Inclusion of Structural Flexibility in Design Load Analysis for Wave Energy Converters

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Abstract—Hydroelastic interactions, caused by ocean wave loading on wave energy devices with deformable structures, are studied in the time domain. A midfidelity, hybrid modeling approach of rigid-body and flexible-body dynamics is developed and implemented in an open-source simulation tool for wave energy converters (WEC-Sim) to simulate the dynamic responses of wave energy converter component structural deformations under wave loading. A generalized coordinate system, including degrees of freedom associated with rigid bodies, structural modes, and constraints connecting multiple bodies, is utilized. A simplified method of calculating stress loads and sectional bending moments is implemented, with the purpose of sizing and designing wave energy converters. Results calculated using the method presented are verified with those of high-fidelity fluid-structure interaction simulations, as well as low-fidelity, frequency-domain, boundary element method analysis.

Index Terms—Wave energy converter, structural flexibility, design, dynamics, and modal analysis

I. INTRODUCTION

Harvesting energy from the ocean has attracted increased research attention in recent decades [1], [2]. Numerous forms of wave energy converters (WECS) have been designed to capture the energy of the ocean surface waves and convert it to electricity [3]. Compared to other renewable energy sources, such as wind and solar, the wave energy resource is less seasonal and offers the highest energy density [4]. However, wave energy technologies are still in the early stages of research and a physical understanding for optimizing WEC reliability and lowering the cost of energy is still evolving.

Structural costs have been identified as one of the primary cost drivers for WECs. As a result, accurate load predictions during the design process are essential to further reduce the cost of WECs. Furthermore, several new concepts incorporating composite materials and flexible WEC components, rather than the traditional steel/rigid-body designs, have recently been proposed as potential cost reduction pathways. Consequently, WEC structural flexibility, as well as the resulting fluid structure interactions (FSI) of the WEC system, have become increasingly important aspects to WEC design. Therefore, when considering the interaction between wave loading and WEC structural dynamics, an accurate and computationally efficient coupled approach for structural and hydrodynamic analyses is needed for WEC design.

The most widely used approach for FSI modeling is to couple computational fluid dynamics with finite element analysis. Another direct method to modeling the structural-hydrodynamic behavior of flexible WEC components is to simultaneously model the frequency-domain hydrodynamic boundary value problem with the elastic finite element response [5]. However, both approaches are computationally expensive and generally unsuitable in the initial phases of WEC design. Alternatively, a reduced-order, generalized modes method may be adopted, in which additional degrees of freedom (DOF) associated with a preselected set of generalized body modes, are included in the frequency-domain hydrodynamic boundary value problem [6]. These generalized modes are typically the natural mode shapes of the deformable structures; however, approximate mode shapes, such as Legendre polynomials, may also be used [6]. Hydrodynamic loads corresponding to radiation (i.e., added mass and damping coefficients) and wave excitation are then evaluated based on a unit modal response. Newman employed the generalized modes method to analyze the deformation of barges and a vertical cylinder using WAMIT, a frequency-domain, boundary-element-based potential flow solver [6], [7]. A similar study was conducted to analyze a very flexible barge in the time domain utilizing hydrodynamic coefficients from a frequency domain potential flow solution and a state-space model to approximate the radiation and excitation impulse response functions [8]–[10].

The objective of this research is to use and verify the generalized modes method to calculate wave-induced structural loads and flexible body dynamics for WEC applications. In this study, the generalized modes method is adopted in WEC-Sim, a radiation-diffraction-based time-domain numerical model that calculates the dynamics of WEC devices comprised of rigid bodies, power-take-off (PTO) systems,
and mooring systems [11]. The generalized modes method is also used to recover body component stress and bending moments, using an approach based on the linear superposition of stresses/bending moments in modal coordinates [12]. The modal stresses/bending moments corresponding to the component modes can be calculated either analytically or by finite element analysis (FEA). After describing the process of implementing modes derived from FEA, and the dynamics model used to combine rigid and flexible body simulations, the resulting dynamic response of the WEC can be simply obtained with closed-form analytic expressions or finite element analysis, and the rigid and flexible bodies may be considered as simplified representations of the structural deformation significantly reduces computation time and modeling complexity. With the assumption of linear structure with small deformation, the modal properties of flexible bodies may be simply obtained with closed-form analytic expressions or finite element analysis, and the rigid body system is discretized using a classical rigid body dynamics approach. The resulting dynamic response of the WEC device then combines the rigid body motions with the flexible deformations.

