Draft: FAST Ice Module Manual

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1. Introduction

FAST is the CAE tool maintained by the National Renewable Energy Laboratory (NREL) for simulating onshore and offshore wind turbine systems. NREL's core CAE tool, FAST (Jonkman 2005, 2013) joins AeroDyn (a rotor aerodynamics module) (Latino 2002, Moriarty 2005), HydroDyn (a platform hydrodynamics module) (Jonkman 2007, 2009) for offshore systems, SubDyn (a multimember substructure finite element module) (Damiani 2013, Song 2013), a control and electrical system (servo) dynamics module, and a structural (elastic) dynamics module to enable coupled nonlinear aero-hydro-servo-elastic analysis in the time domain.

For the recent development of offshore wind in the Great Lakes and future implementation of offshore wind in other ice-covered waters, there is a need for FAST to be capable of modeling all the design-driving loads including the ice load interacting with the offshore wind turbine system during operation. In this manual, a new module of FAST with the name IceDyn for assessing the dynamic response of offshore wind turbines subjected to ice forcing is presented.

The *IceDyn* module includes 6 ice mechanics models that incorporate ice floe forcing, deformation and failure and structure geometry. The six models are: quasi-static ice loading on vertical structure, dynamic ice loading on vertical structure, random ice loading on vertical structure, non-simultaneous ice loading on vertical structure, ice loading on sloping structure and large ice floe impact.

This draft of the Ice Module Manual includes description of models 1 through 6. In each model, a brief theory description is given first. Then the input parameters for that model are listed with explanation. After that, as an example, a time history of ice load is presented.

2. Ice module

The general layout the ice input file is as shown in Figure 1.

```
IceDyn_Input.txt
------------------ IceDyn v1.01.x Input File -----------------------------------
Freshwater Ice of Great Lakes input properties.
---------------------- STRUCTURE PROPERTIES ------------------------------------
 1        NumLegs      - number of support-structure legs in contact with ice
 0        LegPosX      - array of size NumLegs: global X position of legs 1-NumLegs (m)
 0        LegPosY      - array of size NumLegs: global Y position of legs 1-NumLegs (m)
 6        StWidth      - array of size NumLegs: Width of the structure in contact with the
ice, or diameter for cylindrical structures (m)
---------------------- ICE MODELS ----------------------------------------------
 6        IceModel     - Number that represents different ice models. {1: quasi-static load;
2:dynamic ice-structure interaction; 3: random ice load; 4: non-simultaneous ice failure;
5: sloping structure 6: large ice floe impact}
 1        IceSubModel - Number that represents different ice sub models.
---------------------- ICE PROPERTIES -General ---------------------------------
0.5       IceVel       - Velocity of ice sheet movement (m/s)
0.8       IceThks      - Thickness of the ice sheet (m)
1000      WtDen        - Mass density of water (kg/m3)
900       IceDen       - Mass density of ice (kg/m3)
0.0       InitLoc      - Ice sheet initial location (m)
0.0       InitTm       - Ice load starting time (s)
2         Seed1        - Random seed 1
5         Seed2        - Random seed 2
---------------------- ICE PROPERTIES - Ice Model 1, SubModel 1-----------------
2.7       Ikm          - Indentation factor
3.5e6     Ag           - Constant depends only on ice crystal type, used in calculating
uniaxial stress (MPa-3s-1)
65000     Qg           - Activation Energy (Jmol^-1)
8.314     Rg           - Universal gas constant (Jmol-1K-1)
269       Tice         - Ice temperature (K)
---------------------- ICE PROPERTIES -Ice Model 1, SubModel 2------------------
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Figure 1 Ice input file

The general input parameters for all ice models are listed in the Table 1. The input parameters for each ice model are listed after the theory explanation of each model in the following sections.

Table 1. General input parameters

NumLegs	Number of support-structure legs in contact with ice
LegPosX	Array of size NumLegs: global X position of legs 1-NumLegs (m)
LegPosY	Array of size NumLegs: global Y position of legs 1-NumLegs (m)
StWidth	Array of size NumLegs: Width of the structure in contact with the ice, or diameter for cylindrical structures (m)
IceModel	Number that represents different ice models.
IceSubModel	Number that represents different ice sub models.
IceVel	Velocity of ice sheet movement. It has the unit of m/s.
IceThks	Thickness of the ice sheet. It has the unit of m.
WtDen	Mass density of water. It has the unit of kg/m^3.
IceDen	Mass density of ice. It has the unit of kg/m^3.
InitLoc	Ice feature initial location. It has a unit of m. The default value is 0.
InitTm	This is the ice load starting time. It has a unit of s.
Seed1	Random seed 1
Seed2	Random seed 2

2.1 Ice model 1 – quasi-static ice loading

In model 1, we assume that the structure is stiff enough so that its interaction with ice does not affect the magnitude and period of the ice load. In this case, the ice load is prescribed. In model 1, there are two sub-models. The first one is quasi-static creep. The second one is elastic buckling.

<u>Ice model 1 sub-model 1. Creep</u>

In the creep model, the ice force on the structure has the following empirical expression (Korzhavin 1971):

$$F_{\max} = Ikmwh\sigma \tag{1}$$

Where:

I is the indentation factor, has the range of 1 to 3.

k is the contact factor, has the range of 0.3 (for non-simultaneous failure) to 1 (for simultaneous failure, such as creep) for small scale structures. Meanwhile, Sanderson suggests the contact factor k has to be very low (0.02-0.13) for full-scale structures (Sanderson 1988).

m is the shape factor, 0.9 for cylindrical structures and 1 for flat indenters. Michel and Toussaint (Michel and Toussaint 1977) also suggest a different value 2.97 for the product of I, k and m. Ralston (Ralston 1979) suggests a range of 1.15 to 4 depending on the aspect ratio.

w is the width (diameter) of the structure.

h is the thickness of the ice sheet

σ is the uniaxial compressive strength of ice.

The uniaxial compressive strength of ice depends on the strain rate. Michel and Toussaint (Michel and Toussaint 1977) proposed that plots of indentation pressure divided by 2.97 vs. indentation speed divided by four times the indenter width coincide with plots of uniaxial strength of columnar ice vs. strain rate.

In the method they proposed, first from the ice velocity U and the diameter of the structure w, calculate $U/4w$ (Ralston 1979) and adopt this as indentation strain rate $\dot{\varepsilon}$.

Then calculate the uniaxial stress, which would correspond to this strain-rate if it were a uniaxial strain-rate, (Sanderson 1988 Equation 4.6, 4.7) via:

$$\sigma = F_{\sigma}\{\dot{\varepsilon}\}$$

For freshwater granular ice we use

$$\sigma = \left[\frac{1}{A_g}\exp\left(\frac{Q_g}{RT}\right)\dot{\varepsilon}\right]^{1/3} \tag{2}$$

where

$R = 8.314 Jmol^{-1}K^{-1}$ is the universal gas constant

T is temperature in kelvin

Q_g is the activation energy

A_g is a constant which depends only on crystal type.

The constants A_g and Q_g take the following values:

Above 265 K (-8°C): $Q_g = 120 kJ \cdot mol^{-1}$

$A_g = 7.8 \times 10^{16} (MPa)^{-3} s^{-1}$

Below 265 K (-8°C): $Q_g = 78 kJ \cdot mol^{-1}$

$A_g = 4.1 \times 10^{8} (MPa)^{-3} s^{-1}$

For freshwater columnar ice we use:

$$\sigma = \left[\frac{1}{A_c} \exp\left(\frac{Q_c}{RT} \right) \dot{\varepsilon} \right]^{1/3}$$

where

$$Q_c = 65 kJ \cdot mol^{-1}$$

$$A_c = 3.5 \times 10^{6} (MPa)^{-3} s^{-1}$$

Before the ice stress reaches the "yield stress", we assume ice is under elastic strain. The elastic strain when ice begins to "yield" can be calculated as:

$$\varepsilon_e = \frac{Ikm\sigma}{E}$$

where E is the Young's Modulus of ice.

We assume constant strain rate. Then the time when the ice begins to "yield" is

$$T_{rise} = \varepsilon_e / \dot{\varepsilon}$$

The criteria of ice model 1.1 follow:

The indentation speed during the loading phase should be within the ductile range of ice. This means the indentation strain rate should be below 10^{-4}. This gives an upper limit of the ice velocity.

The input parameters for ice model 1 sub-model 1 are listed in Table 2.

Table 2. Input parameters for ice model 1 sub-model 1

Variable name	Symbol	Explanation
Ikm	Ikm	This is the product of all the factors related to ice indentation. I is the indentation factor, has the range of 1 to 3. k is the contact factor. It is approximately 1 for model 1. m is the shape factor, 0.9 for cylindrical structures and 1 for flat indenter. (Korzhavin 1962). Michel and Toussaint (Michel and Toussaint 1977) also suggest a value 2.97 for the product of I, k and m. Ralston (Ralston 1979) suggest a range of 1.15 to 4 depending on the aspect ratio.
Ag	A_g or A_c	This is a constant used to calculate the uniaxial ice stress. This constant only depends on crystal type. For freshwater granular ice, the suggest value is $A_g = 7.8\times10^{16}(MPa)^{-3}s^{-1}$ (Above 265 K (-8°C)), $A_g = 4.1\times10^{8}(MPa)^{-3}s^{-1}$ (Below 265 K (-8°C)). For freshwater columnar ice, $A_c = 3.5\times10^{6}(MPa)^{-3}s^{-1}$.
Qg	Q_g or Q_c	This is activation energy used to calculate the uniaxial ice stress. The suggest value is: for freshwater granular ice, $Q_g = 120kJ\cdot mol^{-1}$(Above 265 K (-8°C)), $Q_g = 78kJ\cdot mol^{-1}$ (Below 265 K (-8°C)). For freshwater columnar ice, $Q_c = 65kJ\cdot mol^{-1}$.
Rg	R	This is the universal gas constant (Jmol-1K-1)
Tice	T	This is the temperature of ice. It has the unit of Kelvin.

Ice model 1 sub-model 2. Elastic buckling

In this model, as shown in the following figure, we assume a truncated wedge-shaped plate of elastic material (ice) floats on an elastic foundation (fresh/sea water) and is loaded at its edge by a load P acting over a width D.

