Short-Term Load Forecasting Error Distributions and Implications for Renewable Integration Studies

Preprint

B.-M. Hodge, D. Lew, and M. Milligan

To be presented at the IEEE Green Technologies Conference Denver, Colorado April 4–5, 2013
NOTICE

The submitted manuscript has been offered by an employee of the Alliance for Sustainable Energy, LLC (Alliance), a contractor of the US Government under Contract No. DE-AC36-08GO28308. Accordingly, the US Government and Alliance retain a nonexclusive royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for US Government purposes.

This report was prepared as an account of work sponsored by an agency of the United States government. Neither the United States government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States government or any agency thereof.

Available electronically at http://www.osti.gov/bridge

Available for a processing fee to U.S. Department of Energy and its contractors, in paper, from:

U.S. Department of Energy
Office of Scientific and Technical Information
P.O. Box 62
Oak Ridge, TN 37831-0062
phone: 865.576.8401
fax: 865.576.5728
email: mailto:reports@adonis.osti.gov

Available for sale to the public, in paper, from:

U.S. Department of Commerce
National Technical Information Service
5285 Port Royal Road
Springfield, VA 22161
phone: 800.553.6847
fax: 703.605.6900
email: orders@ntis.fedworld.gov
online ordering: http://www.ntis.gov/help/ordermethods.aspx

Cover Photos: (left to right) PIX 16416, PIX 17423, PIX 16560, PIX 17613, PIX 17436, PIX 17721

Printed on paper containing at least 50% wastepaper, including 10% post consumer waste.
Abstract—Load forecasting at the day-ahead timescale is a critical aspect of power system operations in the unit commitment process. It is also an important factor in renewable energy integration studies, where the combination of load and wind or solar forecasting techniques create the net load uncertainty that must be managed by the economic dispatch process or with suitable reserves. An understanding of the load forecasting errors that may occur in this process can lead to better decisions about the amount of reserves necessary to compensate for the errors that do occur. In this work, we performed a statistical analysis of the day-ahead (and two-day-ahead) load forecasting errors observed in two independent system operators for a one-year period. Comparisons were made with the normal distribution commonly assumed in power system operation simulations used for renewable power integration studies. Further analysis identified time periods when the load is more likely to be under- or overforecast.

Index Terms—forecasting, load modeling, power system analysis computing, statistical distributions

I. INTRODUCTION

Load forecasting has always played an important role in power system operations, giving an estimate of the amount of generating capacity that must be available at any future point in time. Accurate forecasting in the short-term, defined here as the hours-to-days time frame, can lead to more economic system operation. This increased efficiency is accomplished through more accurate scheduling, which decreases the amount of out-of-merit dispatch caused by the combination of forecasting errors and unit ramping limits, and a reduction in reserves. Scheduling in power system operations is conducted through the unit commitment and economic dispatch process. Unit commitment involves determining which generating units will be turned on during future time periods. It is a necessary process and is often performed at a day-ahead interval because some thermal units require long start-up and shutdown times. Because the unit commitment process determines the availability of units for the next day, accurate day-ahead load forecasts are required to ensure that production can meet demand in an economic manner. Dispatch is the process by which the unit outputs are fine-tuned to match the more accurate load forecasts produced at shorter timescales.

The unit commitment and economic dispatch processes form the basis for renewable power integration study simulations such as the Western Wind and Solar Integration Study [1]. Load forecasting can be included in these studies, to see how the interaction of load and wind or solar forecasting errors impact system operations. Many previous studies have assumed that the load forecast errors follow a normal distribution [2-5]. This assumption has been made for the day-ahead and hour-ahead timescales commonly used to represent the unit commitment and economic dispatch time frames. This assumption is often made without an examination of real forecast data from the area under study and can impact study results.

In this work, we studied the statistical properties of short-term load forecast errors, made comparisons to the commonly assumed normal distribution, and suggested an alternative model distribution.

II. METHODS AND DATA

In this section, we describe the load forecasting data used in this study and the methods used in the analysis. Section II-A describes the load forecasting data used in the study, while Section II-B discusses the statistical background and methods used for the characterization of load forecasting errors.

