Analysis of Variability and Uncertainty in Wind Power Forecasting: An International Comparison

Preprint

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Analysis of Variability and Uncertainty in Wind Power Forecasting: An International Comparison

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***Abstract*—One of the critical challenges of wind power integration is the variable and uncertain nature of the resource. This paper investigates the variability and uncertainty in wind forecasting for multiple power systems in six countries. An extensive comparison of wind forecasting is performed among the six power systems by analyzing the following scenarios: (i) wind forecast errors throughout a year; (ii) forecast errors at a specific time of day throughout a year; (iii) forecast errors at peak and off-peak hours of a day; (iv) forecast errors in different seasons; (v) extreme forecasts with large overforecast or underforecast errors; and (vi) forecast errors when wind power generation is at different percentages of the total wind capacity. The kernel density estimation method is adopted to characterize the distribution of forecast errors. The results show that the level of uncertainty and the forecast error distribution vary among different power systems and scenarios. In addition, for most power systems, (i) there is a tendency to underforecast in winter; and (ii) the forecasts in winter generally have more uncertainty than the forecasts in summer.**

Keywords-wind forecasting; reliability; power systems; uncertainty; variability

I. Introduction

The worldwide nameplate capacity of wind power has reached 282,482 MW as of the end of 2012 [1]. Wind energy in many countries has already achieved a relatively high level of penetration, such as 30% in Denmark, 16% in Spain, and 7.3% in Germany [1]. The variable and uncertain characteristics of wind power mean that short-term forecasting of wind power plays an important role in grid operations at these penetration rates. In addition, uncertainties in the wind forecast significantly impact the integration costs of wind energy; forecast inaccuracies can result in substantial economic losses and reliability issues.

A. *Overview of Wind Forecasting*

Wind forecast models can be broadly divided into two categories [2]: (i) forecasting based on the analysis of historical time series of wind; and (ii) forecasting based on numerical weather prediction models. The first type of forecast model generally provides reasonable results in the estimation of long-term horizons, such as mean monthly, quarterly, and annual wind speed. Measure-correlate-predict is one of the most popular methods used for long-term wind power forecasting [3, 4]. In addition, statistical and machine learning techniques that utilize historical data have been shown to work well for forecast horizons less than one hour [5, 6]. For short-term horizons more than one hour (daily or hourly forecasts), the impact of atmospheric dynamics becomes more important, and numerical weather prediction models become more suitable. Short-term wind power forecasting (between 1 hour and 72 hours) is uniquely helpful in power system planning for the unit commitment and economic dispatch process.

For wind integration studies and stochastic unit commitment models, it is important to characterize wind power forecast errors, especially for large and infrequent forecast errors. A variety of topics on forecast errors have been studied in the literature, including distributions of wind power forecast errors [7-10], uncertainties in wind forecasting [10-12], the economic value of improved wind forecasting [13], and wind power ramp forecasting [14, 15].

B. *Research Motivation and Objectives*

Different wind forecasting strategies are adopted in many power systems. Actions taken by power system operators to compensate for wind forecast errors are affected by many factors, e.g., the location of the power system, wind capacities, and time of year. Understanding forecasting errors and uncertainties in different power systems and scenarios is helpful for (i) developing improved wind forecasting technologies for a specified power system and (ii) better allocating resources to compensate for wind forecast errors. To this end, this paper investigates the uncertainty in wind forecasting at different times of year. In addition, an extensive comparison of wind forecasting is performed using large-scale wind power prediction data from six countries: the United States, Finland, Spain, Denmark, Norway, and Germany. In this study, day-ahead wind power forecasts were supplied for the six countries or balancing areas within a country.

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II. Methodologies

To comprehensively understand the variability and uncertainty in wind forecasting, the following scenarios were analyzed in this study: (i) hourly day-ahead wind power forecast errors throughout a year; (ii) forecast errors at a specific time of day (hour 14:00 in this paper) throughout a year; (iii) forecast errors at peak (7:00 – 22:00) and off-peak hours of a day; (iv) forecast errors during different seasons (e.g., summer or winter); (v) extreme events with large overforecast or underforecast errors (more than 25% wind forecast error normalized by total wind capacity); and (vi) forecast errors when the wind power generation was at different percentages of the total wind capacity: less than 25% of the total wind capacity, between 25% and 75% of the capacity, and more than 75% of the capacity.

