Development of a Trip Energy Estimation Model using Real-World Global Positioning System Driving Data

Preprint

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To be presented at ITS World Congress 2017
Montreal, Canada
October 29 – November 2, 2017

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Conference Paper
NREL/CP-5400-69121
September 2017

Contract No. DE-AC36-08GO28308

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Paper ID # AM-SP1095

Development of a trip energy estimation model using real-world global positioning system driving data

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Abstract
A data-driven technique to estimate energy requirements for a proposed vehicle trip has been developed. Based on over 700,000 miles of driving data, the technique has been applied to generate a model that estimates trip energy requirements. The model uses a novel binning approach to categorize driving by road type, traffic conditions, and driving profile. The trip-level energy estimates can easily be aggregated to any higher-level transportation system network desired. The model has been tested and validated on a vehicle driving data set from Austin, Texas. Ground-truth energy consumption for the data set was obtained from Future Automotive Systems Technology Simulator (FASTSim) vehicle simulation results. The energy estimation model has demonstrated 12.1% normalized total absolute error. The energy estimation from the model can be used to inform control strategies in routing tools, such as change in departure time, alternate routing, and alternate destinations, to reduce energy consumption. The model can also be used to determine more accurate energy consumption of regional or national transportation networks if trip origin and destinations are known. Additionally, this method allows the estimation tool to be tuned to a specific driver or vehicle type.

KEYWORDS:
Energy estimation, drive cycles, green routing

Introduction
The transportation sector accounts for 28.5% of total energy consumption in the United States [1]. To increase the energy efficiency of such a large sector, it is critical to accurately predict the energy required for individual trips and trip segments. Estimation of proposed trip routes has direct application in the green routing and mobility planning areas. Accurate trip energy estimation can also be applied to regional- or national-level transportation energy analysis where trips (i.e., origin – destination pairs) are known, but real-world driving data are unavailable.

Previous work
There has been a wide range of activity in this area in recent years. The activity most pertinent to the present work has been on estimating the energy impact of various road and driving attributes, and previously-developed energy estimation models [2]. Exploration into road and driving attributes such as road grade, vehicle driving speed, and congestion [3] has shown the energy and emissions sensitivities of various drive trains to these features [4-8]. With some understanding of the sensitivities to these road and driving features, researchers have developed models to estimate energy consumption for various applications [9,10]. One such application is in fully electric vehicles. The range and battery state-of-charge for electric vehicles are critical values to accurately design electric vehicles and model their use. To obtain accurate range and state-of-charge values, an accurate energy estimation is necessary [11]. Such models typically take the form of a regression or analytical expression that requires assumptions about the exact driving conditions. Additionally, there is a need to estimate energy regardless of fuel source or drivetrain technology for green routing applications to select the least energy-consuming route from a set of proposed routes. This need is the inspiration for the present work.

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Contributions of present work

Accurate estimation of trip and sub-trip energy requirements without a known speed trajectory is a need that exists for a wide range of applications in the transportation sector [12]. Making these estimates without a set driving trajectory on the proposed route requires large amounts of previously logged data in a variety of driving conditions to inform a prediction model [13]. Metropia, Inc. has created an advanced mobility platform that allows drivers to take full advantage of road network capacity and alternative commute modes [14]. This platform is used to provide real-world driving data to inform the estimation model development.

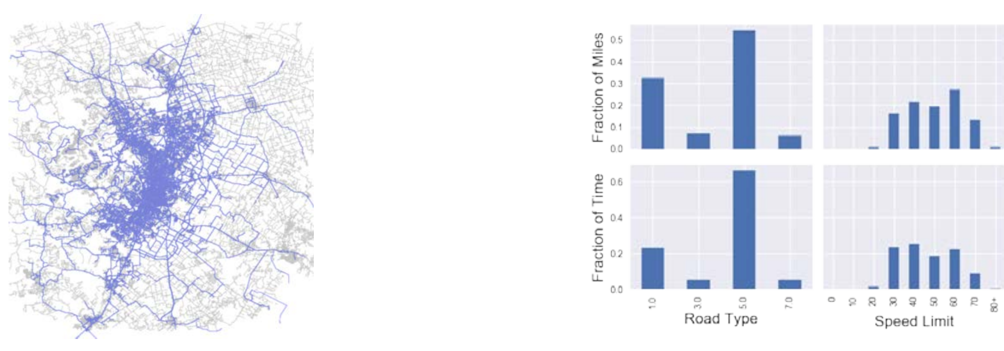
The data-driven model approach provides valuable qualitative and quantitative results. The most important fuel economy determinants for a personal vehicle trip were identified as a first step in this model development. Simply recognizing these determinants establishes principles for minimizing the energy requirements of vehicle travel by choosing less energy-intense routes. However, more thorough validation of the model accuracy makes it a powerful tool for more rigorous transportation system optimization.