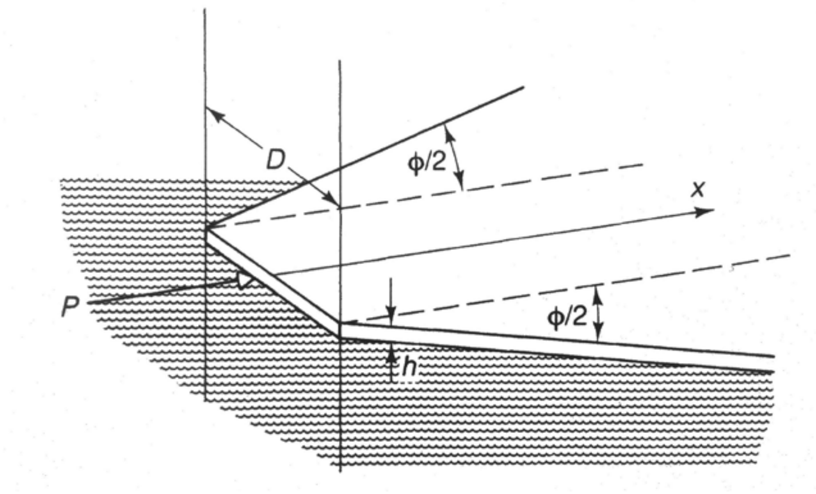

Figure 2 Wedge-shaped ice sheet floating on water (Sanderson 1988)

The wedge angle ϕ is variable. The plate can be a parallel-sided floating plate (uniaxial loading) with $\phi = 0$. As ϕ tends to 180°, the plate approaches an infinite half-plane. According to Sanderson (Sanderson 1988), it is usually observed that, the ice sheet that interacted with structures, would form radial cracks at angles of around 45° before buckling. Therefore, we set the default value of wedge angle to be

$$\phi = 2 \times 45^\circ = 90^\circ$$

The solution given by Kerr (1978) depends on the boundary condition at the loaded edge, where ice and structure interact. For a simply supported edge, which is a more realistic case, the buckling load is given by:

$$P_b = 5.3 B_f \kappa \left(\kappa D + 2 \tan \frac{\phi}{2} \right) \tag{3}$$

where B_f represents the flexural rigidity of the ice cover

$$B_f = \frac{E h^3}{12(1 - \nu^2)}$$

and we define

$$\kappa = \left(\frac{\rho_w g}{4 B_f} \right)^{1/4}$$

and ρ_w is the density of water, g is the gravity acceleration, h is the ice thickness, D is the structure width, ν is the Poisson's ratio of ice and E is the Young's modulus of ice.

Since this is the elastic buckling model, the average stress of ice at buckling and total elastic strain can be calculate as:

$$\sigma_b = \frac{P_b}{Dh}$$

$$\varepsilon_b = \frac{\sigma_b}{E}$$

And the time from loading till buckling is

$$T_b = \frac{\varepsilon_b}{\dot{\varepsilon}}$$

The ice load will linearly build up over time until the buckling load is reached. Then the ice load will drop to zero.

The criteria of our ice model 1.2 should be as follows:

For buckling to occur before crushing occurs, the buckling load should be smaller than the crushing load, which is normally on the order of 5 MPa (Sanderson 1988).

$$P_b < P_c$$

$$h < \frac{\sigma_c^2 \left(1 - \nu^2\right)}{0.59 \rho_w g E}$$

This gives an upper limit for the thickness of the ice sheet.

The input parameters for ice model 1 sub-model 2 are listed in Table 3:

Table 3. Input parameters for ice model 1 sub-model 2

Variable name	Symbol	Explanation
Poisson	ν	This is the Poisson's ratio of ice. The default number is 0.3.
WgAngle	ϕ	This is the wedge angle of the ice sheet.
EIce	E	This is the Young's Modulus of ice. It has the unit of GPa. The default value is 9.5 GPa.

Ice model 1 sub-model 3. Nominal stress

In the third sub-model of ice model 1, the user can input the average ice failure stress. The time history of ice load is similar to sub-model 1. The ice force will linearly increase until the ice stress reaches the critical stress. Then ice force will stay constant. The input parameter of this model is listed in Table 4.

Table 4. Input parameters for ice model 1 sub-model 3

Variable name	Symbol	Explanation
SigNm	σ	This is the nominal ice stress (MPa).

<u>Ice model 1: Examples</u>

Ice model 1.1:

Assume one ice sheet of thickness 0.65m, is moving at a speed of 0.001m/s. It is contacting a wind turbine with a diameter 4m at the water surface. The temperature of the ice is −4°C. The time history of ice load is as shown in Figure 3.

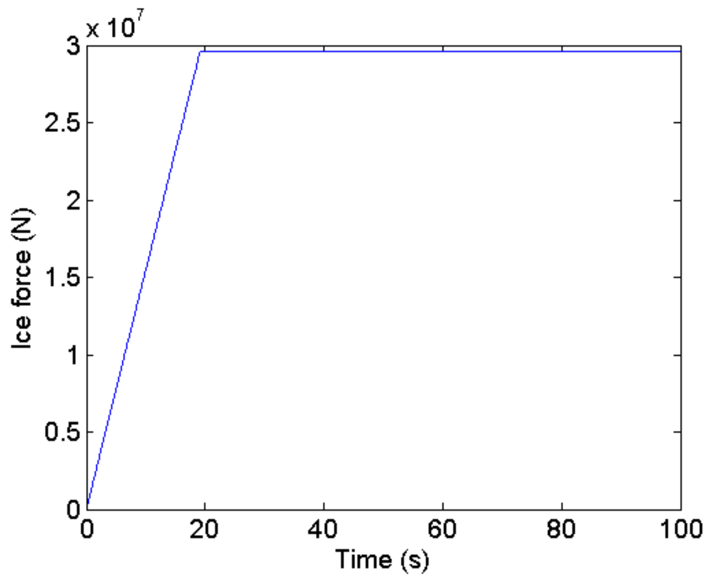

Figure 3 Ice load time history for ice model 1 sub-model 1

Ice model 1.2:

Assume one ice sheet of thickness 0.2m, is moving at a speed of 0.001m/s. It is contacting with a wind turbine of a diameter 4m at the water surface. The temperature of ice is −4°C. The density of the water is 1000 kg/m^3. The time history of ice load is as shown in Figure 4.

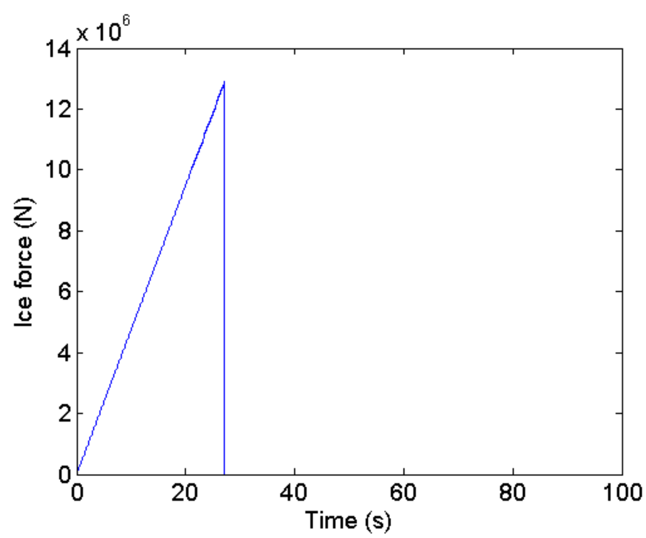

Figure 4 Ice load time history for ice model 1 sub-model 2

Ice model 1.3:

Assume one ice sheet of thickness 0.5m, is moving at a speed of 0.001m/s. It is contacting with a wind turbine of a diameter 4m at the water surface. The nominal stress is 5MPa. The time history of ice load is as shown in Figure 5.

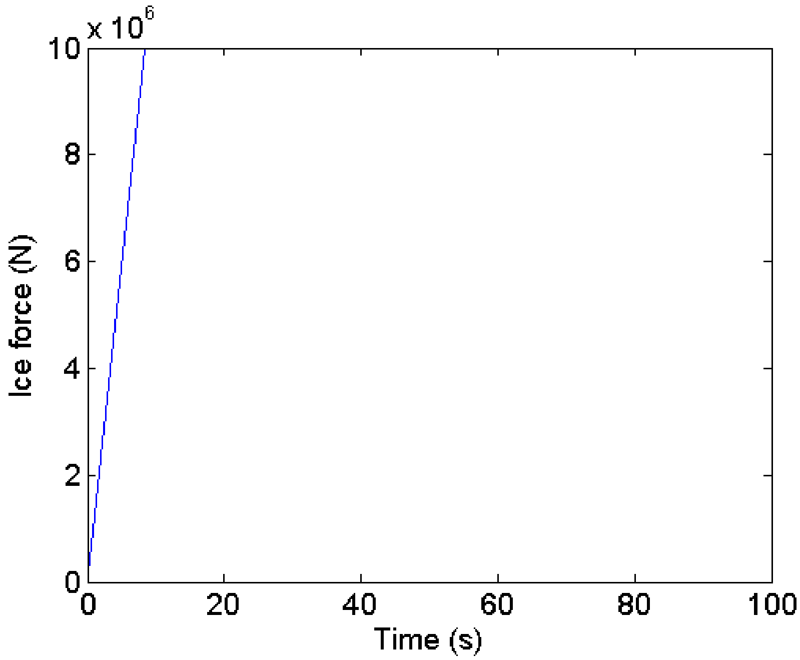

Figure 5 Ice load time history for ice model 1 sub-model 3

2.2 Ice Model 2 – dynamic ice loading

During an ice interaction with a rigid structure, the ice fails in ductile or brittle modes, depending on the indentation speed. For both modes, the ice force can be prescribed. However, when ice interacts with a compliant structure, or the ice loading frequency is comparable to the natural frequency of the structure, the structural response can be very large and have some feedback on the ice load. In this case, the ice force exerted on the structure can no longer be prescribed.

In this model, we use the mechanical model presented by Matlock et al. and Karr et al. to describe the ice-structure interaction process (Matlock 1971, Karr 1993). The ice sheet is represented by a system of brittle elastic bars, as shown in Figure 8.

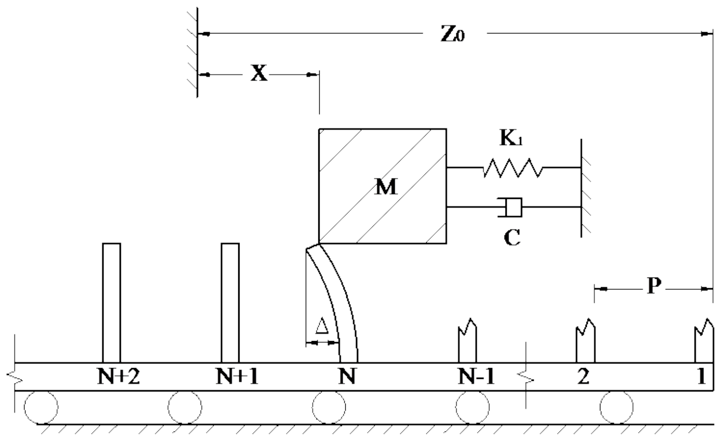

Figure 6 Ice-structure interaction model

In this model, the ice sheet consists of a series of ice teeth. The position of the first ice tooth at the beginning of the simulation is denoted as z_0. The ice sheet is assumed to move at a constant speed $\dot{z} = v_{ice}$. The distance between ice teeth is assumed to be a constant value P. Each ice tooth is assumed to exhibit linear elastic deformation when it contacts the structure and before the maximum deflection $\Delta_{\max}$ is reached.

<u>Ice Model 2 sub-model 1 – dynamic ice loading, single tooth deflection</u>

Assuming the position of the structure at current time is x, the tip deflection of the current Nth ice tooth can be calculated as:

$$\Delta = \dot{z}t + z_0 - x - P(N-1)$$

Denote the stiffness of the ice tooth as K_{ice}, then the ice force becomes $K_{ice}\Delta$. When ice tooth does not contact the structure or when it breaks, the ice force becomes zero. Therefore, the ice force can be expressed as

$$\begin{cases} K_{ice}\Delta & 0 < \Delta < \Delta_{\max} \\ 0 & \Delta \le 0, \Delta = \Delta_{\max} \end{cases} \tag{4}$$

In this model, the user is expected to input the ice sheet thickness h, structure diameter w, average ice brittle strength σ_g, distance between ice teeth $P$and the maximum ice tooth deflection Δ_{max}. The stiffness of ice can be calculated as:

$$F_{max} = wh\sigma_g$$
$$K_{ice} = F_{max} / \Delta_{max} \tag{5}$$

The FAST program will provide the ice module the current simulation time and structure position. Then based on Eqn. (4), the ice module will return the current ice force.

