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Nomenclature

β	Surface tilt, angle from horizontal, 0° to +180°.
β_a	Axis tilt, angle from horizontal of the inclination of tracker axis, 0° to +90°.
γ	Surface azimuth, angle clockwise from north of the horizontal projection of the surface normal, 0° to +360°.
γ_a	Axis azimuth, angle clockwise from north of the horizontal projection of the tracker axis, 0° to +360°. If the axis tilt is greater than zero, the vertex of the angle is at the inclined end of the axis.
γ_s	Solar azimuth, angle clockwise from north of the horizontal projection of a ray from the sun, 0° to +360°.
θ	Incidence angle, angle between a ray from the sun and the surface normal, 0° to +180°.
θ_z	Zenith angle, angle between a ray from the sun and the vertical, 0° to +90°.
R	Rotation angle, angle of rotation of collector about axis when observed from the inclined end of axis, -180° to +180°. Equals zero when the normal to the surface is in the vertical plane, clockwise is positive.

Abstract

An equation for the rotation angle for optimum tracking of one-axis solar trackers is derived along with equations giving the relationships between the rotation angle and the surface tilt and azimuth angles. These equations are useful for improved modeling of the solar radiation available to a collector with tracking constraints and for determining the appropriate motor revolutions for optimum tracking.

1 Introduction

The beam radiation on a tracking surface is maximized by orienting the surface, within the constraints of the tracking apparatus, so that the solar radiation incidence angle is minimized. The incidence angle, θ, is the angle between a ray from the sun and the normal to the surface. Braun and Mitchell (1983) provide expressions for the incidence angle in terms of the surface tilt and azimuth angles for fixed-tilt and for optimally tracking one- and two-axis surfaces.

This paper provides an alternative solution for one-axis trackers with the collector surface parallel to its axis. The minimum incidence angle is solved for by first determining the required rotation of the surface about its axis. Next, the surface tilt and azimuth are determined from the rotation angle and the tilt of the tracker axis. Finally, the value of the incidence angle is calculated from the surface tilt and azimuth angles and the zenith and solar azimuth angles.

Although the rotation angle is an intermediate value for determining the incidence angle, it has applications of its own for the control of tracker movement and for modeling the solar radiation available for a collector. For a motorized tracker with fixed gearing, the tracker rotation is directly proportional to the number of motor revolutions; consequently, the calculated rotation angle can be used to determine the number of motor revolutions to move the tracker to its optimum position. When modeling collector solar radiation, the rotation angle can also be used to account for non-optimum tracking that may occur when the optimum rotation angle exceeds the rotation limits of the tracker.

2 Relationship Between Rotation Angle and Surface Tilt and Azimuth

The surface tilt, β, and the surface azimuth, γ, are functions of the axis tilt, β_a, the axis azimuth, γ_a, and the rotation angle, R. Figure 1 is used to determine the relationship between these angles. For the analysis, γ_a is the azimuth of the tracker axis when observed from the inclined end of the tracker axis, and R, also observed from the inclined end of the tracker axis, is positive for clockwise rotation and negative for counterclockwise rotation. R equals zero when the normal to the surface is in a vertical plane. In Figure 1, this normal is the unit normal represented by the line OA. Line OB is the unit normal rotated angle R about the axis. The triangles formed by the unit normals and the vertical axis are used to derive equations for the surface tilt and azimuth.