The target application for the proposed energy estimation method is trip routing. The navigation and routing sector [15,16] has a direct need to estimate energy requirements before a trip is driven to calculate the optimal route, based on time, distance, and energy consumption priorities. The model will be integrated into Metropia's previously mentioned mobility platform, to inform control mechanisms in a route energy consumption optimization tool. The objective of these control mechanisms is to minimize energy consumption for a proposed trip and subsequently, an entire mobility region. The estimation tool will be deployed in the mobility platform for users to test the control mechanisms on real-world driving.

Real-world driving data

The proposed model relies on high-resolution global positioning system (GPS) driving data from previous trips to accurately estimate energy costs for driving that has not yet taken place. Metropia has provided anonymized 1-Hz GPS data from the Austin, Texas, region for this purpose. The data span five consecutive months and contain over 85,000 vehicle trips covering a total distance of nearly 700,000 miles. This amounts to roughly 100 million GPS points. In addition to the 1-Hz driving data, a routable road network with link-level road attributes such as speed limit and road type was provided. The road network contains roughly 250,000 links and is complete for the region where the driving took place.

Figure 1 shows the GPS trajectories in blue, plotted on top of the road network links in gray. The point distribution shows good coverage of the city, with a high density of urban driving. Figure 1 also shows the distribution of road type and speed limit throughout the data set, to give an idea of the types of driving in this data.



(a) (b)

Figure 1 (a) The 1-Hz GPS points for 5 months of driving in Austin, Texas, are shown in blue, and the road network is shown in gray. (b) Distributions in the data set by fraction of total mileage and fraction of total time are shown for road type (1=freeway, 3=ramp/frontage road, 5=highway, 7=arterial) and speed limit in 10-mph bins.

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The road network is a critical component in developing an accurate model. The model is only as accurate as the inputs provided. To join the road network attributes to the second-by-second GPS points, raw GPS trajectories must be map-matched to links in the road network. This step was performed with Metropia's in-house map-matching software. Road network attributes that are potentially valuable to the energy estimation model include the following:

- Number of lanes
- Free flow speed
- Link type (freeway, arterial, highway, etc.)
- Length
- Speed limit
- Orientation of start and end nodes.

Additionally, traffic prediction data were used to incorporate congestion into the energy estimation model. The traffic data contain estimated travel times and vehicle volumes over the network links that experience some kind of routine congestion (~20% of the network). The predicted values are at 15-minute intervals for every day of driving in the data set. The resulting speed profiles for a link are more accurate than simply assuming the speed limit or free flow conditions correspond to the speed at which a vehicle will traverse a link. Figure 2 shows an example of the predictive traffic data. Two speed profiles are selected along I-35 near downtown Austin. The profiles span one day and show a slowdown in both directions that is worst during the evening peak, from about 4-6 p.m.

Figure 2 Two road network links along I-35 in downtown Austin, with their speed profiles plotted as an example of the traffic prediction data

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Energy estimation modelling process

Overview

The Metropia data set contains nearly 100 million spatial points from real drivers acquired at a 1 Hz frequency. GPS points of this high resolution can cause drive cycle profiles to be noisy, so the drive cycles are cleansed and filtered via National Renewable Energy Laboratory (NREL) standard processing [17] to make them more suitable for the powertrain simulation model. As a part of the processing, the U.S. Geological Survey Digital Elevation Model is used to append road grade information to the drive cycle data [4], and typically GPS points are matched to a road network to obtain other physical features such as length, lane count, etc.