For each scenario, the distribution of forecast errors and the uncertainty in the day-ahead forecasts were estimated. The 95th percentile of 1-hour-ahead wind forecast errors, an important factor for the decision of flexibility reserve requirements, was also compared among different countries.

A. Distribution of Wind Power Forecast Errors

Multiple distribution types have been analyzed in the literature to quantify the distribution of wind power forecast errors, including the hyperbolic distribution [8, 9], kernel density estimation (KDE) [10], the normal distribution [16, 17], and Weibull [18] and beta distributions [7]. KDE was adopted in this paper to model the distribution of wind power forecast errors for different scenarios.

KDE is a nonparametric approach to estimate the probability density function of a random variable. It has been widely used in the wind energy community for wind speed distribution characterization [19, 20] and wind power forecasting [10, 21]. For an independent and identically distributed sample, $x_1, x_2, \cdots, x_n$, drawn from some distribution with an unknown density f , the KDE is defined as [19]

$$\hat{f}(x;h) = \frac{1}{n}\sum_{i=1}^{n} K_h(x - x_i) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right) \qquad (1)$$

In the equation, $K(\cdot) = (1/h)K(\cdot/h)$ has a kernel function K (often taken to be a symmetric probability density) and a bandwidth h (the smoothing parameter).

B. Uncertainty in Wind Forecasting

In this paper, the uncertainty in wind forecasting was evaluated by the *Rényi entropy* and *standard deviation* of forecast errors. An information entropy approach was proposed in the literature [11, 12] for assessing wind forecasting methods. Entropy in information theory is a measure of the uncertainty in a random variable; and a smaller information entropy (or standard deviation) value indicates less uncertainty in the forecasting. In this paper, *Rényi entropy* was adopted to quantify the uncertainty in wind forecasting, which is defined as

$$H_\alpha(X) = \frac{1}{1-\alpha}\log_2 \sum_{i=1}^{n} p_i^\alpha \qquad (2)$$

where α is a parameter that allows for the creation a spectrum of Rényi entropies; and p_i is the probability density of the i^{th} discrete section of the distribution. Larger values of α favor high probability events, whereas smaller values of α weight all of the instances more evenly [11]. The *Rényi entropy* can utilize all of the information present in the forecast error distributions to evaluate the uncertainty, whereas the standard deviation shows only how much variation or dispersion exists from the average or expected value.

C. Heat Maps of Wind Power Forecast Errors

One of the most prevalent concerns associated with integrating a large amount of wind power into the grid is the ability to handle large forecast errors in wind power output. To analyze the ramping characteristics in different power systems, a heat map of wind forecast errors was developed to present the mean forecast error per month and hour of day, which allows the operator to simultaneously see the timing and magnitude of forecast errors.

III. Variability and Uncertainty Analysis From Operational Systems

In this work, we followed the convention that the error (e_w) is equal to the forecast (P_{wf}) minus the actual (P_{wa}) wind power value.

$$e_w = P_{wf} - P_{wa} \qquad (3)$$

The wind power forecast errors from the six countries were observed at both the day-ahead and 1-hour-ahead timescale. The day-ahead forecasts were estimated using different methodologies for the six countries. The 1-hour-ahead forecasts for the six countries were synthesized using a 1-hour-ahead persistence approach. It is important to note the 1-hour-ahead persistence approach was adopted for analyzing power system flexibility but might not be used in some of the analyzed power systems. To compare the forecasts in different countries, the day-ahead and 1-hour-ahead wind power forecast errors were normalized by the total wind capacity in the analyzed power system. The following subsections show the comparisons in the uncertainties in the forecasting, distributions of forecast errors, and heat maps based on day-ahead forecasts; the 95th percentiles of forecast errors were compared based on 1-hour-ahead forecasts.