Ice Model 2 sub-model 2 – dynamic ice loading with two ice teeth bending

The previous sub-model assumes the distance between ice teeth is larger than the maximum elastic tip deflection of one ice tooth. However, in reality, the maximum displacement of one ice tooth and the distance between this ice tooth and the next one are usually random numbers. Therefore, there is possibility that the former number is larger than the later one and two ice teeth bend at once, as shown in Figure 9.

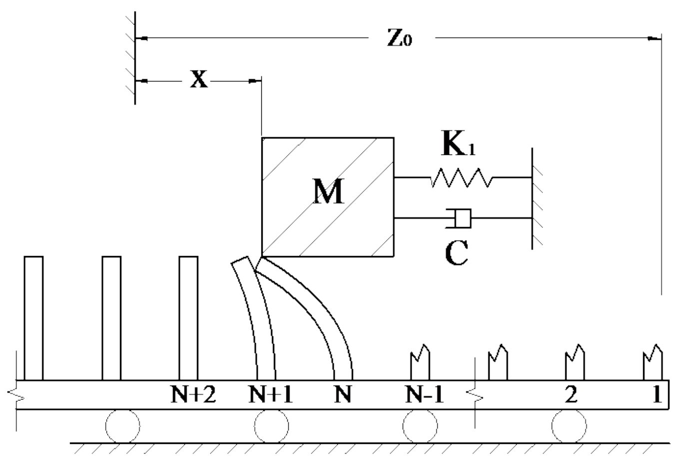

Figure 7 Ice-structure interaction with two ice teeth bending

If we ignore the thickness of the ice tooth, when 2 ice teeth deflect together, the ice force becomes

$$P = K_{ice}\Delta + K_{ice}(\Delta - P)$$

Therefore, the ice force can be expressed as

$$y = \begin{cases} 0 & \Delta \le 0, \Delta = \Delta_{max} \\ K_{ice}\Delta & 0 < \Delta \le Pch \\ K_{ice}\Delta + K_{ice}(\Delta - Pch) & Pch \le \Delta < \Delta_{max} \end{cases}$$

For this model, the user is also expected to input the ice sheet thickness, structure diameter, ice brittle strength, distance between ice teeth and the maximum ice tooth deflection.

The input parameters for ice model 1 sub-model 1 and 2 are listed in Table 5:

Table 5 Input parameters for ice model 2 sub-model 1 and 2

Variable name	Symbol	Explanation
IceStr2	σ_g	This is the average ice brittle strength. It has the units of MPa.
Delmax	Δ_{max}	This is the maximum ice tooth tip displacement. It has the units of m.
Pitch	P	This is the distance between sequential ice teeth. It has the units of m.

Ice model 2: Examples

Ice model 2.1

User inputs are ice thickness $\boldsymbol{h}$ = 0.5m, ice velocity $\boldsymbol{v_{ice}}$ = 0.2m/s, structure diameter of $\boldsymbol{D}$= 4.0m at the water surface and the indentation factors $\boldsymbol{Ikm}$ = 2.7. The ice brittle strength is σ = 5MPa, distance between ice teeth is $\boldsymbol{P}$ = 1.0m, and the maximum elastic deflection is Δ_{max} = 0.5m. The simulated ice force time history is shown below.

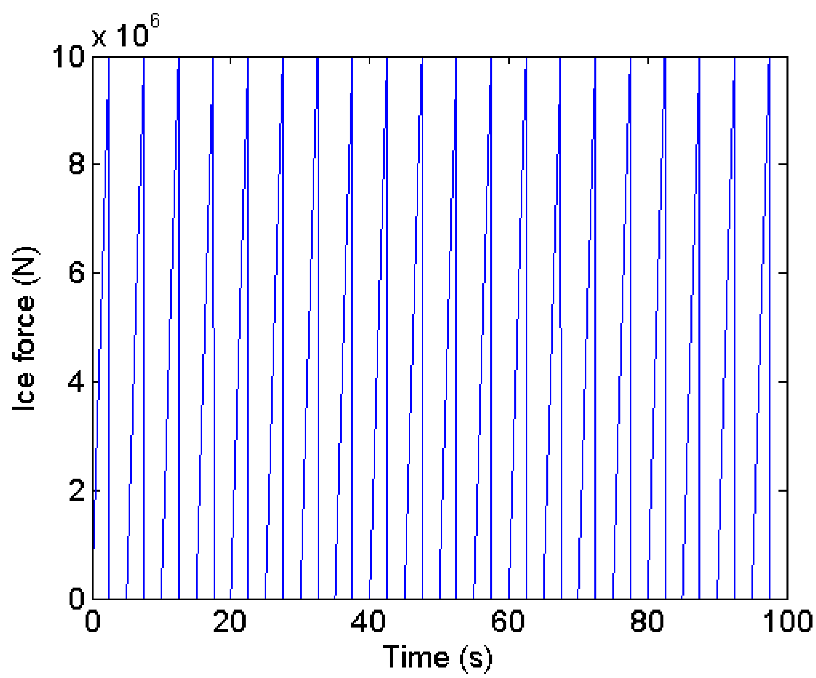

Figure 8 Ice load time history for ice model 2 sub-model 1

Ice model 2.2

User input ice thickness h = 0.5m, ice velocity v_{ice} = 0.2m/s, structure diameter of D= 4.0m at the water surface and the indentation factors Ikm = 2.7. The ice brittle strength is σ = 5MPa, distance between ice teeth is P = 1.0m, and the maximum elastic deflection is Λ_{max} = 1.5m. The simulated tower base moment is shown below.

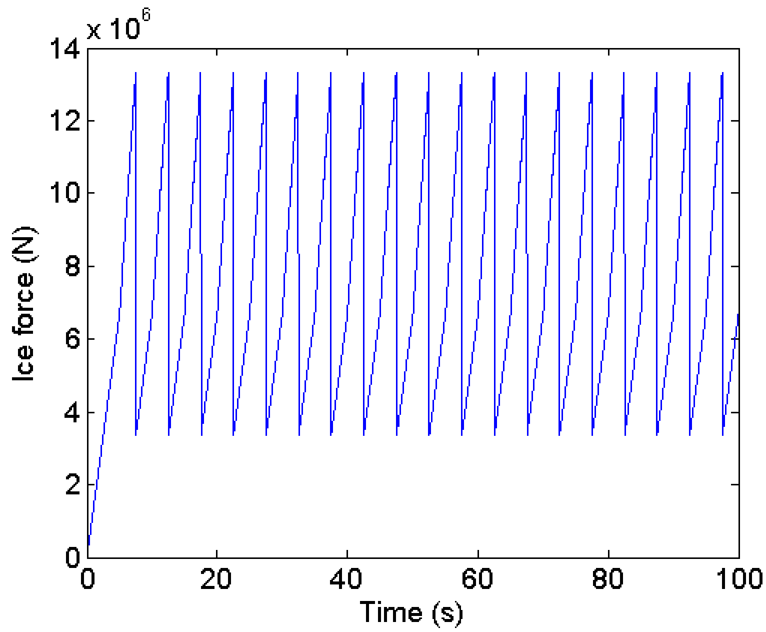

Figure 9 Ice load time history for ice model 2 sub-model 2

2.3 Ice Model 3 – random ice loading

According to ISO standard 19906:2010 (BSI 2011), ice-loading events can be estimated using a deterministic method or a probabilistic method. According to the previous discussions, the ice force can be influenced by many factors, such as ice thickness, ice drifting speed, ice crushing stress, floe sizes, ice temperature, ice-structure interaction width and others. Most of these parameters usually vary randomly. Therefore, to better simulate ice-loading events in reality, a random ice-loading model is needed.

Since there are many factors that influence the ice load, it may become too complicated if we consider random distribution for all of them. As suggested by ISO standard, we only consider the joint probability distribution of the most important parameters, such as ice thickness, ice sheet drifting speed and ice strength.

When generating ice loading events during a period of time, three issues need to be considered: 1) maximum ice load during one loading event; 2) total ice loading time during one event; 3) time between two loading events. In order to simplify and clarify the problem, these following assumptions are made: only a single event is allowed to occur for incremental time intervals; the probability of an event to occur is independent of the probability of any other event; also the probability of an event to occur within a time interval with a given duration must be identical throughout the whole time series.

2.3.1. Maximum ice load within one loading cycle

Ice model 3, sub-model 1 - creep

When ice velocity is low, according to the previous analysis in ice model 1.1 creep indentation, the maximum ice force depends on the ice velocity v_{ice} and thickness h. In this case, we consider these two parameters as random variables and all the other parameters such as D, R, T_{ice} determined as constants. In the current research, v_{ice} and h are assumed independent random variables.

Some previous researchers have studied random ice properties. Both Leira (Leira et al., 2009) and Liu (Liu et al., 2009) used lognormal distribution to model ice thickness data:

$$P_H(h) = \frac{1}{\sigma h\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln h - \mu}{\sigma}\right)^2\right]$$

with a mean value of $\exp\left(\mu+\sigma^2/2\right)$ and a variance of $\left[\exp\left(\sigma^2\right)-1\right]\exp\left(2\mu+\sigma^2\right)$.

Liu (Liu et al., 2009) applied Rayleigh distribution to describe ice velocity:

$$P_V(v) = \frac{v}{\sigma^2}\exp\left(-\frac{v^2}{2\sigma^2}\right)$$

with a mean value of $\sigma\sqrt{\pi/2}$ and a variance $\sigma^2\left(4-\pi\right)/2$.

In this model, for low ice speed case, the user will input the mean value and variance for ice thickness and ice velocity. We apply lognormal distribution for ice thickness and Rayleigh distribution for ice velocity. For each ice loading event, from these random distributions, independently generate a random ice thickness and ice velocity. Then based on ice model 1a, calculate maximum ice force for the current ice-loading event.

Ice model 3, sub-model 2 - crushing

When a level ice sheet moves quickly passing a vertical structure, ice force and structural response both fluctuate randomly. In this condition, ice sheet is believed to fail in a brittle manner and crushed ice pieces flake, causing variation of the ice force. The failure mode is known as continuous crushing (Yue et al., 2009).

Generally speaking, when the ice velocity is high, the frequency of ice loading will become much larger than the natural frequency of the structure. Therefore, the displacement of the structure will not be large enough to influence the ice loading. In this case, the ice load can also be prescribed.

When ice fails in crushing, there is no direct relationship between ice crushing strength and ice velocity. For this reason, we treat ice strength, ice thickness and ice velocity as the most important random variables.

There have been many researchers that studied the random ice strength. In Christensens paper ice properties in the Great Belt in Denmark were analyzed (Christensen and Skourup, 1991). The ice load from ice crushing was modeled as

$$F_{\max} = \alpha w h \sigma \tag{6}$$

where α is a constant related to aspect ratio w/h, w is the width (diameter) of the structure. h is the thickness of the ice sheet and σ is the uniaxial compressive strength of ice.