Once the raw driving data have been cleaned and filtered, the Future Automotive Systems Technology Simulator (FASTSim) is run for all drive cycles to determine the second-by-second fuel consumption for the drive cycle. The backward-/forward-calculating FASTSim model allows for rapid results generation and has the capability to model a range of stock vehicles and powertrain configurations [18]. For this work, the FASTSim fuel consumption results from a model similar to the 2012 Ford Fusion conventional powertrain are taken to be the ground truth for the development of the energy estimation model. FASTSim has been continuously updated and validated for various drivetrains, and is a trusted simulation tool that allows a large set of fuel consumption data to be generated for modelling efforts such as this. Figure 3 provides a concise summary of the workflow. This paper focuses on the last two steps in Figure 3 since the prior steps are all well described in the literature.

Figure 3 The diagram outlines the process for generating the energy estimation model

Data Processing

FASTSim returns fuel consumption estimates for each point along the drive cycle input data. The point-based FASTSim fuel consumption results are then aggregated to the link level, as road network links represent the finest resolution that traffic and road attributes can be reliably obtained from available input data.

We refer to each link-level aggregation of second-by-second FASTSim fuel consumption calculations over the link as a “pass.” The road network attributes are then joined to the pass by a link identification number. The attributes include speed, link type, road grade, link orientation, and traffic prediction data. Speed and link type are simply road network attributes that can be appended to the pass results; however, the other attributes must be calculated. The road grade is computed for each pass as a ratio of "rise" to "run," in which run is the horizontal distance and rise is the change in elevation over the link. The result is taken as a percentage. Link sequencing calculations are performed to determine what kind of turn was made coming into the current link and what needs to be made exiting the link. If available, traffic data are also joined to the pass by link ID and the nearest timestamp to the start time of the pass.

The passes, with appended information, are used to build the fuel consumption rate estimation model. Passes are grouped into “bins” by the link attributes (speed limit, grade, link type, etc.) that are selected for a particular model. Average fuel consumption rate is calculated from the FASTSim fuel consumption results for each bin, which generates the estimation model. Every bin of attributes and conditions has an average fuel consumption rate value. The model can then be applied as a lookup table. For a given pass over a link in a proposed trip, the lookup table provides the appropriate bin to estimate the fuel consumption rate.

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Model selections

Selection of road attributes and driving conditions was an iterative process. The objective was to choose the attributes that best represent distinct vehicle fuel efficiency operating conditions along a road link. All attributes and characteristics must be available for a *proposed* vehicle trip before it is driven (i.e., while real GPS data were used to calculate the expected efficiency results for a given set of conditions, attributes such as vehicle-specific acceleration rates were excluded from the model because these are not known prior to the vehicle actually driving the route). Once the available attributes were designated, the specific selections for the model iterations were made. Too many criteria in the model can spread the supporting input data too thin, which leads to sample sizes that are not representative of all passes that may fall in that bin. Maintaining proper representation in each bin of the model is important when applying the model to other data sets. However, having too few criteria will also result in an inaccurate model due to grouping too many varied operating conditions together. Keeping these pitfalls in mind, we began the iterative process by first implementing the most basic potential model, simply assuming a single average fuel consumption rate for all driving. This single bin approach gives a useful global statistic, but results in high errors when a “one-size-fits-all” fuel consumption rate is used to estimate the fuel requirement for an individual trip. It is clear that more accurate estimates than this are needed, hence the motivation for the present work.

The first attribute selected for consideration in the model is speed. Vehicle speed has a known and significant impact on fuel economy [5]. Since real vehicle speed cannot be known definitively prior to a proposed vehicle trip, a surrogate value must be used. In the initial data set, the options were free-flow speed and speed limit. Free-flow speed is defined as the typical speed on a link when there is no congestion and speed limit is the posted maximum speed. Since the free-flow speed attribute was only present on about half of the road network used in this project, speed limit was chosen as the initial surrogate for average vehicle speed over a link (it enjoyed full coverage in the available road network). The first model using speed as a binning attribute was a simple two bin model with bins of [0–40 mph] and [40 mph+]. These ranges are chosen to roughly separate “city” and “highway” driving conditions. A second model iteration further refined speed limits to 10 mph intervals ranging from 0 to 80+ mph. This model naturally does a more thorough job of separating various driving conditions, beyond just local and highway operation.