A. Data Utilized

In this work, we analyzed load forecasting data from two timescales and geographic locations. The data came from two independent system operators in the United States: the California Independent System Operator (CAISO) and the New York Independent System Operator (NYISO). Day-ahead and two-day-ahead load forecasts for each hour of the day, as well as matching actual load data, were obtained for the entire year 2010 from the CAISO Open Access Same-Time Information System [6]. The mean load value during the course of the year was 26,186 MW, the maximum load value was 47,282 MW, and the minimum load value was 17,890 MW. The NYISO hourly load data and day-ahead load forecasts from 2010 were obtained from the NYISO Market & Operations Data website [7]. The mean load value for the year was 18,664 MW. The minimum and maximum loads were

The authors are with the National Renewable Energy Laboratory, Golden, CO 80020 USA (e-mail: bri-mathias.hodge@nrel.gov, debra.lew@nrel.gov, michael.milligan@nrel.gov).
11,859 MW and 33,452 MW, respectively. Fig. 1 shows the hourly values of the NYISO load during the course of the year 2010.

Fig. 1. NYISO hourly load for the year 2010, in MW.

B. Statistical Background

The range of values that a random variable may take can be described through the use of a probability density function. The normal (or Gaussian) distribution is among the most common and has often been assumed to describe load forecasting errors [2-5]. The truncated normal distribution is often used to provide support over a fixed interval. The normal distribution can be fully described by the first two statistical moments: mean and variance; however, the third and fourth moments—skewness and kurtosis, respectively—can be utilized to provide additional information about an observed distribution. If an observed distribution is well represented by the normal distribution, the skewness and excess kurtosis values should both be close to zero. Skewness is a measure of the asymmetry of a distribution; kurtosis provides a measure of the distribution’s peakedness and the weight of the distribution’s tails. The normal distribution has a kurtosis value of three, and thus the excess kurtosis is the kurtosis value minus three. Because we used the normal distribution as a point of reference in what follows, when we subsequently refer to kurtosis we specifically refer to the excess kurtosis value. A distribution with a high kurtosis value is known as a leptokurtic distribution; a lower kurtosis value is described as platykuritic.

The R statistical computing environment [8] was used in the analysis work performed in this study, and the hyperbFit function of the HyperbolicDist package [9], in particular, was used to characterize the distributions and provide estimates of the hyperbolic distribution parameter values. The Shapiro-Wilk [10] normality test was used to test whether an observed distribution comes from a normally distributed population.

III. RESULTS

To understand the impact of load forecasting errors on system operations and renewable power integration studies, we analyzed and characterized the distribution of load forecast errors that occurred during 2010 in the NYISO and CAISO balancing areas. Section III-A examines the distribution of day-ahead load forecast errors in the CAISO system. A similar analysis of the two-day-ahead load forecast errors in CAISO is presented in Section III-B. The NYISO day-ahead load forecast errors are examined in Section III-C. Section III-D discusses some of the consequences of erroneously assuming that load forecasting errors are normally distributed.

A. CAISO Day-Ahead Load Forecasts

Day-ahead load forecasts are important in power system operations because, through the unit commitment process, they help determine which slow-starting thermal power plants (e.g., coal and nuclear plants) will be on during which hours of the next day. Load forecast errors can therefore cause a suboptimal commitment of thermal generation in the day-ahead market. System operators are accustomed to uncertainty in load and therefore have methods of dealing with forecasting inaccuracies (e.g., regulation reserve and hour-ahead dispatch); however, large load forecast errors can have large negative consequences for system operation. One traditional method of examining statistical distributions is through the plotting of a histogram. A critical parameter in the plotting of a histogram is the choice of the number of bins. Because we are particularly interested in the tails of the distributions—i.e., the large forecast errors—we chose \( n = 200 \), a number that is greater than the recommendation from Scott’s rule [11]. Fig. 2 shows the observed day-ahead load forecast errors for the CAISO system in 2010. The dotted line shows a normal distribution with the same mean and standard deviation as the observed errors. The blue line is a hyperbolic distribution fit to the observed errors. The observed error distribution is more peaked, with narrower shoulders and larger tails than the normal distribution assumption would suggest. One of the most critical features of the observed distribution is the negative mean bias, represented by a mean value of -84.71 MW. The distribution is also positively skewed and leptokurtic. It is also important to note the spread of the forecast errors, with both positive and negative errors of approximately 4 GW. These are very significant errors on a system with a mean load of approximately 26 GW, and would require very large corrective actions before the actualization time, at high economic cost, to prevent reliability issues.
Fig. 2. Histogram of the observed day-ahead forecast errors in the CAISO system in MW, $\mu = -84.71; \sigma = 851.05; \gamma = 0.44; \kappa = 0.92$. The blue line represents a hyperbolic distribution fit to the data, with parameter values: $\pi = 0.274; \zeta = 3.076; \delta = 1,156.279; \mu = -564.815$. The black dashed line represents a normal distribution with the same mean and standard deviation.