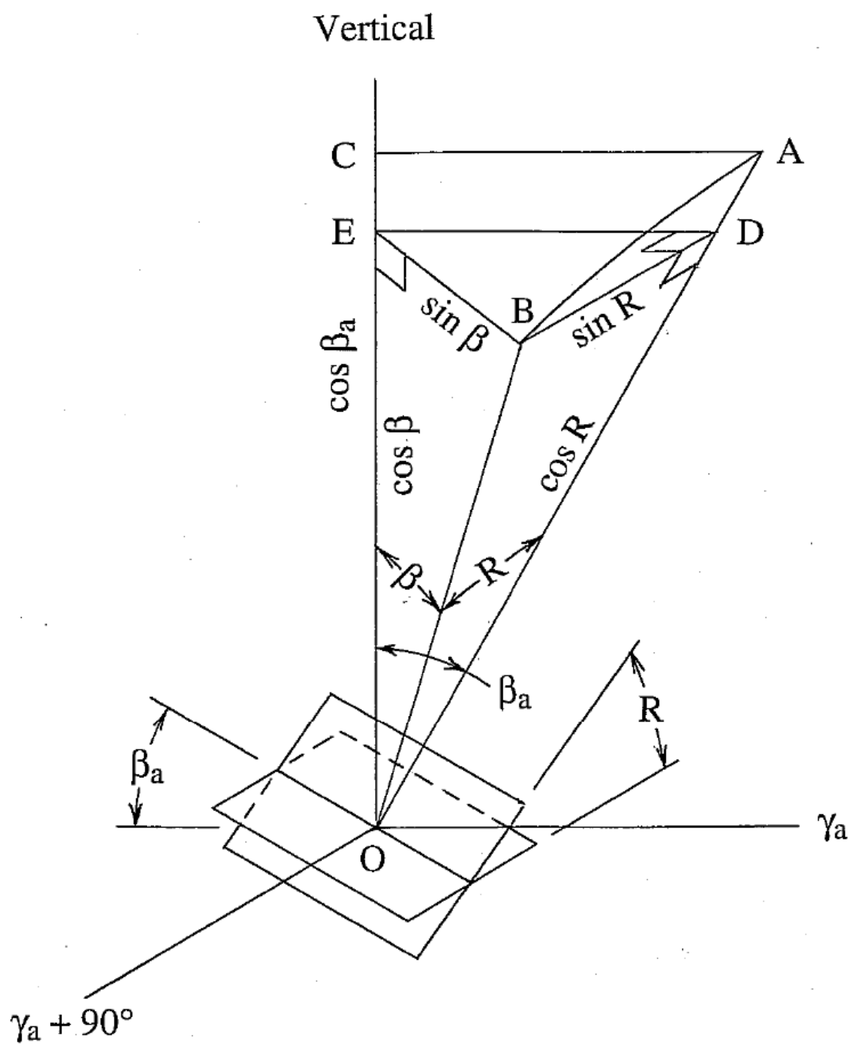

Figure 1. Geometry for one-axis tracking surface

Recognizing that triangles AOC and DOE are similar triangles whose respective sides are proportional, the surface tilt is expressed as:

$$\beta = \cos^{-1}[\cos R \cos \beta_a]. \tag{1}$$

The surface azimuth differs from the axis azimuth by the angle BED. Angle BED equals $\sin^{-1}[\sin R \div \sin \beta]$. Consequently, the surface azimuth is expressed as:

$$\gamma = \gamma_a + \sin^{-1}[\sin R \div \sin \beta] \qquad \text{For } \beta \neq 0, -90° \leq R \leq +90°. \tag{2}$$

If β equals zero (horizontal surface), γ cannot be determined from equation (2). In this case, γ is assigned any value because the surface is horizontal and assumed to have no azimuth response. Equation (2) will also not give the correct solution when R is outside the range of -90° to +90° because it does not distinguish between trigonometric quadrants when performing the arcsine operation. R can fall outside the range of -90° to +90° when the solar azimuth differs by more than 90° from the axis azimuth and the axis tilt is greater than zero. Extreme cases are midnight sun conditions for northernmost locations, where R can range from -180° to +180° as the tracker follows a sun that never sets.

For R values outside the range of -90° to +90°, either equation (3) or equation (4) applies.

$$\gamma = \gamma_a - \sin^{-1}[\sin R \div \sin \beta] - 180° \qquad \text{for } -180° \leq R < -90°. \tag{3}$$

$$\gamma = \gamma_a - \sin^{-1}[\sin R \div \sin \beta] + 180° \qquad \text{for } +90° < R \leq +180°. \tag{4}$$

3 Rotation Angle for Optimum Tracking

Various sources give the following trigonometric relationship for the incidence angle (Iqbal 1983):

$$\cos\theta = \cos\beta\cos\theta_Z + \sin\beta\sin\theta_Z\cos(\gamma_S - \gamma). \tag{5}$$

where θ_Z and γ_S are the zenith and solar azimuth angles, and can be determined from the time and location using various algorithms, such as presented by Michalsky (1988) with Errata (1988).

To introduce angle R into equation (5) and remove β and γ, substitutions for $\cos\beta$, $\sin\beta$, and $\cos(\gamma_S - \gamma)$ are made using equation (1) and equation (2) and various trigonometric identities. This procedure is shown in the appendix. The resulting expression for the cosine of the incidence angle is:

$$\cos\theta = \cos R\,[\sin\theta_Z\cos(\gamma_S - \gamma_a)\sin\beta_a + \cos\theta_Z\cos\beta_a] + \sin R\sin\theta_Z\sin(\gamma_S - \gamma_a). \tag{6}$$

For optimum tracking, the value of R should give the minimum incidence angle, thereby maximizing the value of $\cos\theta$. This value of R is determined by differentiating equation (6) with respect to R, setting it equal to zero, and solving for R.