The second attribute selected for inclusion in the estimation model was road grade, which is also known to significantly impact vehicle fuel economy [4]. Grade is considered in the FASTSim vehicle model calculation of road load and subsequently power required to propel the vehicle, so it is beneficial to consider grade in the binning of FASTSim results to more accurately predict vehicle fuel consumption. A third attribute is link type, which is a classification of the roadway type. The Metropia-provided network included five road type categorizations: freeway, ramp/frontage road, arterial, highway, and local street. These categories provide potentially important distinctions for energy estimation. For instance, stop/start driving on local streets and congested crawling of traffic on freeways may exhibit similar average speeds but differing fuel consumption rates.

Road link sequencing was also considered in the model development. Particularly, sequencing was used to consider the orientation of links relative to each other. Figure 4 shows the orientation hypothesis graphically. The top scenario considers the orientation of the previous link to the current. In this situation where a 90-degree right-hand turn is required; the vehicle will likely be decelerating on the previous link and accelerating on the present link. The result is a higher rate of fuel consumption for the present link than if it were at cruise condition. The lower scenario is the opposite. If the following link requires a 90-degree right-hand turn from the present link, the vehicle would be expected to decelerate on the present link and accelerate on the following link. This result is a lower rate of fuel consumption for the present link than if it were at cruise condition. The road network provides an orientation field for both nodes on each link in the network. These nodes allowed for calculation of the turn direction (left/right) and turn severity (slight/sharp) for any sequence of connected links in the road network.

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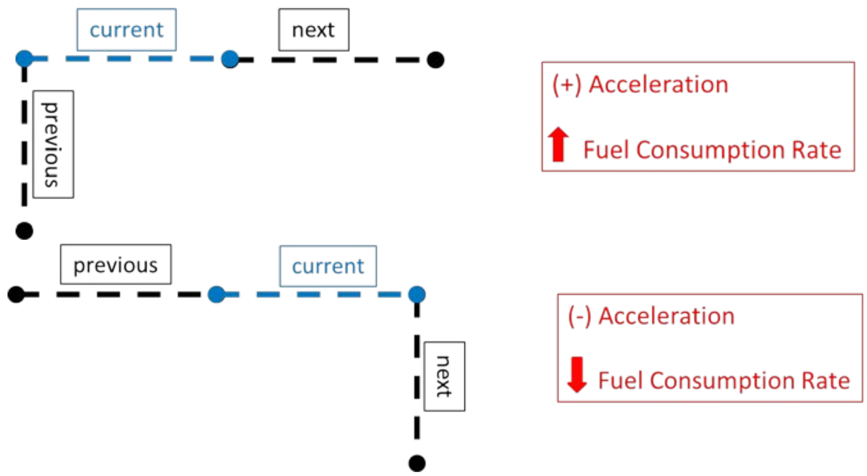

Figure 4 The diagrams show road link sequencing. The upper scenario shows the previous link is skewed and requires a right-hand turn onto the current link. The lower scenario requires a right-hand turn onto the coming network link.

Implementation

The model additions mentioned above were implemented iteratively to record the incremental improvement of each road or driving attribute. The raw driving data from Metropia were stored in a PostgresSQL relational database and worked with in a Python environment. The Pandas data analysis package was used heavily for all in-memory analysis in model development, implementation, and results processing [19]. Model deployment into the Metropia platform will require reconfiguring the data input and output structure as the model will be handling data on a more rolling basis. The current structure for model generation is shown in Figure 5. The model input is the full set of FASTSim results aggregated into link passes. Then the model is built by labelling each pass with the appropriate categorizations. Bins are then created from each unique set of categories, and the passes matching those categorical sets are included in the bin. Finally, the output is a lookup table with average fuel consumption rate values for each bin, to be applied to proposed trips that can be segmented at the road network link level. In the final model deployment, lookup tables can be statically deployed for real-time energy estimation in the Metropia platform. The table values can be periodically updated as more data become available to the model generator.