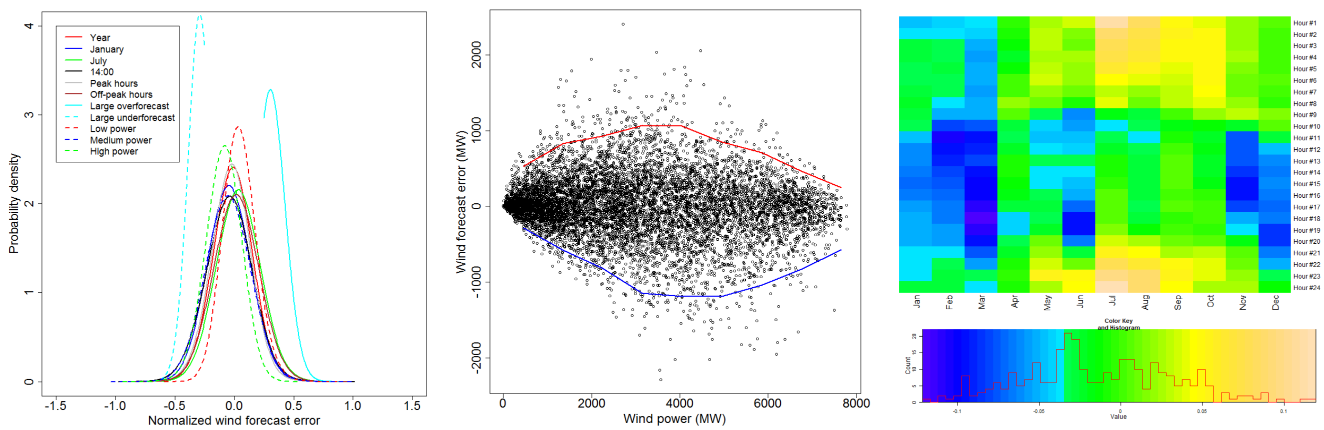
A. United States

Day-ahead forecasts for the United States were taken from the Electric Reliability Council of Texas (ERCOT) interconnection for the year 2010, with a total wind capacity of approximately 9,000 MW. The wind power plants are well dispersed in the state of Texas.

The results showed that the level of uncertainty and the forecast error distribution vary among different scenarios. For the distributions estimated using KDE shown in Fig. 1(a), it was observed that the distribution of forecast errors at peak hours (7:00 – 22:00) followed a similar trend with the distribution estimated using the entire year data. It was observed from the comparison between distributions at peak and off-peak hours that there were relatively more overforecast events at off-peak hours and relatively more underforecast events at peak hours. This indicates that the required amount of "down" reserves to accommodate wind forecast errors may be relatively less than the amount of "up" reserves at off-peak hours, and vice versa at peak hours. By comparing distributions among low- medium-, and high-power levels, most overforecast events were observed in the low-power scenario, followed by medium- and high-power scenarios. This also indicated that more "down" reserves are required for the high-power scenario, which is more likely to happen at peak hours. For extreme events, distributions of overforecast and underforecast events were estimated separately.

Figure 1(b) shows wind forecast errors versus power output with 95% confidence interval bands. Confidence intervals, represented by red and blue lines, can be used to determine "up" and "down" reserve requirements to compensate for a certain percentage of occurrences. The range of wind power (horizontal axis in Fig. 1(b)) was divided into 10 groups; for each group, the average power was calculated as well as the confidence intervals that cover 95% of forecast errors. The confidence interval bands were then interpolated from the group averages. It was observed that more reserves are required when wind power generation is approximately 3,500 MW 4,500 MW.

The mean hourly day-ahead wind power forecast error per month of the ERCOT power system is shown in Fig. 1(c). We observed that it tends to underforecast in winter, and tends to overforecast at nights in summer. Power system operators could mitigate the effects of wind integration on power system reliability by adopting appropriate statistical corrections of wind forecasting errors based on the pattern of forecasting accuracies. It is important to note that the color key in the heat map of different countries is represented by different scales, with each provided at the bottom of the figure.



(a) Distributions of day-ahead forecast errors. (b) 95th percentiles of 1-hour-ahead forecast errors. (c) Heat map of day-ahead forecast errors.