Among the above factors for calculating $F_{\max}$, α and w was set as deterministic in order to simplify the problem. Therefore, the random variables were h and σ. Cristinsen argued that since σ and h were both related to the temperature, they are not fully independent. To circumvent this problem, the product of σh was split into temperature-dependent and temperature independent parts:

$$\sigma h = \sigma_0 x$$

where σ_0 is a constant reference strength independent of temperature and x is the product of all the temperature-dependent parts of the product σh. The distributions of σ_0 and x can be combined under an assumption of no correlation.

The difficulty of applying Christensen's method is that when calculating the ice loading time within each loading event, the value of ice strength is needed, but Christensen's method does not estimate ice strength directly. Here we use a statistical model of $F_{\max}$ and generate a statistical model of σ from that of $F_{\max}$:

$$\sigma = F_{\max} / \alpha D h$$

Since σ and h are not independent, and their correlation has not been quantified yet, it is difficult to generate the statistical model of σ from the statistical data of $F_{\max}$ and h. For this reason, also since ice-loading time is important for deciding structural response, we may decide to assume ice strength and thickness as independent random variables.

A Weibull distribution was fitted to the joint set of data on reference strengths of Great Belt in Denmark.

$$F_W(\sigma_0) = 1 - \exp\left[-\left(\frac{\sigma_0}{\beta}\right)^k\right]$$

In Suyuthi's research (Suyuthi 2012), the maximum ice load within one loading event was also assumed to have Weibull distribution. The c.d.f of ice load y

$$F_Y(y) = 1 - \exp\left\{-\left(\frac{y}{\theta}\right)^k\right\}$$

The shape parameter of the Weibull's distribution was estimated to be $k = 0.99$, while the scale parameter was estimated to be $\theta = 21.03$.

According Jordaan (Jordaan 1993), the distribution of ice-induced pressure on ship hull can be represented by an exponential distribution:

$$F_X(x) = 1 - \exp\left[-(x - x_0)/\alpha\right]$$

where x_0 and α are constants related to contact area.

For longer time duration, the distribution might tend to the double-exponential (Gumbel) form:

$$F_X = \exp\left\{-\exp\left[-\left(x - x_0\right)/\alpha\right]\right\}$$

In Kamio's research (Kamio 2003), the experimental data of fracture strength of notched ice specimens conformed to the simplest form of two-parameter Weibull distribution.

$$F_\sigma(\sigma) = 1 - \exp\left[-\left(\sigma/\sigma_0\right)^\delta\right]$$

where scale parameter $\sigma_0 = 0.356MPa$ and the shape parameter $\delta = 3.111$.

For high ice speed cases, the user will input the mean value and variance for ice thickness, velocity and ice strength. We may then apply a Weibull distribution for ice strength, lognormal distribution for ice thickness and Rayleigh distribution for ice velocity. From these random distributions, for each ice loading event we may independently generate a random ice thickness, velocity and strength. Then based on equation (6), calculate the maximum ice force for events.

2.3.2. Loading time

According to Sodhi (Sodhi 1998), elastic deformation is dominant in continuous crushing. Therefore, we can assume that for both low and high ice velocity, before the ice stress reaches the creep/crushing strength, ice is under elastic strain. The elastic strain when ice begins to fail can be calculated as:

$$\varepsilon_e = \frac{\alpha\sigma}{E}$$

where E is the Young's Modulus of ice.

We assume constant strain rate. Then the time when the ice begins to crush is

$$T_{rise} = \varepsilon_e / \dot{\varepsilon}$$

$$T_{rise} = \frac{4D\varepsilon_e}{v_{ice}}$$

In the failure mode the continuous crushing, the crushing strength can be assumed independent of ice velocity.

2.3.3. Time between two loading events

According to Suyuthi (Suyuthi 2012), only a single loading event is allowed to occur for incremental time intervals. The probability of an event to occur is independent of the probability of any other event. Under these assumptions, the duration t between sequence events should follow an exponential distribution with p.d.f and c.d.f as follows:

$$f_T(t) = \lambda e^{-\lambda_t}$$

$$F_T(t) = 1 - e^{-\lambda_t t}$$

where $1/\lambda_t$ is defined as the expected time between subsequent event.

Ice Model 3 sub-model 3 – random dynamic ice loading

As stated in the previous sub-model, the properties of each ice tooth are not uniform. They are usually random numbers and differ between ice teeth. Since we calculate elastic stiffness of ice tooth from ice brittle strength as in model 2.1, we apply the Weibull distribution to describe ice brittle strength, as suggested in ice model 3 and calculate K_{ice} based on equation (5). We apply normal distribution to describe distance between ice teeth and the maximum ice tooth deflection. The ice force exerted on the structure when it is in contact with the *Nth* ice tooth can be expressed as:

$$\begin{cases} 0 & \Delta \le 0 \\ K_N \Delta & 0 < \Delta < \Delta_{\max,N} \\ K_N \Delta + K_{N+1}(\Delta - P_N) & P_N \le \Delta < \Delta_{\max,N} \end{cases}$$

where K_N is the stiffness of the *Nth* ice tooth, K_{N+1} is the stiffness of the *(N+1)th* ice tooth. P_N is the distance between the Nth and the *(N+1)th* ice teeth. $\Delta_{\max,N}$ is the maximum tip deflection of the *Nth* ice tooth.

For this model, the user is expected to input the mean value and the variance of the ice brittle strength, distance between ice teeth and the maximum ice tooth deflection and determined values of ice sheet thickness and structure diameter.

The input parameters common for ice model 3 sub-models 1 and 2 are listed in Table 6. The input common for ice model 3 sub-model 2 and 3 are listed in Table 7.

Table 6 Input parameters for ice model 3 sub-model 1 and 2

Variable name	Symbol	Explanation
ThkMean	μ_h	This is the mean value of ice thickness. It has the unit of m.
ThkVar	v_h	This is the variance of ice thickness. It has the unit of m^2.
VelMean	μ_v	This is the mean value of ice velocity. It has the unit of m/s.
VelVar	v_v	This is the variance of ice velocity. It has the unit of m^2/s^2.
TeMean	μ_T	This is the mean value of ice loading event duration time. It has the unit of s.

Table 7 Input parameters for ice model 3 sub-model 2,3

Variable name	Symbol	Explanation
StrMean	μ_σ	This is the mean value of ice strength. It has the unit of MPa.
StrVar	v_σ	This is the variance of ice strength. It has the unit of MPa^2.

The input parameters for ice model 3 sub-model 3 are listed in Table 8.

Table 8 Input parameters for ice model 2 sub-model 3

Variable name	Symbol	Explanation
DelMean	μ_Δ	This is the mean value of the random maximum ice tooth tip displacement. It has the units of m.
DelVar	v_Δ	This is the variance of the random maximum ice tooth tip displacement. It has the units of m^2.
PMean	μ_P	This is the mean value of is the random distance between sequential ice teeth. It has the units of m.
PVar	v_P	This is variance of the random distance between sequential ice teeth. It has the units of m.

Ice model 3 examples

Ice model 3.1 Creep

Assume the ice sheet thickness has a mean value of 0.5m and a variance of 0.25 m^2. Ice velocity has a mean value of 0.001 m/s and a variance $1*10^{-6}$ m^2/s^2. Ice loading event duration has a mean value of 50 s. The wind turbine has a diameter 4.0m at the water surface. The temperature of the ice is −4°C.

The FAST simulation result is shown in Figure 12.

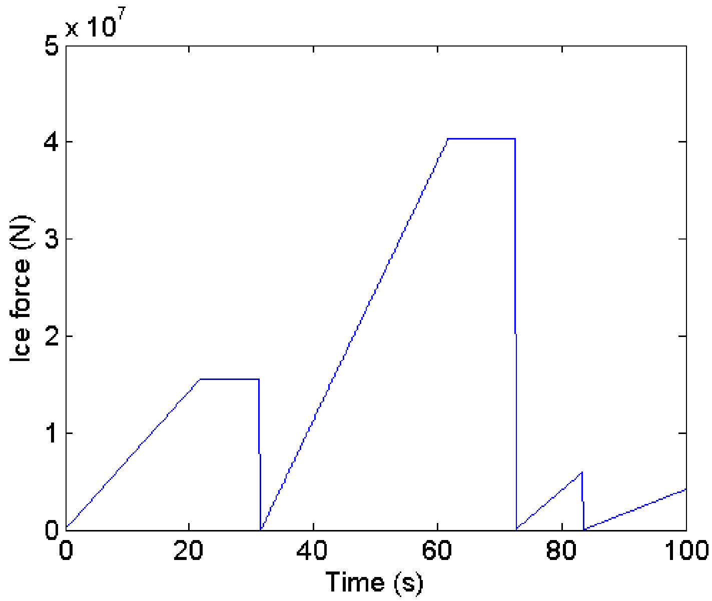

Figure 10 Ice load time history for ice model 3 sub-model 1

Ice model 3.2 Crushing

Assume the ice sheet thickness has a mean value of 0.5m and a variance of 0.0025 m^2. Ice velocity has a mean value of 0.1 m/s and a variance 0.01 m^2/s^2. Ice loading event duration has a mean value of 1 s. The ice strength has a mean value of 5MPa and a variance of 1 MPa^2. The wind turbine has a diameter 2.7m at the water surface. The temperature of the ice is −4°C.

The FAST simulation result is shown in Figure 11.

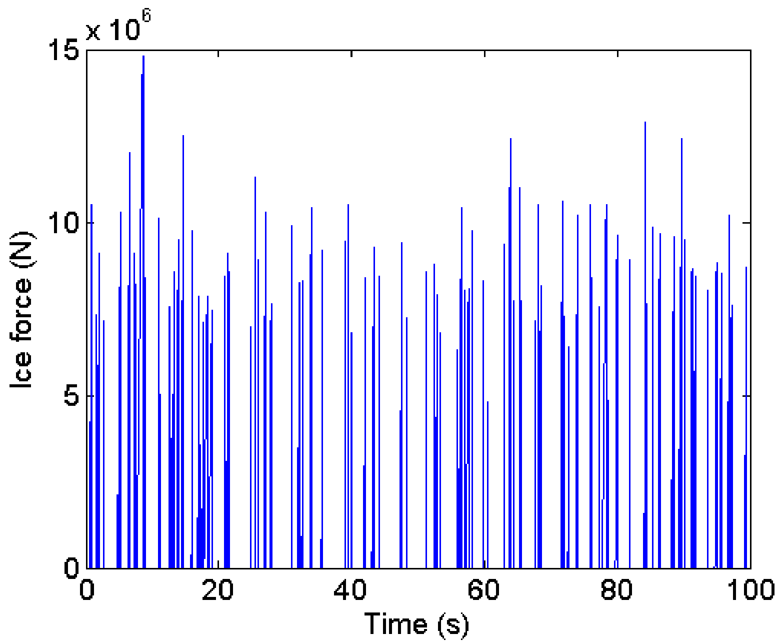

Figure 11 Ice load time history for ice model 3 sub-model 2

Ice model 2.3

User input ice thickness h = 0.5m, ice velocity v_{ice} = 0.2 m/s, structure diameter of D= 4m at the water surface. The average ice brittle strength has a mean value of σ = 5MPa, distance between ice teeth has a mean value of P = 0.1 m, and a standard deviation of 10% of its mean value. The maximum elastic deflection has a mean value of Δ_{max} = 0.2m and a standard deviation of 10% of its mean value. The ice force time history is shown in Figure 12.