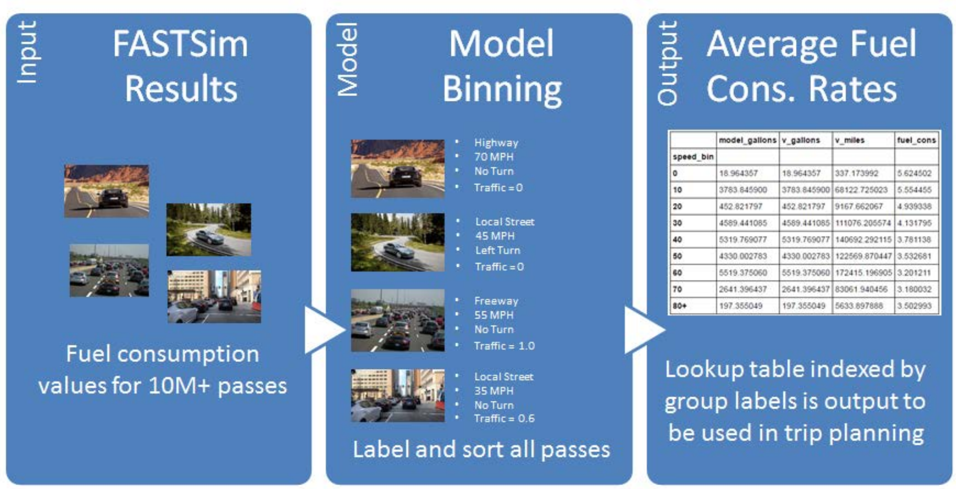

Figure 5 The flowchart illustrates the model creation process from data input to one example of the data output

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Results

Estimation accuracy

To compare the incremental improvement of the energy estimation model iterations as more attributes are added and to understand the overall model accuracy, a performance metric was needed. Since the working unit of driving that this model will be applied to is the trip level, trips were selected to quantify error. The normalized total absolute error (NTAE) between estimated trip energy and ground truth trip energy is calculated for the full set of all trips after the model has been applied. The NTAE is simply the ratio of total absolute error in energy consumption from all trips to the total energy consumption of all trips. Equation (1) defines NTAE:

$$NTAE = \frac{\sum_{i=1}^{n} |(f_i - y_i)|}{\sum_{i=1}^{n} y_i}, \tag{1}$$

where n is the total number of trips in the data set for a given trip, i, f_i is the trip estimated energy requirement, and y_i is the ground truth trip energy. The NTAE provides a metric of accuracy relative to the entire set of regional data against which the energy estimation model is applied. This is a key feature of the metric because the intended purpose of the model is to reduce energy use at a system-wide level via the Metropia mobility application.

Model performance

Table 1 lists the final set of tested energy estimation models. Iteratively adding model features allowed attributes to be excluded from the next iteration of the model if they did not provide significant incremental NTAE improvement. The number of bins and the specific values used in binning each attribute were also swept to find more optimal binning schemes.

The performance of each model in Table 1 is plotted in Figure 6. The NTAE values give an idea of which model additions have the greatest impact on accuracy. It is clear that the addition of road grade to the model provided the most significant decrease in NTAE, with a reduction of about 3%. It is anticipated that with further additions to this model, such as traffic prediction data, the NTAE value with continue to fall. For now data quality concerns with the traffic prediction data prevented out including it in this publication, but it is our intention to add it in the future. To provide context for the NTAE values, the dashed red line in Figure 6 represents a model with inputs that are more accurate indicators of vehicle energy consumption. These inputs are vehicle average speed, average acceleration, and road grade. While this line is not necessarily a theoretical minimum error, it is an acceptable target for the energy estimation model that does not consider actual second-by-second vehicle driving data.

Table 1 Models generated and run in this work are tabulated with their corresponding model number used in later figures

Model #	Model Description
1	10-mph Speed Bins
2	Speed Bins + Link Type
3	Speed Bins + Road Grade
4	Speed Bins + Road Grade + Previous Link Orientation