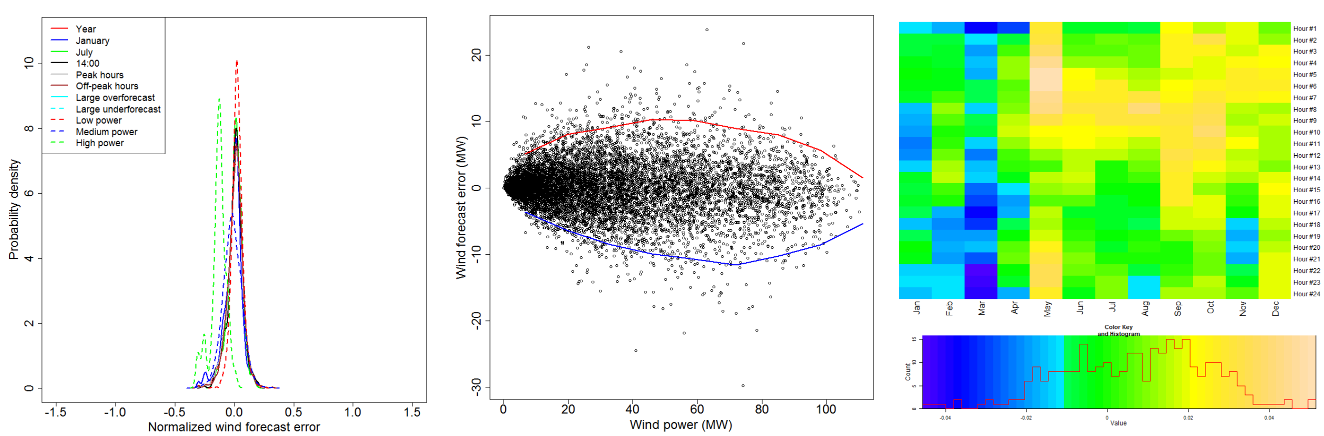
Figure 1. Analysis of wind forecast errors of the power system in the United States.

B. Finland

The wind capacity analyzed for Finland was the smallest in the study, with 130.6 MW of rated power spreading throughout 23 sites in Finland. The forecasting horizon was 12 hours to 36 hours, which can be used for day-ahead trading purposes.

Figure 2 shows the results of wind forecast error analysis for the year 2012. As with the U.S. ERCOT power system, the distribution of forecast errors at peak hours followed the distribution using the entire year's data. Most forecast errors for the high-power scenario were negative, and this tendency can help power system operators determine appropriated corrections beforehand with high levels of wind power generation. The distribution of forecast errors in July was also similar with the distribution using the entire year's data.

The 95th percentiles of 1-hour-ahead forecast errors in Fig. 2(b) indicated large reserve requirements with medium levels of wind power integration. From the heat map of forecast errors in Fig. 2(c), most underforecast errors were observed from January to March, and most overforecast events occurred in May, and from September to December.



(a) Distributions of day-ahead forecast errors. (b) 95th percentiles of 1-hour-ahead forecast errors. (c) Heat map of day-ahead forecast errors.

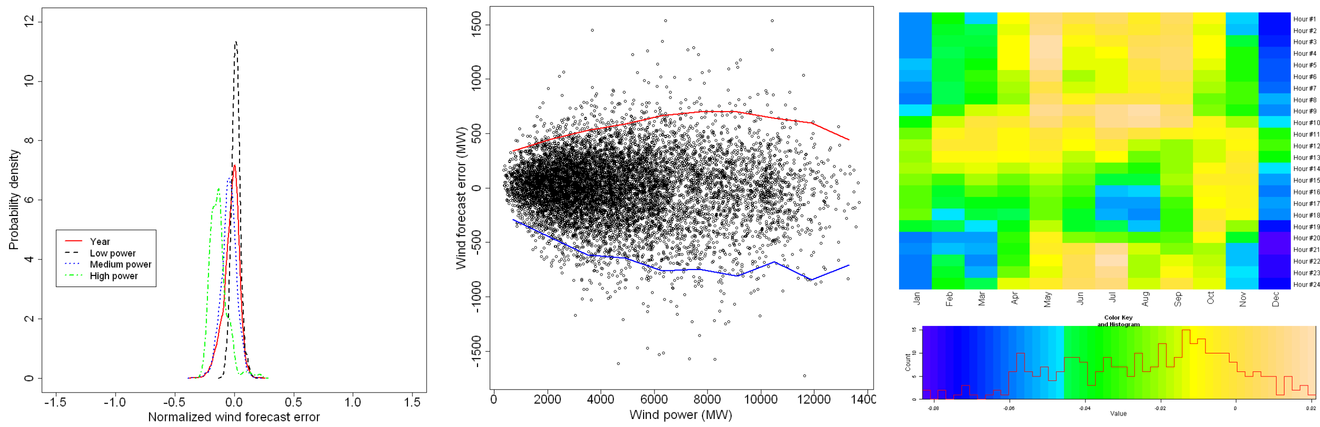
Figure 2. Analysis of wind forecast errors of the power system in Finland.

C. Spain

The Spanish system data included 14,000 MW of total wind capacity for the year 2011. The wind power plants are well dispersed in the country. Figure 3 shows the results of wind forecast error analysis in Spain.