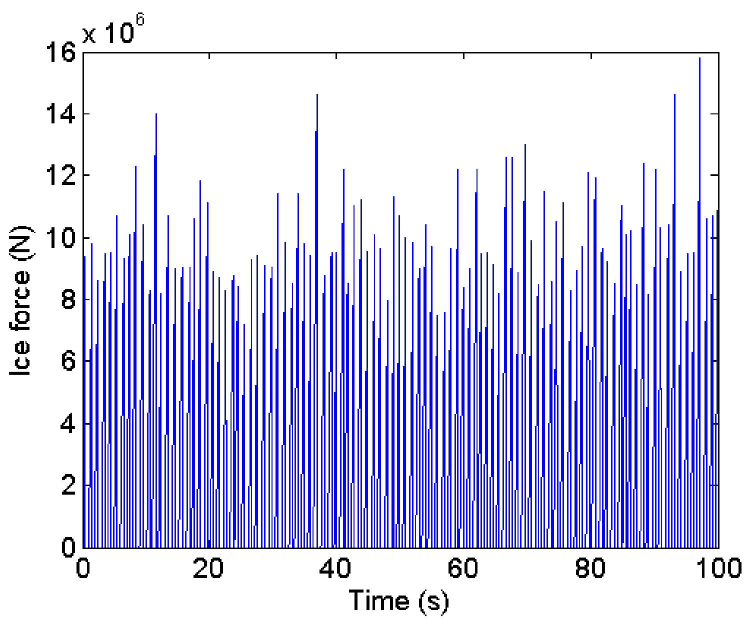

Figure 12 Ice load time history for ice model 3 sub-model 3

2.4 Ice Model 4 – non-simultaneous ice failure

All of the previous models assume perfect contact between the ice sheet and the structure. However, as shown in Figure 15, the actual contact zones are small areas that may have much higher pressure than the global average contact stress. Here we assume that there are the ice has $n_d$contact zones of width $d_r$interacting with a structure of width D.

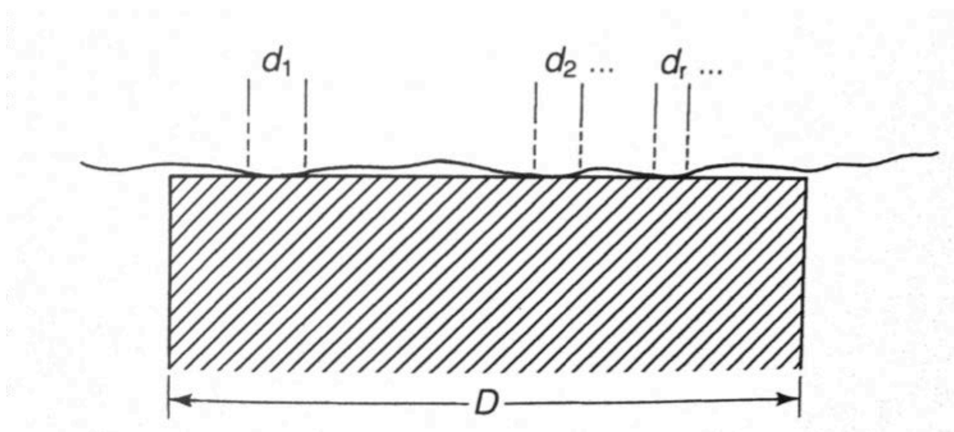

Figure 13 Imperfect contact between ice and structure (Sanderson 1988)

Many pervious researches have addressed this problem in a statistical manner (Kry 1980, Slomski and Vivatrat 1983). These analyses used the hypothesis that large-scale failure occurs by successive fracture of independent zones. The treatment was calculating the statistical sum of individual stress time series for a large number of zones. The calculation lead to a conclusion that the peak stress over a large multi-zone area should be lower than over the area of a single zone (Sanderson 1988). This probabilistic approach can be simulated using ice model 3 in this module by providing a reduced mean value for ice strength.

In our ice model 4, we also apply a model presented by Ashby et al, (1986). In this model, we first consider an irregular ice block of thickness h in contact with an indenter of width D. We assume the contact area can be simplified as a set of cubical independent cells of dimension $L_i \times L_i \times L_i$, as shown in Figure 16.

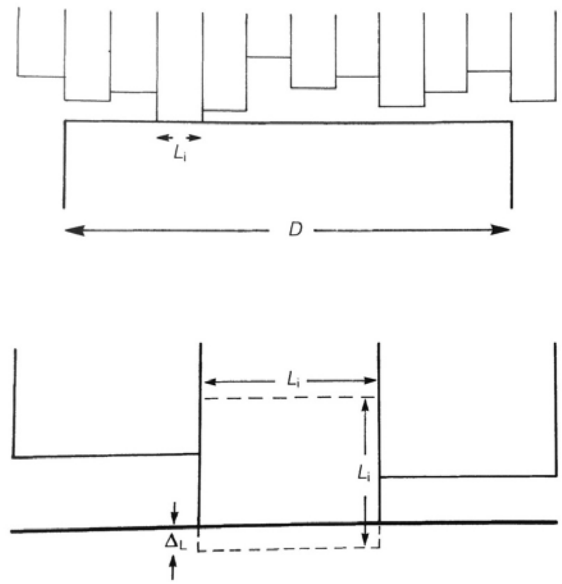

Figure 14 Independent failure zones

As each cell comes into contact with the indenter, the cubical cell will be first under elastic deformation. When the deformation reaches a critical value Δ_L, the cell fails. At one time instance, only a subset of all the independent zones is in contact with the indenter. This results a reduction of average ice pressure.

In Ashby's work, this model was presented in order to calculate an average ice force for design purpose. In our ice module, we need to provide a time history of ice load. Therefore, we developed a time-dependent ice model based on the independent failure zone theory. In our ice model 4, first a random ice-structure contact face profile is generated. Based on the user-input independent failure zone number along contact width N_1 and height N_2, the single zone size becomes:

$$L_1 = w / N_1$$

$$L_2 = h / N_2$$

The distance between sequential ice teeth L_3 is also according to user input. In order to satisfy the independent failure zone theory, the value of L_1, L_2 and L_3 should be approximate.

If we assume the mean position of the contact face is zero, the face position of each zone y_i is assumed to have normal distribution, with the user-input standard deviation.

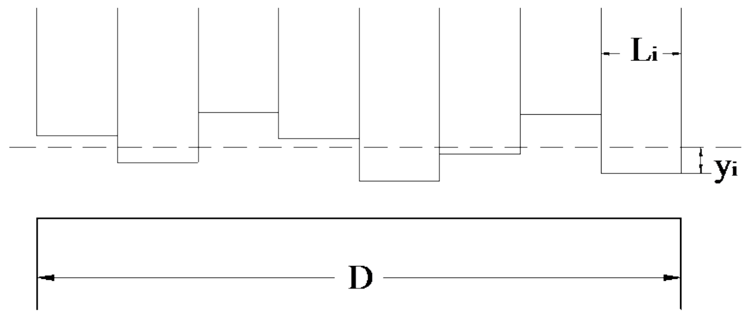

Figure 15 Random contact face profile

When contact face profile is generated, we put the most upfront zone just in contact with the structure at the beginning of the simulation. Therefore, the initial position of each failure zone becomes

$$Y_i = y_i - y_{\max}$$

Then as the indentation begins, when the deformation of one failure zone exceeded its limit ΔL, an ice cubic cell of size L_i^3 fails and gets out of the way. This failure zone will not be contacting the structure until it proceeds a distance of L_i. In this way, we apply the mechanical model proposed by Matlock et al. (1971) and Karr et al. (1995) and treat each failure zone as a series of ice teeth, with L_i as the distance between successive teeth.

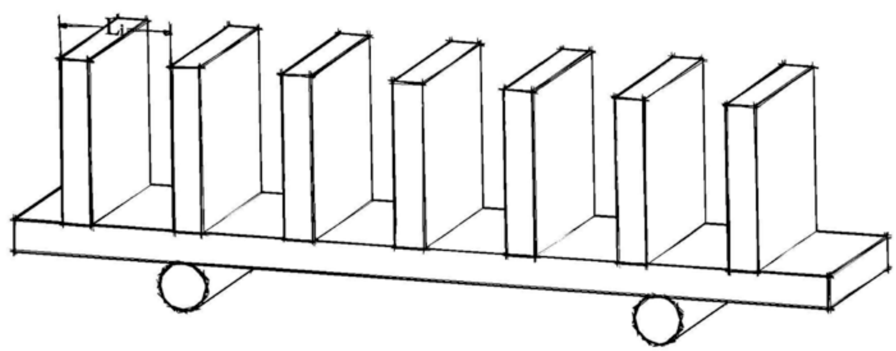

Figure 16 Ice teeth model of independent failure zones

For the i_{th} failure zone, similar as in ice model 2, we assign a number n_i as the current ice tooth number. We can calculate the stiffness of the ice tooth as

$$K_i = L_1 L_2 \sigma / \Delta_L$$

where σ is the ice uniaxial compressive strength. The deformation of the current ice tooth is

$$\Delta = y_i - y_{\max} + v_{ice} t - L_i (n_i - 1)$$

Then the quasi-static ice force exerted on the i_{th} failure zone is

$$F_i = \begin{cases} K_i\left[y_i - y_{\max} + v_{ice}t - L_i(n_i - 1)\right] & 0 < \Delta \le \Delta_L \\ 0 & \Delta \le 0 \end{cases}$$

When Δ reaches its limit, the current ice cell fails and $n_i = n_i + 1$.

Then the total ice force at time t is the sum of all the local ice force

$$F = \sum_{i=1}^{N_f} F_i$$

The input parameters for ice model 4 are listed in Table 9.

Table 9 Input parameters for ice model 4

Variable name	Symbol	Explanation
PflMean	μ_y	This is the mean value of contact face position. It has the unit of m.
PflSig	s_y	This is the standard deviation of ice contact face position. It has the unit of m.
ZoneNo1	L_1	This is the number of failure zones along contact width. It has the unit of m.
ZoneNo2	L_2	This is the number of failure zones along contact height/thickness. It has the unit of m.
ZonePitch	L_3	This is the distance between sequential ice teeth. It has the unit of m.
IceStr	σ	This is the ice failure stress within each failure zone. It has the unit of MPa.
Delmax	Δ_{max}	This is the ice teeth maximum elastic deformation. It has the unit of m.

Ice model 4 example

The user input: ice velocity 0.01m/s, ice block thickness 0.99m, structure width 2.7m. The standard deviation of ice contact face position is 0.02m. The independent failure zone size is 0.27m. The ice failure stress is 5MPa. The critical elastic deformation:

$$\Delta_L = 0.1L_i = 0.27m$$

The ice contact face profile change over time is illustrated in the following Figure 17. The resulting time history of total ice force is shown in Figures and 18.