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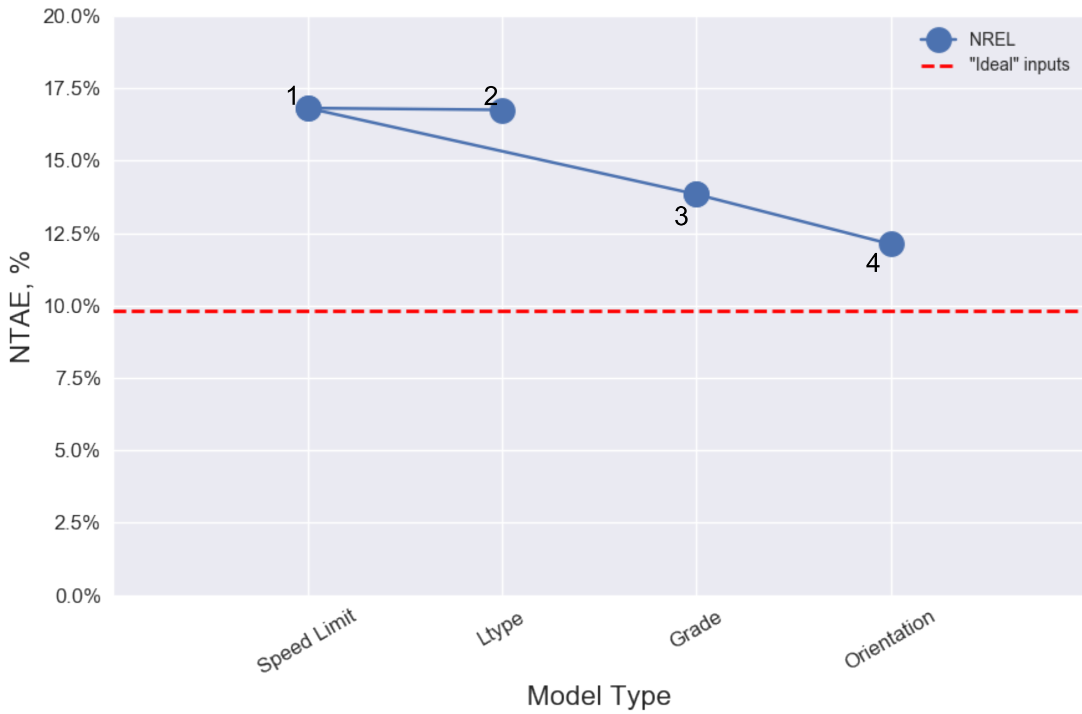

Figure 6 NTAE values for each of the four models listed in Table 1 are plotted to show the relative improvement of successive model additions. Points connected by links imply that they build on one another. Model points are a composite of all points to their left that are connected by a branch. The dashed red line represents model performance when it considers "ideal" parameters: vehicle average speed, average acceleration, and road grade.

The overall NTAE results from the model generation process are obviously of interest, but the impact of each model attribute on fuel consumption rate is also of interest. Figure 7 shows trends for four model attributes: speed limit, road grade, link type, and previous link orientation. A threshold of 500 miles was set for each of these bins to exclude outliers and non-representative bins from the plots. The speed limit vs. fuel consumption rate curve is encouraging because it shows the same trend as vehicle average speed vs. fuel consumption rate would show [7]. Note that the fuel consumption rate for the modeled vehicle reaches a minimum somewhere in the vicinity of 60 mph. The upper right plot in the figure shows road grade impacts segmented by speed limit. While some speed lines have low density due to not meeting the mileage threshold for each bin, the overall trend is as expected for each curve, with fuel consumption increasing as hill climbing demands increase. The lower left link type plot shows a significant difference in fuel consumption rate between freeway and non-freeway link types. This difference suggests that there is a difference in driving style on freeway versus non freeway links that possess similar speed limits. Finally, the lower right plot shows model results with previous link orientation included, which provided much more significant model improvement than did next link orientation. The plot shows turns off of the previous link binned by severity and direction. The results show that severe left turns lead to a higher fuel consumption rate on the present link than similar right turns. This creates the asymmetry of the speed lines for the plot and is attributed to a higher likelihood of idling before executing a left turn.

