A chance-constrained MPC-based algorithm is proposed for a HEMS capable of responding to grid DR events. Chance constraints were incorporated into the optimization problem to ensure the DR request and home temperature preferences are satisfied with a high probability given uncertainty in both the solar and outdoor temperature forecasts. The chance constrained optimization solution was validated with Monte Carlo simulations. Simulation results show the proposed HEMS algorithm responded to the DR request by coordinating flexible devices during the DR notice period prior to DR event.

Future work includes validating the proposed HEMS on hardware in the loop testbeds such as Foresee [2]. Additionally, incorporating other flexible loads such as a dishwasher, water heater, and electric vehicles (EVs) into the proposed HEMS framework may allow for further load flexibility during the DR notice and DR event period to ensure both the DR request and users' thermal comfort are satisfied with a high probability given uncertainties in PV availability and outdoor temperature. Further work on a stochastic HEMS algorithm also includes incorporating uncertainty in home occupancy schedules, which is not effectively modeled by a Gaussian distribution in the chance constraint formulation. While we assume the random variables in this work are Normally distributed, the use of chance constraints can be extended to non-Gaussian distributions using existing techniques in the literature [11].

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