$$d(\cos\theta)/dR = -\sin R\,[\sin\theta_Z\cos(\gamma_S - \gamma_a)\sin\beta_a + \cos\theta_Z\cos\beta_a] + \cos R\sin\theta_Z\sin(\gamma_S - \gamma_a) = 0$$

$$\sin R/\cos R = [\sin\theta_Z\sin(\gamma_S - \gamma_a)] / [\sin\theta_Z\cos(\gamma_S - \gamma_a)\sin\beta_a + \cos\theta_Z\cos\beta_a]$$

$$R = \tan^{-1}(X) + \psi, \tag{7}$$

where:

$X = [\sin\theta_Z\sin(\gamma_S - \gamma_a)] / [\sin\theta_Z\cos(\gamma_S - \gamma_a)\sin\beta_a + \cos\theta_Z\cos\beta_a]$

$\psi = 0°$ if $X = 0$, or if $X > 0$ and $(\gamma_S - \gamma_a) > 0$, or if $X < 0$ and $(\gamma_S - \gamma_a) < 0$

$\psi = +180°$ if $X < 0$ and $(\gamma_S - \gamma_a) > 0$

$\psi = -180°$, if $X > 0$ and $(\gamma_S - \gamma_a) < 0$.

The variable ψ places R in the correct trigonometric quadrant. For determining which value of ψ to use in equation (7), the difference $\gamma_S - \gamma_a$ is evaluated as the angular displacement between two vectors so that it falls within the range of -180° to +180°. For example, if $\gamma_S = 20°$ and $\gamma_a = 210°$, $\gamma_S - \gamma_a$ is evaluated as 20° + 360° - 210° = 170° and not 20° - 210° = -190°.

4 Procedure for Determining Incidence Angle

The use of the preceding equations to determine the beam incidence angle on a one-axis tracking surface might be considered a series of steps:

- Step 1 - Calculate θ_Z and γ_S using algorithms such as those presented by Michalsky (1988) with Errata (1988)
- Step 2 - Calculate R using equation (7)
- Step 3 - Calculate β using equation (1)
- Step 4 - Calculate γ using either equation (2), equation (3), or equation (4), as appropriate
- Step 5 - Calculate θ using equation (5).

For concentrating collectors, the procedure can be simplified slightly by using equation (6) to calculate the incidence angle after R has been determined in Step 2. The surface tilt, β, although necessary to model the diffuse radiation for a flat-plate collector, has no role in determining the beam radiation for a concentrator; consequently, steps 3–5 are replaced by the use of equation (6).

The use of R permits better modeling of tracker performance because it can be compared to a tracker's design parameters to see if R is within the range of the tracker's physical rotation limits. If not, an additional step in the procedure can set R equal to the limit of the tracker rotation range before completing steps 3–5. A similar step could be performed using β, but R is more convenient because for a given tracker the physical limits of rotation, unlike the surface tilt, do not change if the axis tilt is changed.

The use of tracker rotation limits were evaluated for Boulder, Colorado, latitude = 40.0°N, and Barrow, Alaska, latitude = 71.3°N, for a flat-plate one-axis tracker with a south facing axis azimuth and tilted from the horizontal at an angle equal to the site latitude. Monthly and yearly radiation available to the collector were modeled using typical meteorological year hourly data (Marion and Urban 1995), the Perez diffuse radiation model (Perez et al. 1990), and the incidence angles calculated when two different rotation limits are imposed: -180° to +180° for unrestricted rotation and -70° to +70° to represent the physical limits of the tracker.

Compared to the use of unrestricted rotation limits, the use of the -70° to +70° rotation limits for the Barrow site reduced the available yearly radiation for the collector by 3.3% with a maximum monthly reduction of 5.5% occurring in June. For Boulder, the differences for the two rotation limits were minimal. The yearly collector radiation was reduced 0.3% and the June collector radiation was reduced 0.8%. Barrow showed larger differences for the two rotation limits because its more northerly location results in a wider range of spring and summer solar azimuths, which require a wider range of rotation limits for optimum tracking.

5 One-Axis Tracker With Horizontal Axis

Large one-axis trackers often have a horizontal axis because its simpler construction offers cost advantages. For a horizontal axis, the equations can be simplified because the range for R is -90° to +90°, and sin β_a and cos β_a can be evaluated for $\beta_a = 0$. This results in the following equations for R, β, and γ:

$$R = \tan^{-1} [\tan \theta_Z \sin (\gamma_S - \gamma_a)] \quad (8)$$

$$\beta = | R | \quad (9)$$

$$\gamma = \gamma_a + \sin^{-1}[\sin R \div \sin \beta] \qquad \text{For } \beta \neq 0 . \quad (10)$$

If β equals zero, γ can be assigned any value because the surface is horizontal. The tracker axis azimuth, γ_a, can be assigned from either end of the tracker.