Although the examination of the histogram of day-ahead load forecast errors seems to indicate that the distribution is non-normal, additional assurance was provided by utilizing a normal quantile-quantile (Q-Q) plot. Fig. 3 shows a normal Q-Q plot of the CAISO day-ahead load forecast errors. The line drawn passes through the first and third quartiles of the observed errors and should pass through all of the data points if the data is from a normal distribution. However, we noticed significant deviations from the normal distribution, most pointedly in the tails of the observed error distribution. Additional assurance of non-normality was provided by performing a Shapiro-Wilk [10] test on the data. The null hypothesis of the CAISO day-ahead forecast error data coming from a normal distribution was rejected at a significance level of $\alpha = 0.000001$, i.e., the 99.9999% confidence interval.

Fig. 3. Normal Q-Q plot for the CAISO day-ahead forecast errors. The line in the graph passes through the first and third quartiles of the observed data and should pass through all of the data points if the distribution is normal.

Another means by which distributions may be compared is through the plotting of their cumulative distribution functions. A cumulative distribution plot—like that shown in Fig. 4—also allows us to compare the fit of the proposed hyperbolic distribution to both the observed errors, and a normal distribution with the same first two statistical moments. One can observe that the normal distribution shows significant deviations from the observed errors in the tails and shoulders of the distribution. On the other hand, the hyperbolic distribution fit to the data parallels the observed error distribution almost exactly, with only minor deviations in the transition area between the shoulders and tails of the distributions.

Fig. 4. Cumulative distribution plot for the day-ahead forecast errors in the CAISO system during a 12-month period. The red line represents a normal
distribution with the same mean and standard deviation. The blue line represents a hyperbolic distribution fit to the data with: \( \pi = 0.274, \zeta = 3.076, \delta = 1,156.279, \mu = -564.815 \).

Although the figures above are useful for examining the whole range of errors observed, they do not give any indication as to the timing of the errors. The histogram, Q-Q, and cumulative distribution plots provide an aggregated view of the errors that is not particularly useful for improving forecasts. By disaggregating the data into hourly and monthly pieces, we can analyze when the forecasts are most accurate and inaccurate, with the goal of considering the temporally-specific conditions at those times to improve the forecasts. Fig. 5 provides a heat map of the mean hourly forecast error per month. This indicates that load forecasting tends to underforecast the demand during the morning load ramp, especially during the summer months. The forecasting also tends to underforecast significantly during the early evening hours of the winter months. Another interesting result is that the summer late night months tend to be overforecast. That the summer months provide the most trouble for the forecast is not surprising; they have the largest daily variability, as shown in Fig. 6, which shows the hourly CAISO load for the entire year 2010. Based on the results of examining the heat maps, we noticed that the forecasts have difficulty forecasting the timing of the morning and evening up- and down-ramps, a problem that is most pronounced during the summer months.