Distributions of large underforecast and overforecast errors were not adequately characterized because of fewer points for the two scenarios. As shown in Fig. 3(a), most forecast errors for the high-power scenario were negative (as with the analysis results in Finland), and this tendency can help power system operators determine appropriated corrections beforehand.

As shown in the figure of 95th percentiles of 1-hour-ahead forecast errors, the maximum "down" reserve requirement was needed when the wind power reached the total wind capacity. It was observed from the heat map that, as with the U.S. ERCOT power system, it tended to be underforecast in December and January, and overforecast during nights and early morning from May to September.



(a) Distributions of day-ahead forecast errors. (b) 95th percentiles of 1-hour-ahead forecast errors. (c) Heat map of day-ahead forecast errors.

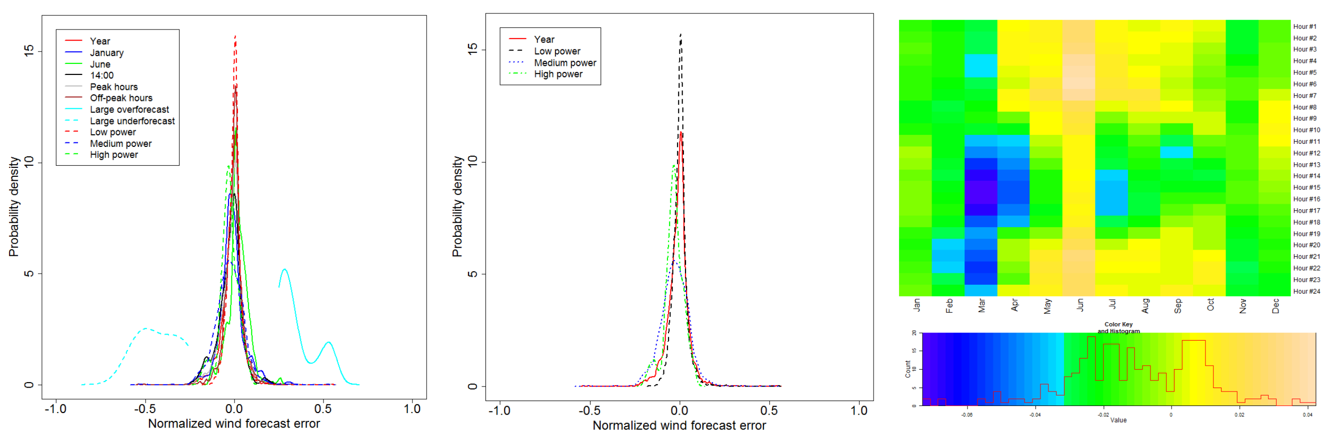
Figure 3. Analysis of wind forecast errors of the power system in Spain.

D. Denmark

The Danish system data included 3,265 MW of total wind capacity for the year 2012. The wind power plants are well dispersed in the country, including all onshore wind power plants in Demark. The forecasting horizon was 12 hours to 36 hours.

In Fig. 4(a), a multimodal distribution of forecast errors was observed in the scenario of a large overforecast. By comparing the distributions of forecast errors in January and June, we observed that the overall distribution in January was on the left side of the distribution in June. This observation indicated relatively more underforecast events in January and more overforecast events in June for the Danish power system. As shown in Fig. 4(b), more underforecast events were observed in the medium-power scenario than that in the high-power scenario, which differs with power systems in other countries.

The heat map of forecast errors presents most overforecast events at nights and early morning between May and September. Most forecasting errors between November and March were negative, except during the time period between 7:00 and 13:00 in December. Overall, the overforecast time period in the heat map was more continuous than that in other countries, and this observation also applied to underforecast time periods.



