Rolling Element Bearing Stiffness Matrix Determination

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Motivation

• Limited work on stiffness matrix in the literature
  o Diagonal matrix approximation typically used
• Elastic deformation of race causes nondiagonal terms

\[
\begin{bmatrix}
  k_{xx} & k_{xy} & k_{xz} \\
  k_{xy} & k_{yy} & k_{yz} \\
  k_{xz} & k_{yz} & k_{zz}
\end{bmatrix}
\]

Symmetric

\[
\begin{bmatrix}
  k_{xx} & k_{x\theta_x} & k_{x\theta_y} & 0 \\
  k_{x\theta_x} & k_{yy} & k_{y\theta_y} & 0 \\
  k_{x\theta_y} & k_{y\theta_y} & k_{zz} & 0 \\
  0 & 0 & 0 & k_{\theta_x \theta_x}
\end{bmatrix}
\]

Coupling between the radial and rotational displacement

Coupling between the axial and rotational displacement

Coupling between the axial and radial displacement
Finite Element/Contact Mechanics Model

• Three-dimensional finite element model includes microgeometry

• Analyze contact between rolling elements and races
  - Contact searched at every time instant as bearing rotates

Radial Ball Bearing  Cylindrical Bearing  Contact Grid
Accuracy Order of Finite Element Analysis

• Numerical Jacobian used to compute $K$
• Order of Jacobian approximation formula should be comparable to the accuracy order of finite element analysis (FEA)
• Method to obtain the accuracy order of FEA

\[
V_h = V + c_1 h^{p_1} + c_2 h^{p_2} + \ldots
\]

\[
e_{h_3} - e_{h_2} = \frac{V_{h_3} - V_{h_2}}{V_{h_2} - V_{h_1}} \approx \frac{h_2^{p_1} - h_3^{p_1}}{h_1^{p_1} - h_2^{p_1}}
\]

\[
log\left\{\left(\frac{V_{h_3} - V_{h_1}}{V_{h_3} - V_{h_2}}\right) \frac{\log\frac{h_2}{h_3}}{\log\frac{h_1}{h_3}}\right\}
\]

\[
p_1 = \frac{\log(h_1)}{\log(h_2)}
\]

$V_h$ : finite element solution  
$V$ : exact solution  
$h$ : finite element size  
$p_1$ : order of accuracy

<table>
<thead>
<tr>
<th>Accuracy Order</th>
<th>FEA</th>
<th>FEA/Contact</th>
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</thead>
<tbody>
<tr>
<td>p1</td>
<td>1.11</td>
<td>1.94</td>
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</table>
Comparison Against Published Experiments

- Calculated stiffnesses by FEA agree with experiments
  
  (Royston et al. 1998)
  
  (Kraus 1987)
Comparison Against Theoretical Models

- FEA stiffness agrees with the theoretical model
  - Only with unrealistic races that match Harris’s assumptions
- Theoretical models predict higher stiffness with design dimensions
Bearing Stiffness Is Time-Varying

• Number of rollers in contact changes periodically
• Can excite gearbox vibration

![Graph showing the change in radial bearing stiffness over the ball pass period](image)

- **Mean Stiffness**
- **Low Stiffness**
- **High Stiffness**

- **Ball pass period**
- **Four Rollers in Contact**
- **Five Rollers in Contact**

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**Note:**

The graph illustrates the variation of radial bearing stiffness over the ball pass period, showing the number of rollers in contact changing periodically between four and five.
Off-Diagonal Stiffnesses Affect Gear Vibration

• Gear dynamics with off-diagonal stiffnesses differ from gear dynamics with a diagonal stiffness matrix
  o Need to include off-diagonal stiffnesses
Conclusions

• A method developed to determine bearing stiffness matrices

• Method validated by experiments

• Comparison against theoretical models expose their limitations

• Bearing contact is nonlinear and time-varying

• Bearing microgeometry affects stiffness

• Off-diagonal bearing stiffnesses affect gear dynamics
Thank You!

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