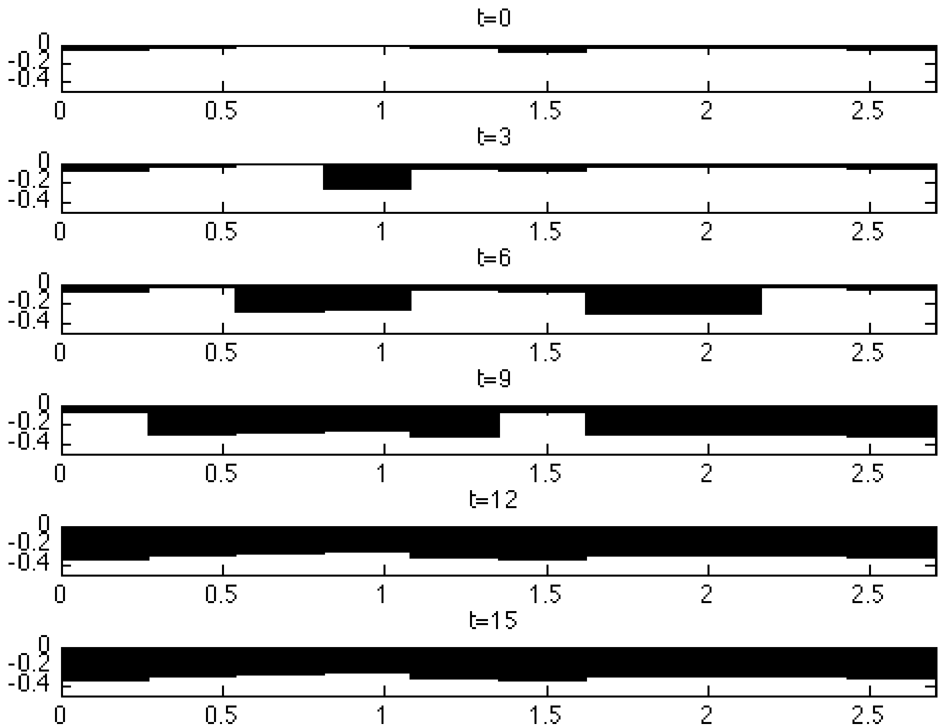

Figure 17 Ice-structure contact profile progressing over time

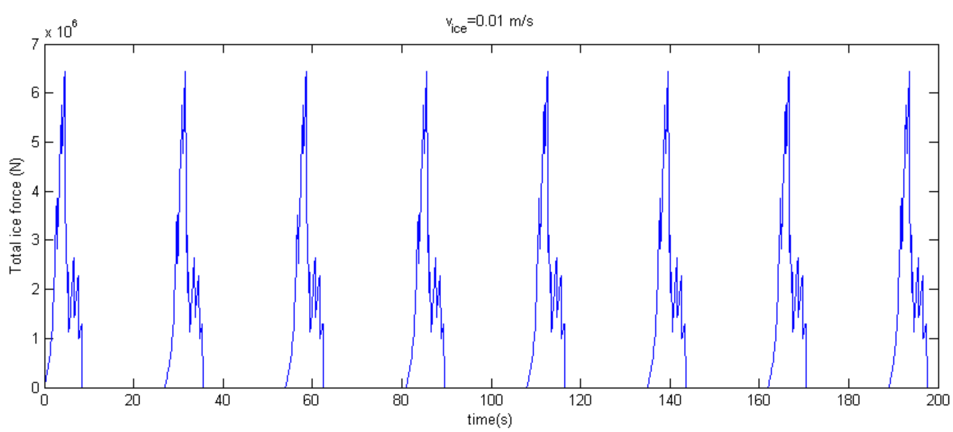

Figure 18 Time history of ice force

2.5 Ice Model 5 – ice loading on sloping structure

All the previous ice models studied ice interaction with vertical structures. For sloping structures, ice may fail in bending rather than simply crushing (Bruun, 2006) (Duan et al., 2002), as shown below. Since ice tensile fracture strength is much smaller than compressive fracture strength, ice fails in bending may result a reduced stress on the structure.

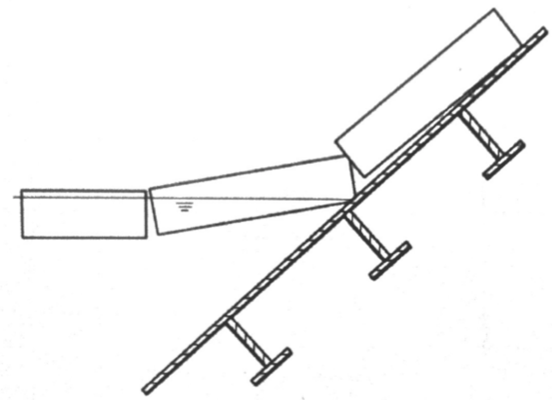

Figure 19 Ice bending on sloping structure (Sanderson 1988)

Depending on different theories of calculating ice breaking force, we have two sub-models here. In model 5a we apply Ralston's method by applying plastic limit analysis to calculate ice force on conical structures (Ralston 1980). In model 5b, we apply a model proposed by Yu et al. (Yu 2014)

2.5.1 Magnitude of ice force at breakage and the breaking length

2.5.1.1 Ice model 5 sub-model 1

The Ralston's method calculates ice load by considering circumferential and side cracks formation, elastic foundation reaction, ice deformation and ice bubble ride up on the conical structure. The horizontal ice force can be calculated as

$$R_H = \left[A_1 \sigma_f h^2 + A_2 \rho_i g h D^2 + A_3 \rho_i g h_R \left(D^2 - D_T^2 \right) \right] A_4$$

where:

$$A_1 = \frac{1}{3}\left[\frac{\lambda}{\lambda - 1} + \frac{1 - \lambda + \lambda \ln \lambda}{\lambda - 1} + 2.422 \frac{\lambda \ln \lambda}{\lambda - 1} \right]$$

$$A_2 = \left(\lambda^2 + \lambda - 2 \right) / 12$$

$$A_3 = \frac{1}{4}\left[\frac{1}{\cos\alpha} + \frac{\mu E(\sin\alpha)}{\sin\alpha} - \mu \frac{f(\alpha,\mu) g(\alpha,\mu)}{\tan\alpha} \right]$$

$$A_4 = \frac{\tan\alpha}{1 - \mu g(\alpha, \mu)}$$

And

σ_f is the flexural strength of ice;

h is the ice thickness;

D is the structure waterline width/diameter;

ρ_i is the weight density of ice;

h_R is the thickness of ride-up ice;

D_T is the top diameter of the cone;

μ is the friction coefficient between ice and the cone;

α is the uprising angle of the cone;

$\lambda = A/R$, A is the circumferential crack diameter and $R = D/2$ is the cone waterline radius.

$$g(\alpha, \mu) = \left(\frac{1}{2} + \frac{\alpha}{\sin\alpha}\right) \Big/ \left(\frac{\pi}{4}\sin\alpha + \frac{\mu\alpha\cos\alpha}{\sin\alpha}\right)$$

$$f(\alpha, \mu) = \sin\alpha + \mu\cos\alpha\,\mathrm{F}(\sin\alpha)$$

$$\mathrm{F}(\sin\alpha) = \int_0^{\pi/2} \frac{1}{\sqrt{1 - \sin^2\alpha\sin^2\theta}} d\theta \approx \frac{\pi}{2} + \frac{\pi}{8}\frac{\sin^2\alpha}{1 - \sin^2\alpha} - \frac{\pi}{16}\frac{\sin^4\alpha}{1 - \sin^4\alpha}$$

$$\mathrm{E}(\sin\alpha) = \int_0^{\pi/2} \sqrt{1 - \sin^2\alpha\sin^2\theta}\, d\theta = \frac{\pi}{2}\sum_{n=0}^{\infty}\left[\frac{(2n)!}{2^{2n}(n!)^2}\right]^2 \frac{\sin\alpha^{2n}}{1 - 2n}$$

The vertical ice force becomes:

$$R_V = B_1 R_H + B_2 \rho_i g h_R \left(D^2 - D_T^2\right)$$

where

$$B_1 = \frac{h(\alpha, \mu)}{\frac{\pi}{4}\sin\alpha + \frac{\mu\alpha}{\tan\alpha}}$$

$$B_2 = \frac{1}{4}\left[\frac{\pi}{2}\cos\alpha - \mu\alpha - \frac{f(\alpha,\mu)h(\alpha,\mu)}{\frac{\pi}{4}\sin\alpha + \frac{\mu\alpha}{\tan\alpha}}\right]$$

$$h(\alpha,\mu) = \cos\alpha - \frac{\mu}{\sin\alpha}\left[\mathrm{E}(\sin\alpha) - \cos^2\alpha\,\mathrm{F}(\sin\alpha)\right]$$

The breaking length is defined as the distance between the zone of ice edge/structure contact and the first circumferential crack (Feng 2003). Breaking length is an important parameter that directly relates to the period of the ice force in dynamic analysis.

According to Ralston (Ralston 1980), the value of $\lambda = l_b / R$ is calculated using upper bound plastic limit analysis to minimize the horizontal ice force. It is the solution of the following equation:

$$\lambda - \ln\lambda + 0.0922\frac{\rho_i g h D^2}{\sigma_f h^2}(2\lambda+1)(\lambda-1)^2 = 1.369$$

In our module, we apply the Ralston model, while users have the freedom to input their own breaking length.

2.5.1.2 Ice model 5 sub-model 2

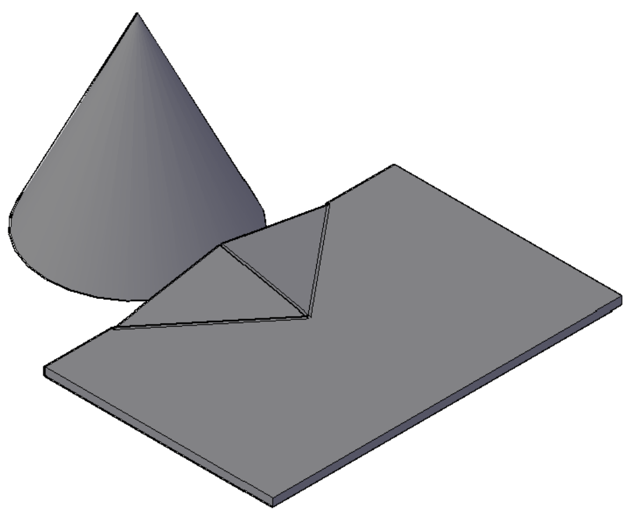

Fig 20 Approximation of ice deformation when contacting steeper cone

In this sub-model, the floating ice sheet is modeled as a rigid-plastic structure supported by elastic foundation. In this method, we first assume a plastic displacement field of the ice sheet. Then according to Augusti's analysis (Augusti 1970), by minimizing the difference between ice sheet stress fields generated from equilibrium with external forces and from plastic flow rule, a relation between ice-structure contact force and ice sheet displacement field can be established. Then by establishing a limit strain or strain rate criterion for the ice fracture failure, the ice breaking force and breaking length can be calculated.

For a steep cone, the loading boundary is a narrow line with length of the order of the ice thickness. The ice sheet deforms in bending in both lateral directions (Feng et al., 2003). To approximate this failure mode, we assume the contact between ice and the structure is at a single point. As shown in Fig. 4.5, the displacement field of ice sheet is approximated with three hinge lines.