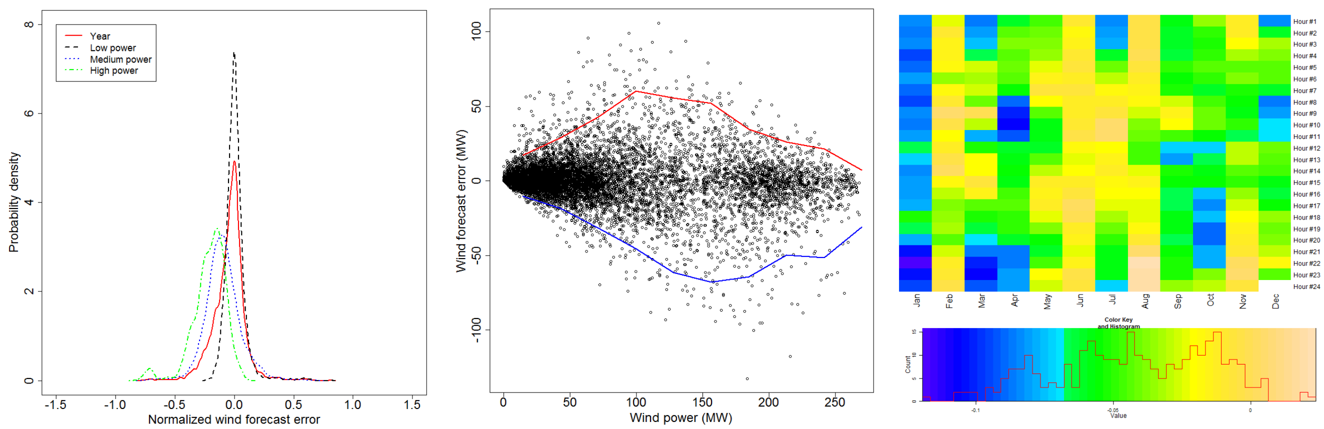
(a) Distributions of day-ahead forecast errors. (b) Distributions at different power levels. (c) Heat map of day-ahead forecast errors.

Figure 4. Analysis of wind forecast errors of the power system in Denmark.

E. *Norway*

The Norwegian system data included 284 MW of total wind capacity spreading throughout four sites for the year 2011. The forecasting horizon is 12 hours to 36 hours.

Figure 5 shows the results of wind forecast error analysis. A multimodal distribution of forecast errors was also observed in the case of large overforecast as with the Danish power system, which is not shown in Fig. 5. Similar to most power systems, most underforecast events were observed in the high-power scenario, followed by medium- and low-power scenarios. Figure 5(b) shows wind forecast errors versus power output with 95% confidence interval bands for the Norwegian power system. Maximum "up" reserves are required with 100 MW to 150 MW wind power generation, whereas maximum "down" reserves are required with a slightly larger wind power generation, approximately 140 MW to 190 MW. It was observed from the heat map in Fig. 5(c) that there were significantly more underforecast events than overforecast events throughout the year.



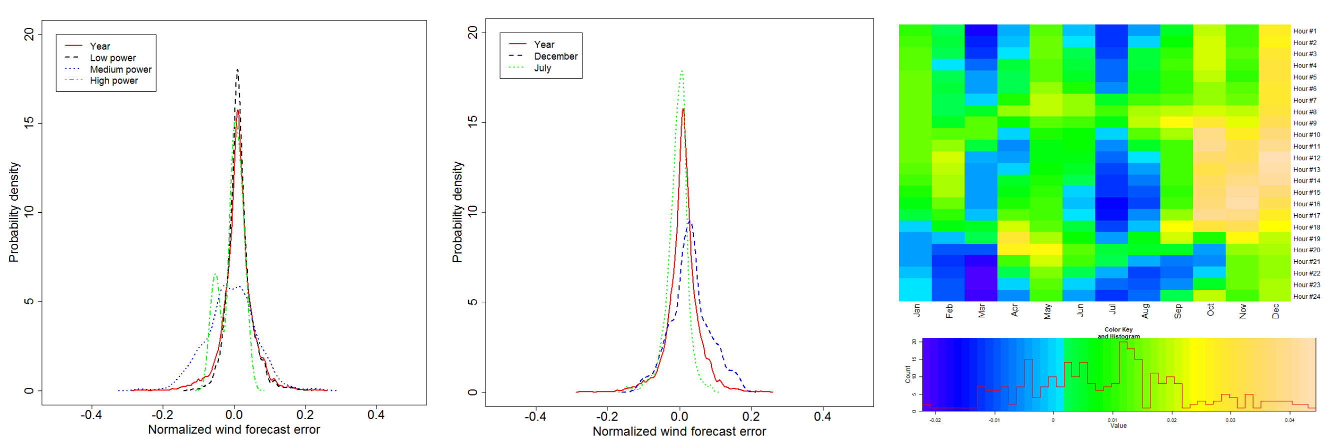
(a) Distributions of day-ahead forecast errors. (b) 95th percentiles of 1-hour-ahead forecast errors. (c) Heat map of day-ahead forecast errors.

Figure 5. Analysis of wind forecast errors of the power system in Norway.