6 Conclusions

An equation for the rotation angle for optimum tracking of one-axis solar trackers has been derived along with equations giving the relationships between the rotation angle and the surface tilt and azimuth angles. General equations were presented for one-axis trackers with any axis tilt and azimuth as well as simplified equations for one-axis trackers with a horizontal axis.

The use of the rotation angle permits improved modeling of tracker performance because it can be compared to a tracker's design parameters to see if it is within the range of the tracker's physical rotation limits. If not, the rotation angle can be set equal to the limit of the tracker rotation range before calculating the surface tilt, azimuth, and incidence angles. As shown by the example, a tracker's rotation limits are more significant for locations farther from the equator.

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Appendix

To introduce angle R into equation (5), substitutions for $\cos\beta$, $\sin\beta$, and $\cos(\gamma_S - \gamma)$ are made using equation (1) and equation (2) and various trigonometric identities.

From equation (1),

$$\cos\beta = \cos R \cos\beta_a .$$

From the identity $\sin^2 a + \cos^2 a = 1$,

$$\sin\beta = [\,1 - \cos^2\beta\,]^{1/2} = [\,1 - \cos^2 R \cos^2\beta_a\,]^{1/2} .$$

From equation (2) and the identity $\cos(a - b) = \cos a \cos b + \sin a \sin b$, where $a = \gamma_S - \gamma_a$ and $b = \angle\, BED$,

$$\cos(\gamma_S - \gamma) = \cos(\gamma_S - \gamma_a)\cos(\angle\, BED) + \sin(\gamma_S - \gamma_a)\sin(\angle\, BED) .$$

From equation (2),

$$\begin{aligned} \sin(\angle\, BED) &= \sin R/\sin\beta = \sin R/[\,1 - \cos^2\beta\,]^{1/2} \\ &= \sin R / [\,1 - \cos^2 R \cos^2\beta_a\,]^{1/2} . \end{aligned}$$

From the identity $\sin^2 a + \cos^2 a = 1$,

$$\begin{aligned} \cos(\angle\, BED) &= [\,1 - \sin^2(\angle\, BED)\,]^{1/2} = [\,1 - \sin^2 R / (1 - \cos^2 R \cos^2\beta_a)\,]^{1/2} \\ &= [\,1 - \sin^2 R / (1 - \cos^2 R \cos^2\beta_a)\,]^{1/2} \end{aligned}$$

Using a common denominator

$$= [\,(1 - \cos^2 R \cos^2\beta_a - \sin^2 R\,) / (1 - \cos^2 R \cos^2\beta_a)\,]^{1/2} .$$

Replacing $\sin^2 R$ with $1 - \cos^2 R$

$$\begin{aligned} &= [\,(1 - \cos^2 R \cos^2\beta_a - 1 + \cos^2 R) / (1 - \cos^2 R \cos^2\beta_a)\,]^{1/2} \\ &= [\,- \cos^2 R \cos^2\beta_a + \cos^2 R) / (1 - \cos^2 R \cos^2\beta_a)\,]^{1/2} \\ &= [\cos^2 R\,(\,1 - \cos^2\beta_a) / (1 - \cos^2 R \cos^2\beta_a)\,]^{1/2} \\ &= [\cos^2 R \sin^2\beta_a / (1 - \cos^2 R \cos^2\beta_a)\,]^{1/2} \end{aligned}$$

$$= \cos R \sin \beta_a / [1 - \cos^2 R \cos^2 \beta_a]^{½} \; .$$

Therefore,

$$\cos(\gamma_s - \gamma) = [\cos(\gamma_s - \gamma_a) \cos R \sin \beta_a + \sin(\gamma_s - \gamma_a) \sin R] / [1 - \cos^2 R \cos^2 \beta_a]^{½} \; .$$

Replacing these expressions for $\cos \beta$, $\sin \beta$, and $\cos(\gamma_s - \gamma)$ in equation (5) yields the following expression for the cosine of the incidence angle:

$$\cos \theta = \cos R \cos \beta_a \cos \theta_z + \sin \theta_z [\cos(\gamma_s - \gamma_a) \cos R \sin \beta_a + \sin(\gamma_s - \gamma_a) \sin R] \; .$$

Rearranging terms,

$$\cos \theta = \cos R [\sin \theta_z \cos(\gamma_s - \gamma_a) \sin \beta_a + \cos \theta_z \cos \beta_a] + \sin R \sin \theta_z \sin(\gamma_s - \gamma_a) \; .$$