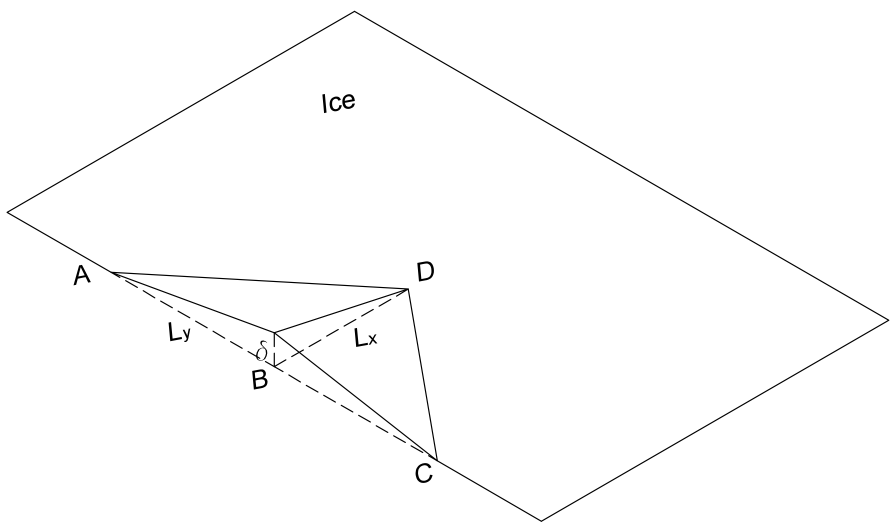

Fig 21. Geometry of the approximate ice displacement field

By applying Agusti's method (Augusti 1970, Yu et al. 2014), the ice force-displacement relationship is in the following form:

$$L_x = \sqrt{\frac{24M_u}{K\delta}}, L_y = \sqrt{\frac{48M_u}{K\delta}}$$

$$P = 8\sqrt{2}M_u$$

where L_x, L_y are lengths of the hinge lines as shown in Fig. δ is the lifted distance of the ice-structure contact edge as shown in Fig. K is the reaction force from the change of buoyancy per displacement per unit area. In this case, $K = \rho_w g$ and ρ_w is the mass density of water. M_u is the ultimate bending moment per unit length at the hinge line.

$$M_u = \frac{\sigma_f h^2}{4}$$

and σ_f is the flexural strength of ice and h is the ice thickness.

Since ice has the characteristic that it is a ductile material at low strain rate and brittle material at high strain rate, it is unrealistic if the plastic deformation or δ increase to a significant level if the strain rate is large. A breaking mechanism should be assumed. Here we use both limit strain and limit strain rate as ice breaking criteria.

We assume that once the limit strain rate $\dot{\varepsilon}_{\mathrm{lim}}$ is reached, the behavior of ice is in the brittle region and ice fails in fracture if its strain exceeds the limit strain $\varepsilon_{\mathrm{lim}}$.

The ice-structure contact force, hinge line lengths at limit strain rate is:

$$L_{x,\mathrm{lim},1} = \frac{3\sqrt{6}}{8}\frac{\dot{\delta}}{\dot{\varepsilon}_{\mathrm{lim}}}$$

$$L_{y,\mathrm{lim},1} = \frac{3\sqrt{3}}{4}\frac{\dot{\delta}}{\dot{\varepsilon}_{\mathrm{lim}}}$$

$$P_{\mathrm{lim},1} = 8\sqrt{2}M_u$$

The ice-structure contact force, hinge line lengths at limit strain is:

$$L_{x,\mathrm{lim},2} = \sqrt{6}\sqrt[3]{\frac{M_u}{K\varepsilon_{\mathrm{lim}}}}$$

$$L_{y,\mathrm{lim},2} = 2\sqrt{3}\sqrt[3]{\frac{M_u}{K\varepsilon_{\mathrm{lim}}}}$$

$$P_{\mathrm{lim},2} = 8\sqrt{2}M_u$$

The criterion that returns the smaller L_x should be applied. The breaking length is then calculated:

$$L_{br} = \frac{L_x L_y}{\sqrt{L_x^2 + L_y^2}}$$

2.5.2 Ice force time history

When one ice sheet drifts against one conical structure, we assume it fails in bending within a very short time after it contacts the structure. After it breaks, the broking ice piece is pushed by the rest of the ice sheet and rides up the cone. Here we ignore the shape of the cone and assume it as a flat sloping structure. Then we have a simplified 2D model, as shown in Figure 23. The calculation of the ice

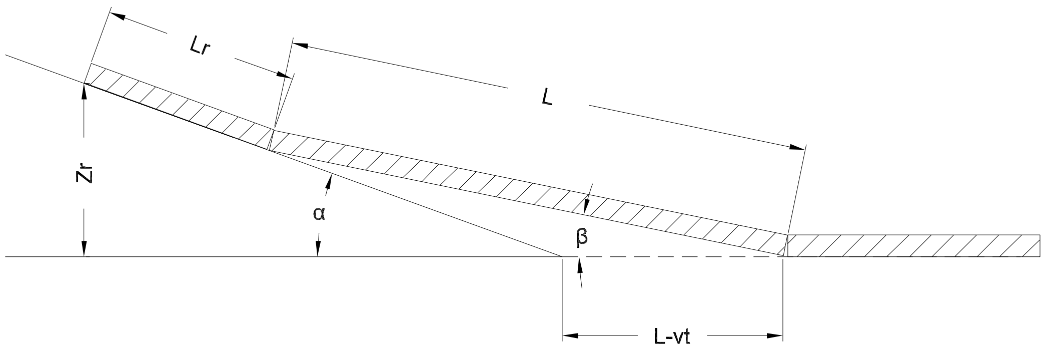

Figure 22 2D model of ice sheet riding up the cone

The ice that contacts the structure is comprised of two parts. One is the ice piece after flexural failure, with the length of the breaking length L. The other is the ride-up piece that comes from previous bending failure, with the length of L_r. We assume the ice can ride up to a height of Z_r. The ice above that height is cleared up due to some mechanism. According to the geometry, the angle $\beta(t)$ can be solved from the following equation:

$$L\sin\beta\cot\alpha + L - vt = L\cos\beta$$

where α is the cone uprising angle; v is the ice drifting speed and t is time.

Then from the total force equilibrium, the horizontal and vertical ice forces R_H and R_V can be calculated:

$$R_H = \left(P_{N1} + P_{N2}\right)\left(\sin\alpha + \mu\cos\alpha\right)$$
$$R_V = \left(P_{N1} + P_{N2}\right)\left(\cos\alpha - \mu\sin\alpha\right)$$

where

$$P_{N1} = W_r\cos\alpha = \rho_i g D h_r\left(Z_r - L\sin\beta\right)\cos\alpha / \sin\alpha$$

$$P_{N2} = \frac{\frac{1}{2}W_L L\cos\beta + W_r L\,\mathrm{g}\left(\alpha,\beta\right) - F_B X_B}{L\mathrm{f}\left(\alpha,\beta\right)}$$

And

$$f\left(\alpha,\beta\right) = \sin\beta\left(\sin\alpha + \mu\cos\alpha\right) + \cos\beta\left(\cos\alpha - \mu\sin\alpha\right)$$

$$g\left(\beta\right) = \left(\sin\alpha + \mu\cos\alpha\right)\sin\left(\alpha - \beta\right)$$

$$F_B = \begin{cases} \rho_w g D\left[L\left(\frac{\rho_i}{\rho_w}h - L\tan\beta\right) + \frac{1}{2}L^2\tan\beta\right] & \beta < \frac{\rho_i h}{\rho_w L} \\ \rho_w g D\left[\frac{1}{2\tan\beta}\left(\frac{\rho_i}{\rho_w}h\right)^2\right] & \beta \geq \frac{\rho_i h}{\rho_w L} \end{cases}$$

$$X_B = \begin{cases} \frac{L}{3}\left(\frac{3h\frac{\rho_i}{\rho_w} - L\tan\beta}{2h\frac{\rho_i}{\rho_w} - L\tan\beta}\right) & \beta < \frac{\rho_i h}{\rho_w L} \\ \frac{1}{3}\frac{\rho_i h}{\rho_w \sin\beta} & \beta \geq \frac{\rho_i h}{\rho_w L} \end{cases}$$

The input parameters for ice model 5 are listed in Table 10.

Table 10 Input parameters for ice model 5

Variable name	Symbol	Explanation
ConeAgl	α	This is the slope angle of the cone. It has the unit of degree.
ConeDwl	D	This is the cone waterline diameter. It has the unit of m.
ConeDtp	D_T	This is the cone top diameter. It has the unit of m.
RdupThk	h_r	This is the thickness of the ride-up ice. It hast the unit of m.
mu	μ	This is the friction coefficient between structure and ice.
FlexStr	σ_f	This is the flexural strength of ice. It has the unit of MPa.
StrLim	ϵ_{lim}	This is the limit strain for ice fracture failure.
StrRtLim	$\dot{\epsilon}_{lim}$	Limit strain rate for ice brittle behavior. It has the unit of s^-1.

Ice model 5 example

Ice model 5.1, Ralston's method.

Consider the following case: The cone has an uprising angle of $\alpha = 55°$. The diameters at waterline and at top are 8m and 1m respectively. The friction coefficient between ice and the cone is 0.3. The ice velocity is 0.2 m/s. The ice thickness and ride-up ice thickness are 0.3m. The weight density is 900 kg/m^3. The flexural strength is 700 KPa.

The ice force time history is shown in Figure 23.

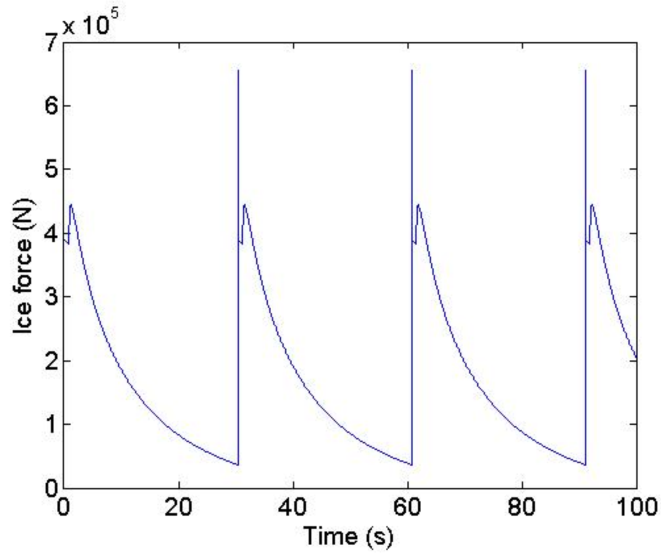

Figure 23 Ice force time history of ice model 5 sub-model 1

Ice model 5.2, new theoretical method.

Consider the following case: The cone has an uprising angle of $\alpha = 55^\circ$. The diameters at waterline and at top are 8m and 1m respectively. The friction coefficient between ice and the cone is 0.3. The ice velocity is 0.2 m/s. The ice thickness and ride-up ice thickness are 0.3m. The weight density is 900 kg/m^3. The flexural strength is 700 KPa. The limit strain for ice to fracture is 0.1 and the limit strain rate is 0.01 s^{-1}. The ice force time history is shown in Figure 24.