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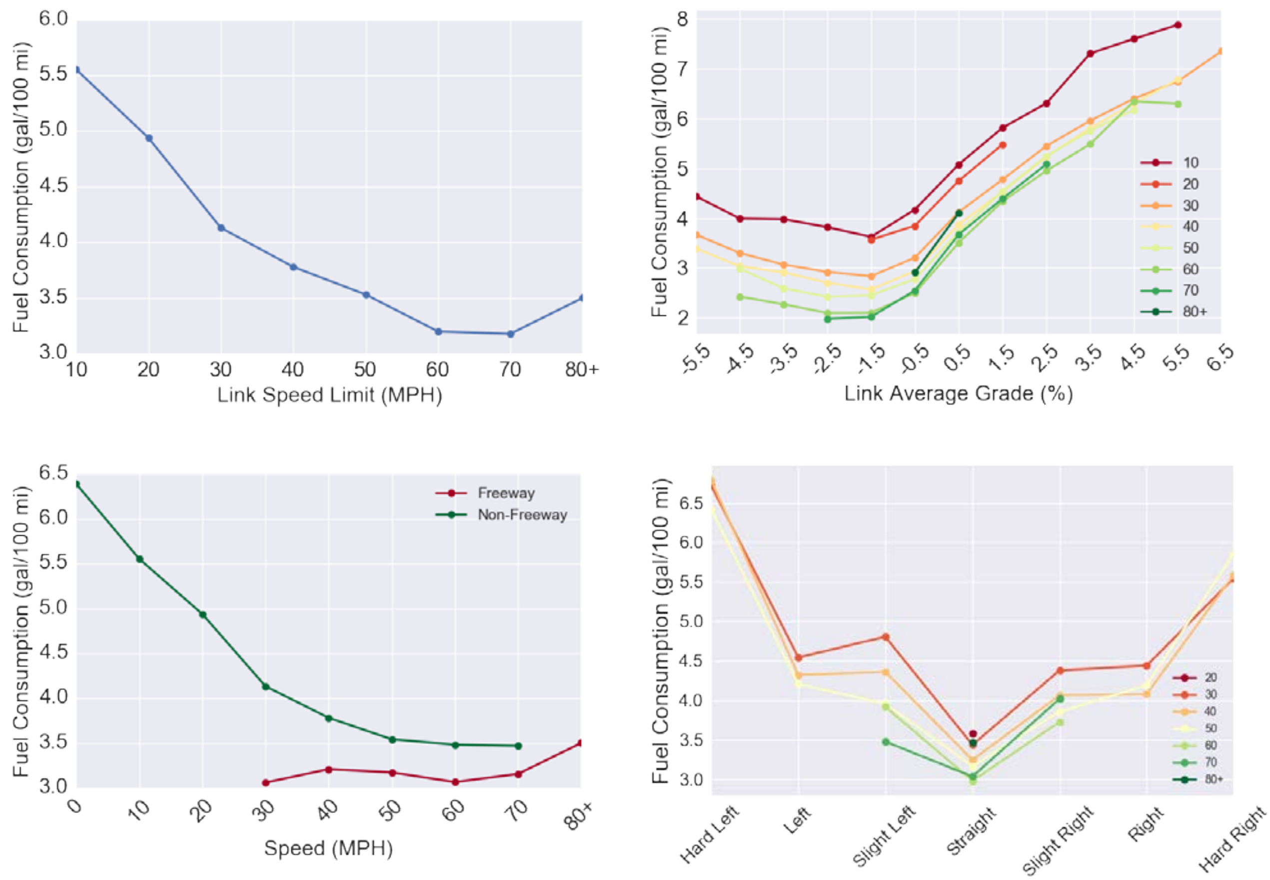

Figure 7 The subplots show binned results from successive model iterations. The upper left plot shows link speed limit vs. fuel consumption rate. The upper right plots link average road grade vs. fuel consumption, disaggregated by link speed limit as well. The lower left shows link speed vs. fuel consumption rate disaggregated into freeway and non-freeway link types. The lower right plots the previous link orientation binned into severity of left or right turns vs. fuel consumption, disaggregated by speed limit. Turn severity is defined as: straight<10°, 10°<slight left/right<80°, 80°<left/right<100°, 100°<hard left/right<180°.

Benchmark comparison with MOVES

The energy estimation model proposed in this work has been benchmarked against the U.S. Environmental Protection Agency's (EPA's) MOtor Vehicle Emission Simulator (MOVES) [20]. MOVES can be applied at various scales and is commonly used by state and local governments to perform energy and emissions inventories for the transportation sector. At the national and county scales, MOVES uses default distributions of drive cycles for the vehicle population and segments the cycles into average speed bins to estimate energy consumption. There are 16 speed bins at 5-mph increments. Each bin has an assigned energy rate associated with it for a particular vehicle type. The energy rates for a default gasoline-powered passenger car were output from MOVES2014a and used to estimate energy consumption for the Metropia driving data used in this work. Figure 8 shows the fuel consumption rates used for the two models by average speed bin. When the MOVES model was applied to the Metropia data set for testing the result was NTAE = 16.4%. This is about 4.3% higher than the NTAE result for the best energy estimation model proposed in this paper. This equates to a 26% proportional reduction in error over the MOVES model for this particular application.