II. COMPONENT SPECIFICATIONS

Two flexible structures are considered in this study, a barge and a column, each of which is based on Newman’s original examples in [6] and further considered in [1]. Both structures may be considered as simplified representations of WEC components—a “Wave-Carpet”-type device and a monopile for a single-body point absorber. Parameters for these structures are listed in Table I.

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Barge</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>kg</td>
<td>4.000 × 10^6</td>
<td>3.142 × 10^6</td>
</tr>
<tr>
<td>Density</td>
<td>kg/m^3</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Cross-Sectional Area</td>
<td>m^2</td>
<td>100.000</td>
<td>341.159</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>MPa</td>
<td>30.720</td>
<td>4800.000</td>
</tr>
<tr>
<td>Area Moment of Inertia</td>
<td>m^2</td>
<td>833.333</td>
<td>7853.982</td>
</tr>
<tr>
<td>Length</td>
<td>m</td>
<td>80.000</td>
<td>200.0</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td></td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

III. FLEXIBLE MULTIBODY DYNAMIC MODELING

This section describes the formulation used to model flexible multibody dynamics for WEC devices. The system considers both the rigid-body motions and the structural deformations of WEC components. The dynamic, structural deformations of WEC bodies are evaluated in modal coordinates, whereas rigid body dynamics are calculated in physical coordinates. Assuming the rigid-body motions may be decoupled from the structural deformation significantly reduces computation time and modeling complexity. With the assumption of linear structure with small deformation, the modal properties of flexible bodies may be simply obtained with closed-form analytic expressions or finite element analysis, and the rigid body system is discretized using a classical rigid body dynamics approach. The resulting dynamic response of the WEC device then combines the rigid body motions with the flexible deformations.

The equation of motion of the hydro-elastic-multibody dynamics of WEC is formulated as

\[ (M + A_\infty)\ddot{x} + \int_0^t D(t - \tau)\dot{x}(\tau)d\tau + C\dot{x} - \frac{1}{2} \rho AC D \ddot{x} + \mathbf{K} \mathbf{x} = \mathbf{F} \]

where \( C = C_M + C_{PTO} + C_{id} + C_s \), \( \mathbf{K} = K_M + K_{PTO} + K_B + K_s \), and \( \mathbf{F} = F_{ext} + F_{ME} \). Throughout this paper, \( M, C, \) and \( \mathbf{K} \) denote mass, damping, and stiffness matrices. \( \mathbf{F} \) is the excitation force vector. The subscripts \( f, r, \) and \( s \) represent rigid body, flexible body, and their hybrid system, respectively. Subscripts \( M, PTO, ME, ext, \) and \( B \) denote mooring, power take-off, Morison, excitation, and buoyancy. \( A_\infty \) is the added mass terms for the structure.

\[
M = \text{diag}[M_{r,1}, M_{f,1}, M_{r,2}, M_{f,2}, \ldots, M_{f,N}]
\]

\[
K_s = \text{diag}[K_{r,1}, K_{f,1}, K_{r,2}, K_{f,2}, \ldots, K_{f,N}]
\]

\[
C_s = \text{diag}[C_{r,1}, 0, C_{r,2}, 0, \ldots, C_{r,N}, 0]
\]

\[
\mathbf{F} = [F_{r,1}, F_{f,1}, F_{r,2}, F_{f,2}, \ldots, F_{r,N}, F_{f,N}]
\]

where \( M_f = \Phi^T M_f \Phi \) and \( K_f = \Phi^T K_f \Phi \) are diagonal matrices. Excitation force in the modal coordinates is \( \mathbf{F}_f = \Phi^T \mathbf{F}_f \).