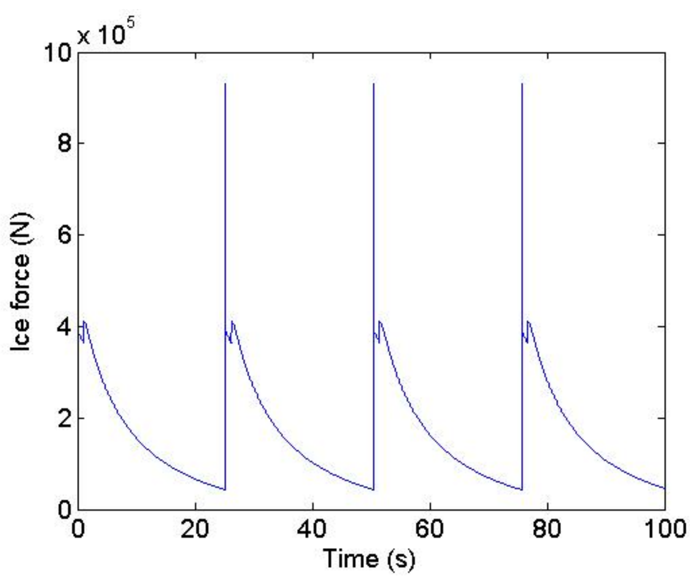

Figure 24 Ice force time history of ice model 5 sub-model 2

2.6 Ice Model 6 – isolated ice floe impact

Impact from an isolated ice floe is a dynamic phenomenon. As the floe collides head-on with the structure, local ice failure takes place at the ice structure interface, and the floe decelerates from its initial velocity. The penetration, area of contact and the ice load increase while the floe velocity decreases until one of the following happens (Bhat 1988):

(1) The ice floe stops in front of the structure before fully enveloping the structure.

(2) The ice floe stops in front of the structure after full envelopment or continues to move against the structure with reduced velocity causing continuous crushing ice loads on the structure.

(3) The ice floe splits into two or more large pieces that rotate around the structure. This may happen before or after full envelopment.

Case (1) results in the "limit momentum" load, case (2) in the "limit stress" load and case (3) in "splitting load." In our model 3, we have addressed the "limit stress" case. In the model 6, the other two types of loads are addressed

Assume a rectangular isolated ice floe, with length l, width w and constant thickness h. It impacts our wind turbine tower with an initial velocity v_0. The tower waterline section has a radius of R, as shown in Figure 26.

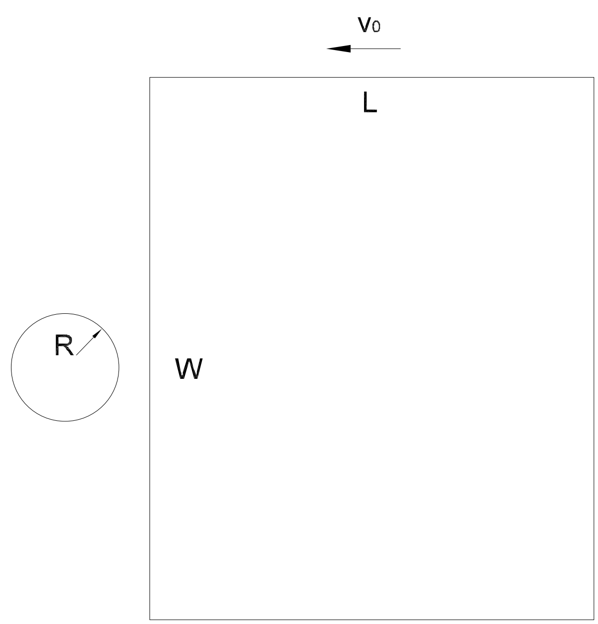

Figure 25 Isolated ice floe impacting wind turbine tower

When the ice flow collides with the structure, there will be local crushing at the ice-structure interface. As the structure penetration depth x increases, as shown in Figure 27, the ice structure contact area width b increases as well.

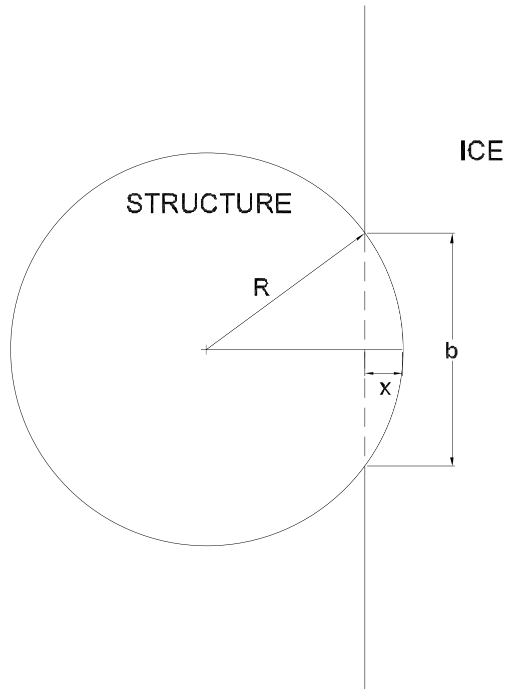

Figure 26 Ice structure interface

According to design rule ISO 19906 (BSI, 2011), the ice crushing strength is a function of contact area. The pressure-area relation has a general form:

$$P = CA^d$$

The ice-structure contact area depends on the structure penetration depth $x(t)$, which is the relative displacement of ice and structure:

$$x(t) = x_{ice}(t) - x_{str}(t)$$

Then the contact area can be calculated as

$$A(t) = b(t)h$$

$$b(t) = 2\sqrt{R^2 - [R - x(t)]^2}$$

Therefore, the time-dependent ice force can be calculated as

$$F_{ice} = \begin{cases} CA(t)^{d+1} & x_{ice} > x_{str} \\ 0 & x_{ice} \leq x_{str} \end{cases}$$

$$F_{ice} = \begin{cases} C\left\{2h\sqrt{R^2 - [R - (x_{ice} - x_{str})]^2}\right\}^{d+1} & x_{ice} > x_{str} \\ 0 & x_{ice} \leq x_{str} \end{cases}$$

The ice displacement x_{ice} is decided from the ice floe motion, which is governed by equations:

$$m_{ice}\ddot{x}_{ice} = \begin{cases} -C\left\{2h\sqrt{R^2 - [R - (x_{ice} - x_{str})]^2}\right\}^{d+1} + F_{dr} & x_{ice} > x_{str} \\ F_{dr} & x_{ice} \le x_{str} \end{cases}$$

$$x_{ice}(t = 0) = 0$$

$$\dot{x}_{ice}(t = 0) = v_0$$

where F_{dr} is the external driving force and m_{ice} is the mass of the ice floe. It can be calculated as

$$m_{ice} = \rho_{ice} lwh$$

Meanwhile, the structure displacement x_{str} is calculated by the FAST main program and read by our ice module as an input.

As the structure penetrates the ice, the contact force between ice and structure increases as the contact area increases. If the force reaches a value large enough to cause the splitting failure, the ice force will drop to zero after the ice splits and rotate around the structure.

Many previous researches have studied the limit force when the splitting happens. According to the study of Wierzbicki and Karr (Wierzbicki and Karr, 1987), macro crack initiation can occur at extremely low loads. Therefore, it can be assumed that short radial cracks will occur in any real ice structure interaction in the brittle regime (Bhat, 1988). Then the splitting failure happens when the propagations of those initial cracks become unstable. Here we applied the general form of splitting force *P* given in the research by Baht et al. (Bhat, 1988) (Bhat et al., 1991):

$$P = \tilde{P} h K_{IC} \sqrt{L}$$

where *h* is the ice thickness. *L* is the length a square floe or can be the radius of a circular floe. K_{IC} is the fracture toughness of ice. $\tilde{P}$ is the non-dimensional splitting load. It can different values based on different material assumptions and the conditions of indentation. To make a conservative prediction of ice load, we set a default value $\tilde{P}$ = 3.3, which is given by Bhat et al. (Bhat, 1988) generated from Finite Element Method and returns the largest value of splitting load. But the users have the freedom to input the value of $\tilde{P}$.

The input parameters for ice model 6 are listed in Table 10.

Table 11 Input parameters for ice model 6

Variable name	Symbol	Explanation
FloeLth	L	This is the ice floe length. It has the unit of m.
FloeWth	w	This is the ice floe width. It has the unit of m.

CPrAr	C	This is the constant in ice crushing strength pressure-area relation.
dPrAr	d	This is the order in ice crushing strength pressure-area relation.
Fdr	F_{dr}	This is the constant external driving force. It has the unit of MN.
FspN	$\tilde{P}$	This is the non-dimensional splitting load.
Kic	K_{IC}	This is the fracture toughness of ice. It has the unit of kNm^(-3/2).

Ice model 6 example 1
Consider the following case:

An ice square floe has a length of 1000 *m*. The ice floe has an initial velocity of 0.5 *m/s* and is under an external driving force of 11 *MN*. C = 5, d = -0.5. The structure has a diameter of 6 *m*. K_{IC} =140. $\tilde{P}$ = 3.3.

The ice force time history is shown in Figure 27.

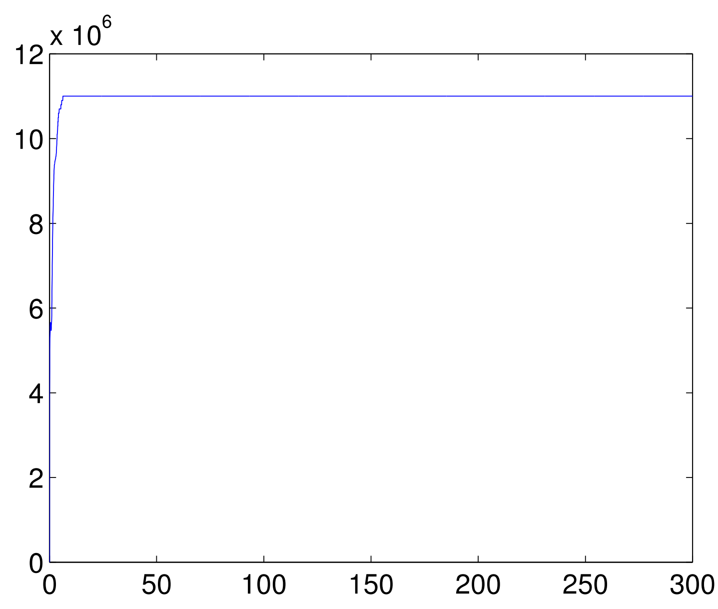

Figure 27. Ice force time history of ice model 6, example 1

Ice model 6 example 2 (Ice fails in splitting)
Consider the following case:

An ice square floe has a length of 800 *m*. The ice floe has an initial velocity of 0.1 *m/s* and is under an external driving force of 9 *MN*. C = 5, d = -0.5. The structure has a diameter of 6 *m*. K_{IC} =140. $\tilde{P}$ = 3.3.

The ice force time history is shown in Figure 28.

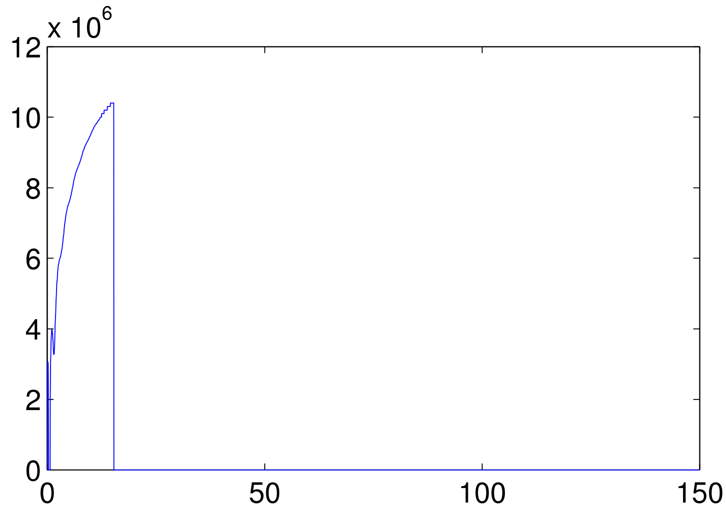

Figure 28. Ice force time history of ice model 6, example 2.

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