**Fig. 5.** Heat map of the mean CAISO day-ahead forecast errors for each hour of the day per month of the year.

**Fig. 6.** CAISO hourly load for the year 2010, in MW. Notice the increased daily variability during the summer months.

### B. CAISO Two-Day-Ahead Load Forecasts

Two-day-ahead load forecasts are not as widely utilized as day-ahead forecasts, but they can play an important role in reliable system operations. Sometimes a look ahead greater than 24 hours is used in the unit commitment and economic dispatch process to prevent time boundary issues with slow-starting thermal units. In this case, a two-day-ahead load forecast enables more efficient commitment through the generation of a 48-hour schedule, where only the first 24 hours are binding. Alternatively, the two-day-ahead forecast can be used to ensure supplemental resource availability from units that have longer than day-ahead start or minimum run times. Although small errors in the two-day-ahead forecast are not important (because of the subsequent additional unit commitment performed at the day-ahead timescale), large errors could result in insufficient reserve capacity availability. For this reason, the tails of the two-day-ahead load forecast errors are the most important feature of the distribution. Fig. 7 shows a histogram of the two-day-ahead load forecasting errors observed during one year in the CAISO system. Compared to the day-ahead forecast errors, the two-day errors were more biased, with a mean value of approximately -180 MW, and had a slightly higher standard deviation. They also were less skewed (or more symmetric) and more leptokurtic. Interestingly, the maximum positive forecast error was greater for the day-ahead forecast (4,163 MW vs. 3,380 MW), while for the negative forecast errors the two-day-ahead forecast error is larger (-3,838 MW vs. -4,972 MW), as expected, because the day-ahead forecast can incorporate more recent information.

**Fig. 7.** Histogram of the two-day-ahead load forecasting errors observed during one year in the CAISO system.
Examining the Q-Q plot of the two-day-ahead forecasts in Fig. 8, the deviations from normality are clearly demonstrated when a comparison was made between the tails of the observed distribution and a line through the first and third quartiles. Again, a Shapiro-Wilk test confirmed the non-normality with the null hypothesis of the CAISO two-day-ahead forecast error data coming from a normal distribution being rejected at a significance level of \( \alpha = 0.000001 \), i.e., the 99.9999% confidence interval.

As shown in the cumulative distribution plot of the two-day-ahead forecasts in Fig. 9, the fit hyperbolic distribution provided a more accurate representation of the observed errors than did the normal distribution. This is particularly evident in the tails of the distribution, which are underrepresented by the assumption of normality.

The same general trends that were noted in the hourly timing of day-ahead forecasting errors were also apparent in the two-day-ahead forecasts, as shown in Fig. 10. The load was often underforecast during the morning ramp—especially during the summer months—and the significant overforecasting events tended to occur late at night and, again, were most severe during the summer months. There were also significant underforecasting tendencies noticeable during the early evening hours in the winter. These results intuitively made sense because the times of greatest errors tended to occur during the times of greatest variability in load, which coincided with the times of greatest uncertainty.
C. NYISO Day-Ahead Load Forecasts

Examining the statistical moments of the NYISO day-ahead load forecast error distribution revealed some interesting results. For example, the mean of the data was nearly -450 MW, indicating a very significant negative mean bias in the forecast. Additionally, the distribution was significantly leptokurtic, with a kurtosis value of 3.8. This is clearly visible in Fig. 11, where the normal distribution is much less peaked than the observed error distribution. There was also significantly more kurtosis than was evident in the CAISO day-ahead load forecasts. In addition, the observed error distribution was thinner through the shoulders than the normal distribution and has heavier tails. The spread of the forecast errors was similar to that of the CAISO errors, but the NYISO system had a significantly lower mean yearly load. The high kurtosis value indicated that although the forecasts were more accurate than expected from the normal distribution much of the time, when there were very significant forecasting errors, they were greater than they would have been assumed with the normal distribution. The distribution also had significant positive skewness.

A Q-Q plot of the forecast errors provided evidence of non-normality, as shown in Fig. 12. The observed errors form the $S$ shape commonly seen in heavy-tailed distributions. A Shapiro-Wilk test confirmed the non-normality with the null hypothesis, that the NYISO day-ahead forecast error data came from a normal distribution, being rejected at a significance level of $\alpha = 0.000001$. 

![Fig. 11. Histogram of the observed day-ahead forecast errors in MW for the NYISO system, $\mu = -447.26; \sigma = 665.21; \gamma = 0.38; \kappa = 3.80$. The blue line represents a hyperbolic distribution fit to the data, with parameter values: $\pi = -4.189 \times 10^{-5}$, $\zeta = 0.023$, $\delta = 10.076$, $\mu = -446.904$. The black dashed line represents a normal distribution with the same mean and standard deviation.](image)

![Fig. 12. Normal Q-Q plot for the NYISO day-ahead forecast errors. The line in the graph passes through the first and third quartiles of the observed data and should pass through all of the data points if the distribution is normal.](image)
The cumulative distribution plot in Fig. 13 shows significant differences between the normal distribution and observed errors for the NYISO day-ahead load forecast errors. Most important were those observed in the tails of the distribution. There were also noticeable, though much smaller, differences between the fit hyperbolic distribution and the observed errors. This demonstrates that although the hyperbolic distribution shows significant improvements from the normal distribution in use as a model distribution, it was not an exact representation of the observed forecast errors. Fig. 14 shows an alternative means of comparing the fit of the hyperbolic distribution, a Q-Q plot between the observed data and the fit hyperbolic distribution. Note the improved fit compared to Fig. 12.