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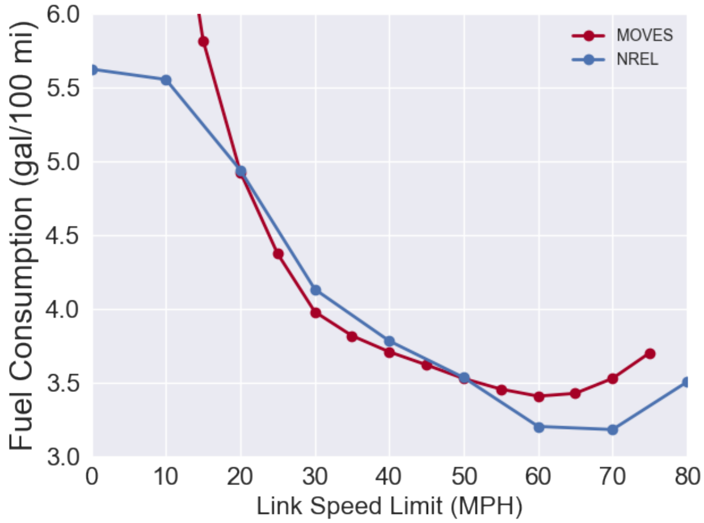

Figure 8 The plot shows fuel consumption rate values for speed bins in the MOVES and NREL-developed energy estimation models

Discussion

There are two main takeaways from the results of this work. First, the energy estimation model can, on average and at the stage of development shown here, estimate the energy requirements of a vehicle trip within 12.1% error. Second, the model accurately accounts for the fuel economy impacts of speed, road grade, road type, and link orientation at the link-pass level. This suggests that even with some amount of error in quantifying energy requirements, the model can still accurately compare proposed trip routes and predict when energy savings would occur. For example, if there are two route options for a planned trip, and one has significantly more grade fluctuation, the model will accurately indicate that the lower grade fluctuation route has lower energy requirements. Clearly, more complex examples exist that involve each of the model attributes. The proposed model is able to incorporate these kinds of considerations for contemplated trip routes in the absence of high-resolution GPS data that would only become available after a driver selects and drives one of the routes.

As discussed previously, planned additions to the model will further improve its overall accuracy and the range of considered variables that influence vehicle trip fuel efficiency. The first addition will be quality traffic prediction data to provide a more accurate estimate of the average speed a vehicle will experience on each road link. Subsequent anticipated additions include real-time weather considerations and customizing the model to account for individual driving behavior differences. Such further model additions will also be informed by initial deployment and performance in the Metropia platform.

Conclusions

The present work outlines the approach taken to develop a data-driven energy estimation model for proposed vehicle trips before they are driven. Five months of high-resolution GPS trajectories from over 800 drivers were processed and used to inform the energy estimation model. Promising results have been presented that quantify the accuracy of the current model in estimating the energy consumed by a vehicle trip. The results also indicate that the model is fit for its intended application—that is, informing control mechanisms in a mobility application to reduce vehicle energy consumption in a region by proposing the least energy consuming trips to users. The model has been shown to outperform EPA's MOVES model for this application, and it has the capacity for further improvement with the addition of more input data as they become available. The demonstrated energy estimation methodology generates a computationally inexpensive model for applying in real-time applications and benefits from an extensible framework. Additions to the model are on-going and will continue to be refined as initial versions are deployed to real-world drivers.

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Acknowledgements
The authors would like to acknowledge the ARPA-E sponsors who have supported this effort through their TRANSNET program award to the NREL team. The authors would also like to thank others at NREL and Metropia who supported the project by providing data, documentation, advice, and brainstorming. The U.S. Government retains and the publisher, by accepting the article for publication, acknowledges that the U.S. Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this work, or allow others to do so, for U.S. Government purposes.

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