Here, \( \mathbf{x} = \{z_1, z_2, \ldots, z_N\}^T \), where \( N \) is the total number of bodies in the system. Each displacement vector consists of rigid and flexible DOFs as

\[
z = \{p, q\} = \{x, y, z, \theta_x, \theta_y, \theta_z, q_1, q_2, \ldots, q_m\}
\]

In the system environment, \( m \) is a user-defined variable that depends on the body selection. In other words, the number of flexible modes for each body can be different. The total DOFs for the system, therefore, equals \( 6N + \sum_{j=1}^N m(j) \) including \( 6N \) rigid-body DOFs and \( \sum_{j=1}^N m(j) \) flexible-body DOFs.

Assuming the dynamic response, \( \mathbf{u} \), is the linear superposition of system’s eigenvectors, such as

\[
\mathbf{u} = \Phi \mathbf{q}
\]

where \( \mathbf{q} = \{q_1, q_2, \ldots, q_m\}^T \), \( \Phi = [\Phi_1, \Phi_2, \ldots, \Phi_m] \), and \( m \) is the total number of eigenvectors.

Similar to Eq. 7, the dynamic stress response of any flexible WEC body, \( \Xi \), is the linear superposition of the system’s stress distribution in the modal coordinate, such as

\[
\Xi = \Upsilon \mathbf{q}
\]

where \( \Upsilon \) is the stress vector of the flexible body in the modal coordinates.

The bending moment of the same flexible body, \( \mathbf{T} \), is

\[
\mathbf{T} = \Lambda \mathbf{q}
\]

where \( \Lambda \) is the bending moment vector of the flexible body in the modal coordinate.
In WEC-Sim, Eq. 1 has been formulated in a state-space form as follows

\[
\dot{\mathbf{y}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\
-M^{-1}\mathbf{K} & -M^{-1}\mathbf{C} \end{bmatrix} \mathbf{y}^{+} 
+ \begin{bmatrix} \mathbf{M}^{-1}\mathbf{A}_{\infty}\mathbf{x} - \mathbf{D}(t - \tau)\mathbf{x}(\tau)d\tau + \frac{1}{2}\rho\mathbf{A}_{D}\mathbf{D}|\mathbf{x}|\mathbf{x}^{+} + \mathbf{F}_{ext} + \mathbf{F}_{ME} \end{bmatrix}
\]

\( (10) \)

where \( \mathbf{y} = [\mathbf{x}; \dot{\mathbf{x}}]^T \) and \( \mathbf{AA} \) is the time-invariant coefficient matrix. \( \mathbf{BB} \) is the time-varying excitation vector. Terms \( \mathbf{A}_{\infty}\dot{\mathbf{x}}, \int_{0}^{t}\mathbf{D}(t - \tau)\mathbf{x}(\tau)d\tau, \) and \( \frac{1}{2}\rho\mathbf{A}_{D}|\mathbf{x}|\mathbf{x}^{+} \) are approximated using solutions at previous time steps. Eq. 10 is solved using ODE4 in MATLAB.

The proposed modeling approach has been implemented in WEC-Sim. Solving the WEC dynamic response consists of multiple major steps, including 1) modal analysis of the studied WEC to identify a set of system natural frequencies and corresponding mode shapes, 2) construct discretized mass and impedance matrices using these structural modes, 3) include these additional degrees of freedom in WAMIT as generalized modes to calculate the additional hydrodynamic coefficients to capture the structural-fluid interaction, and 4) import the hydrodynamic coefficients to WEC-Sim and conduct dynamic analysis of the hybrid rigid and flexible body system. These key steps are also illustrated in Fig. 4.