Fig. 15 shows a heat map of the mean hourly forecast error per month. The figure shows a tendency to strongly underforecast the load during the morning ramp, with the largest errors occurring during the month of March. The variable weather during this month also led to the strongest tendency to overforecast, namely during the late-night hours. Generally speaking, load was often underforecast in the NYISO system, with the mean forecast error of approximately -450 MW. By comparison, the CAISO day-ahead forecast errors showed a mean bias less than -100 MW.

D. Implications of Non-Normality

We have shown that day-ahead and two-day ahead load forecasting errors do not follow a normal distribution for these individual independent system operators, but what are some of the consequences of this finding for power system operations and renewable power integration studies? Because the distribution of observed errors is more leptokurtic than the
normal distribution, this implies that assuming the normal distribution underrepresents the tails of the distribution. Therefore, there will be more frequent large forecasting errors that occur than are assumed, which can lead to undercommitting generating units in the day-ahead market. This suboptimal commitment will lead to more expensive, fast-acting units having to come online after the unit commitment phase, or perhaps during the economic dispatch stage, instead of cheaper baseload units. This will raise the cost of electricity during these time frames. In addition, both systems examined show significant negative mean bias in the forecasts. This contrasts sharply with the zero mean bias normal distribution assumption commonly made in the renewable integration study literature. This consistent underforecasting of load can have significant economic consequences, as if the forecasts were zero mean biased more of the load could be fulfilled through lower-cost baseload units, and more expensive, fast-acting units would be utilized less to accommodate the underforecasting errors.

IV. CONCLUSION

In this study, we analyzed and characterized the load forecasting errors that occur in relatively large balancing areas at the day-ahead and two-day-ahead timescales. The forecast error distributions have been shown to not follow a normal distribution, and the hyperbolic distribution has been proposed as a more accurate means of modeling the distribution. This is consistent with findings from wind and solar power forecasts, both at the day-ahead, and smaller timescales, and for many different geographic locations [12-16]. In addition, the forecasting errors were disaggregated into months and hours of the day, and the times where large forecasting errors are most likely to occur were identified. These results can be utilized in power system operations studies, including wind and solar integration studies, to more accurately model the variability and uncertainty faced in system operations. The significant non-normality and mean biases of the load forecasting error distributions could have significant economic consequences in these studies. The examination of the operating cost results from simulations run with the zero mean bias normal distribution forecast error assumption and those performed with forecast error results based on historical data for the system under study is an area for future examination.

V. REFERENCES


Bri-Mathias Hodge (M’10) received a B.S. degree in chemical engineering from Carnegie Mellon University. He received an M.S. from Åbo Akademi and completed a Ph.D. in chemical engineering at Purdue University. He is currently a research engineer in the Transmission and Grid Integration Group at NREL. His research interests include energy systems modeling, simulation, and optimization.

Debra Lew (M’08) graduated with M.S. and Ph.D. degrees in applied physics from Stanford University, and with B.S. degrees in physics and electrical engineering from the Massachusetts Institute of Technology. Lew presently works as a senior project leader in the Transmission and Grid Integration Group at NREL. Her fields of interest include high penetrations of wind power in power systems and transmission planning to accommodate wind power.

Michael Milligan (M’98, SM’10) received a B.A. degree from Albion College, in Albion, Michigan, and M.A. and Ph.D. degrees from the University of Colorado at Boulder. He is part of the Transmission and Grid Integration Group at NREL, where he has worked on various aspects of electricity supply related to wind energy since 1992. He has worked on issues such as the ancillary service impacts of wind generation, the value of accurate wind forecasting, optimal selection of geographically dispersed wind power plants, modeling wind plant variability, and reliability contribution of wind power plants. He has authored or coauthored more than 120 reports and book chapters and has served on numerous technical review committees for wind integration studies around the United States.