F. *Germany*

The German system data included 26,000 MW of total wind capacity for the year 2010. The wind power plants are well dispersed in the country. The forecast of Germany is a sum of forecasts for four transmission system operators. The forecast of each control zone is a combination of approximately 3 to 6 different wind power forecast systems based on different numerical weather prediction models. The forecasts are used for the trading activities on the day-ahead market and are consequently generated between 08:00 and 11:00 every morning.

Figure 6 shows the results of wind forecast error analysis. The distribution of forecast errors in the German power system was narrower than that in other countries, which shows a better wind forecasting skill in the German power system. Therefore, extreme events in the German power system are defined as more than 10% wind forecast error by total wind capacity. As shown in Figs. 6(a) and 6(b), (i) the distribution of forecast errors for the high-power scenario presented a multimodal characteristic and (ii) the overall distribution in July was on the left side of the distribution in January, which presents an opposite trend with power systems in other countries. The heat map in Fig. 6(c) shows that there were relatively more overforecast events than underforecast events throughout the year.



(a) Distributions of day-ahead forecast errors. (b) Distributions at different seasons. (c) Heat map of day-ahead forecast errors.

Figure 6. Analysis of wind forecast errors of the power system in Germany.

G. Comparison of Uncertainty in Wind Forecasting

The uncertainty in wind forecasting is evaluated by the *Rényi entropy* and *standard deviation* of forecast errors. Figure 6 shows the values of *Rényi entropy* and *standard deviation* for different scenarios. For the calculation of *Rényi entropy*, the number of bins for probability estimation and the value of α are set to be 100 and 2, respectively. In Fig. 6, black and red points represent the *Rényi entropy* and *standard deviation,* respectively; the left axis is *Rényi entropy* and right is *standard deviation.* According to the *Rényi entropy* metric, the wind forecasting in the Danish power system maintains a relatively lower level of uncertainty for most scenarios. Based on the standard deviation values of wind power forecast errors, there is the least uncertainty in the forecasting for the German power system, followed by the Danish power systems. The forecast error variability is relatively lower for Denmark, Germany, and Spain, which all have a significant amount of well-dispersed wind power. It was also observed that, for most power systems, the forecasts in winter generally had more uncertainty than the forecasts in summer.

The *Rényi entropy* of power systems in the United States, Finland, Spain, Denmark, Norway, and Germany, respectively, varied 10.5%, 84.2%, 69.6%, 175.7%, 28.6%, and 37.7% among the eleven scenarios. The variation in the standard deviation values among the eleven scenarios was more significant than that in the *Rényi entropy.*

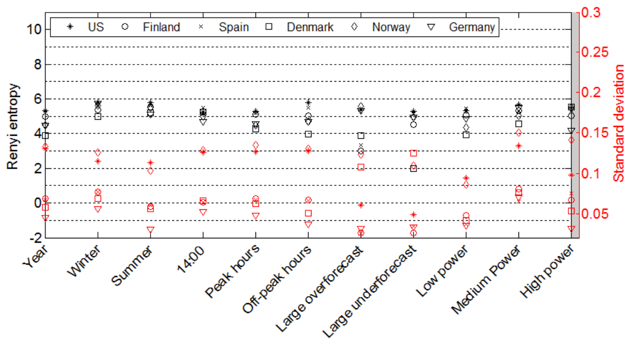


Figure 6. Uncertainty in wind forecasting of different scenarios.

IV. Conclusion

This paper compared the variability and uncertainty in wind forecasting for multiple power systems from six countries. For each power system, eleven scenarios were analyzed to estimate distributions of forecast errors. Multimodal characteristics were observed in the extreme overforecast scenarios in the Danish and Norwegian systems, and in the high-power scenario in the German system. The distribution of forecast errors in the German power system was relatively narrower than that in other countries. For most power systems, there were more underforecast events observed in the high-power scenario than in the low- and medium-power scenarios.

The 95th percentiles of 1-hour-ahead forecast errors showed that maximum "up" and "down" reserves were required when actual wind power generation was at medium to high percentages of the total wind capacity. For most systems, the forecasts in winter presented an underforecast tendency and more uncertainty. There was generally less uncertainty in forecasting when wind power plants were dispersed throughout a wide geographic area.

Future work will (i) investigate multiple years of wind forecasting data to obtain a general trend of forecast errors, and (ii) compare the different methodologies in the forecasting systems in different countries and seek to identify the possible sources of bias and errors in the forecasts.

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