![Fig. 2. Finite element model of the barge established in ANSYS](image)

![Fig. 3. Finite element model of the column established in ANSYS](image)

Fig. 1. Process of a structural-fluid analysis of WEC devices

### IV. FINITE ELEMENT MODELING

Finite element models of the barge and column are created in ANSYS as shown in Fig. 2 and Fig. 3, respectively. Both models were built using beam elements (BEAM188), with three nodes per element. Mesh convergence studies were performed, resulting in a total of 80 beam elements for each structure. The free-free boundary conditions are applied to the barge model and the first four modes are then calculated with modal analysis. For the column, free-fixed boundary conditions are used to model the rigid connection to the seafloor, and modal analysis is used to obtain the first four modes. Although both structures could potentially be modeled with either beam or solid elements, the beam-element model is advantageous in this application, because the element rotational DOFs may be exported along with the load and moment distributions. The modal mass and stiffness matrices, \( \mathbf{M}_{f} = \mathbf{\Phi}^T\mathbf{M}_{f}\mathbf{\Phi} \) and \( \mathbf{K}_{f} = \mathbf{\Phi}^T\mathbf{K}_{f}\mathbf{\Phi} \), as input in WAMIT, are listed in Table II.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Barge</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>Stiffness (N/m)</td>
<td>Mass (kg)</td>
</tr>
<tr>
<td>1st</td>
<td>1.079 × 10^6</td>
<td>7.504 × 10^6</td>
</tr>
<tr>
<td>2nd</td>
<td>1.210 × 10^6</td>
<td>8.267 × 10^6</td>
</tr>
<tr>
<td>3rd</td>
<td>2.392 × 10^6</td>
<td>8.821 × 10^6</td>
</tr>
<tr>
<td>4th</td>
<td>1.831 × 10^6</td>
<td>9.597 × 10^6</td>
</tr>
</tbody>
</table>

![TABLE II](image)
V. RESULT CORRELATION WITH HIGH-FIDELITY CODES AND EXPERIMENTAL DATA

The presented method is validated by correlating results with full-scale computational fluid dynamics analysis considering FSI in STAR-CCM+ and the frequency-domain analysis in WAMIT. STAR-CCM+ [13], one of the highest-fidelity computational fluid dynamics-finite element analysis (FEA) codes, was utilized in this study. FSI is modeled by implicitly coupling the unsteady RANS solver with a FV A solver, within the same STAR-CCM+ simulation.

WAMIT performs boundary element analysis for simulating WEC’s linearized hydrodynamic response in the frequency domain. The steps of using generalized body modes to estimate wave loads on deformable bodies are documented in the WAMIT user manual [7]. Details on these two computational models of the studied articles are discussed in [1].

A. Flexible barge

The lowest bending mode shapes of the flexible barge are illustrated in Fig. 4. These modes were calculated using analytical formulation (Timoshenko beam theory) and FEA in ANSYS. The mode shapes have been normalized to unity such that \( \Phi^T \Phi = I \). The FEA predicts slightly larger deformation than the analytical approach for higher modes because the finite element model considers the cross-couplings among different DOFs whereas the analytical approach does not. This cross-coupling is not negligible for flexible bodies. These modes, as shown in Fig. 4, were selected as candidate flexible body DOFs in WAMIT. WAMIT calculates the response of these flexible DOFs directly in the frequency domain. It also provides associated hydrodynamic coefficients, which are later used in the time-domain model built in WEC-Sim.

The response amplitude operator (RAO) and bending stress of the proposed model for the flexible barge are compared against WAMIT and STAR-CCM+ in Figs. 5 and 6, respectively. From Fig. 5, it is evident that only the first two bending modes are significant for the barge. Good agreement among three approaches is evident and the result correlation between WAMIT and WEC-Sim is excellent. Table III compares the values of bending RAOs at the aforementioned four modes between WAMIT and WEC-Sim at a wave period of 7.5 s. The largest error is less than 1.4% for the third and fourth modes. The error at the first dominating mode is less than 0.5%. The maximum RAO for the bending DOF occurs at the 7.5 s wave period near where the pitch DOF reaches the maximum and heave DOF is the minimum. This phenomena could be a result of the interaction between flexible and rigid DOFs.

Fig. 6 compares the maximum bending stress on the top of the barge along its length between the WEC-Sim and STAR-CCM+ at wave periods of 5, 7.5, 10, and 12 s. These two approaches are in good agreement. The stress profiles over the barge length is nearly parabolic, primarily driven by the first mode in Fig. 4. At wave periods of 5 s and 12.5 s, the stress profiles include the effects of the second and higher modes.

B. Flexible column

Representing a single-point absorber, the bottom of the column is considered fixed to the seabed. Therefore, a clamped-free boundary condition is applied for the modal analysis. The lowest bending mode shapes of the flexible column calculated...
using the analytical and FEA approaches are illustrated in Fig. 7. The Timoshenko beam theory predicts almost the identical modal response of the column as FEA in ANSYS. Because the column is much stiffer than the barge, the cross-coupling among different DOFs is very little. Thus, the analytical results of the column match the FEA results very well.

The RAO of the proposed model for the flexible column is compared against WAMIT and STAR-CCM+ in Fig. 8. Viscous damping effects with $C_D = 1$ along the structure have been considered for the column case. As shown in Fig. 8, the first bending mode dominates the response for the column compared to higher modes (at least one order of magnitude higher) and reaches the maximum near the $8.4\text{ s}$ wave period. The result correlation among all three approaches is excellent.

Fig. 9 compares the bending stress profiles of the column along its height among WEC-Sim, WAMIT, and STAR-CCM+ at wave periods of $5.0$, $6.7$, $8.4$, and $10.1\text{ s}$. The agreement among all three approaches is good. The stress profiles are primarily determined by the first bending mode in Fig. 7 with small effects from higher modes. Small differences are present between the high-fidelity STAR-CCM+ results and generalized body modes approach in WAMIT and WEC-Sim particularly near the top of the column. There are multiple reasons that contribute to the result differences. For instance, the STAR-CCM+ model considers nonlinear wave effects, whereas the other two do not, and STAR-CCM+ requires a longer transient time to reach the system’s steady state. Additional discussions on the potential causes of these wave effects can be found in [1].

Fig. 10 compares the time history of the oscillating bending stress of the column calculated using WEC-Sim and STAR-CCM+ at a wave period of $6.7\text{ s}$ under a regular wave with a time ramp up to $50\text{ s}$. The agreement between these two approaches is reasonable, with slightly stronger oscillation in the STAR-CCM+ results. Being able to calculate these stress cycles in the time domain is an important step for the life calculations of WEC structures.

VI. CONCLUSIONS

A flexible multibody dynamics modeling approach is formulated and implemented in WEC-Sim, the open-source, time-domain simulation tool of WECs. This time-domain approach considers the structural flexibility of WEC components and provides a fast solution for WEC dynamic response, stress, and loads under the effects of wave loading.
This method is applied to two representative structures to calculate dynamic response and bending stress. Results calculated using the presented method are correlated with full-scale computational fluid dynamics analysis considering the fluid-structure interaction and the counterparts of the frequency-domain analysis. The agreement among these approaches is good.

Future research will be focused on using this method to simulate WEC devices with multiple bodies. It is also important to consider nonlinear wave forces, mooring forces, and PTO forces to predict realistic responses and loads during WEC operation. A sensitivity study on the required number and types of candidate modes will be conducted for improving result accuracy. Finally, this study will become part of the WEC design-loads/extreme-condition-model framework [14] and the calculation of WEC fatigue life.

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