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Control Method for Improved Energy Capture below Rated Power

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CONTROL METHOD FOR IMPROVED ENERGY CAPTURE BELOW RATED POWER

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ABSTRACT

To maximize energy capture, a variable-speed wind turbine should operate continuously at the tip-speed-ratio that results in the maximum power coefficient (C_{po}) and, therefore, extracts the maximum energy from the wind. This is the main idea behind improved energy capture from variable-speed operation. However, this goal is only partially achievable due to rapid variations in wind speed and the inertia of the wind turbine rotor. Although it is not possible to operate continuously at maximum efficiency, improvements in energy capture during variable-speed operation can be gained by improved tracking of C_{po}.

In this paper the aerodynamic torque, estimated by an observer, and rotor speed are used to improve the energy capture of a variable-speed turbine. Two methods are used. The first method uses the torque error for control. The second method is formulated such that the estimated percent power loss is used directly for control. Also, the use of blade pitch below rated power is investigated. A small improvement in energy capture is realized by use of the described control methods. For turbines with a sharp C_p peak or slower time constant, greater improvement would be observed.

INTRODUCTION

To improve the energy capture capability of a variable-speed turbine it is desired that the turbine operate near the point for optimum efficiency. Improvements in energy capture during variable-speed operation can be gained by improved tracking of C_{po} (Conner and Leithead, 1993, Eklelund, 1997, Bongers and Dijkstra, 1988). Under varying wind conditions it is possible to use the generator torque to improve energy capture by forcing the rotor to operate nearer the maximum energy capture point. It may also be possible to improve efficiency by pitching the blades when operating off of the desired tip-speed-ratio. The improvement that can be realized using these methods depends on the turbine, the variability of the wind, and the control system used.

An ADAMS model of the Variable-Speed Test Bed turbine has been developed to investigate control methods for this machine. The variable-speed test bed turbine modeled is a three-bladed, downwind, free-yaw machine with a rotor diameter of 9 m. The short, twisted, tapered blade set was used in this study. This blade set has not yet been operated on the machine. In high wind the blades pitch collectively to regulate power. The turbine has a direct-drive generator with a very stiff drive train. Ten-minute data sets of simulated turbulent wind are used to evaluate the fatigue damage and energy capture improvement for controlled variable-speed operation. The control methods used in this study are implemented in discrete time, written in FORTRAN, and linked to ADAMS.

With a computer simulation we have unique control over otherwise uncontrollable parameters. It is possible to repeatedly use the same wind as input to a model while varying control system gains, control methods, or other turbine operating parameters. Changes in system response or loading can then be directly attributed to changes made in the control system or other system parameter changes. This provides for a relatively quick means of evaluation, with a direct comparison of before and after system change response. Evaluation of such effects on an operating turbine requires large amounts of data and statistical methods to determine effects.

NOMENCLATURE

C_p	Rotor Power Coefficient
I	Rotating System Inertia
p	Blade Pitch
R	Rotor Radius
T_{aero}	Aerodynamic Torque
T_e	Torque Error
T_{gen}	Generator Torque
V	Wind Speed
λ	Tip-Speed-Ratio
ρ	Air Density
ω	Rotor Speed

METHODS

From the basic equations governing a wind turbine the aerodynamic torque is given by

$$T_{aero} = \frac{1}{2}\rho C_P(\lambda)\pi R^5 \frac{1}{\lambda^3}\omega^2 \tag{1}$$

For steady-state operation at the tip-speed-ratio for optimum power coefficient, C_{po}, the equation then reduces to

$$T_{aero} = \frac{1}{2}\rho C_{Po}\pi R^5 \frac{1}{\lambda_o^{\,3}}\omega^2 = k\omega^2 \tag{2}$$

This is the torque-speed equation often used for open loop variable-speed control of a turbine. In constant wind the turbine will achieve the tip-speed-ratio for optimum operation. However, in varying wind the inertia of the rotor prevents continuous operation at C_{po}. If the wind speed increases the aerodynamic torque is in excess of the generator torque and the rotor speed increases so that the turbine asymptotically approaches λ_o. A similar process occurs for a decrease in wind speed. During these rotor speed changes the turbine is not operating at C_{po}. From the above equation it is apparent that for optimal operation, the aerodynamic torque, T_{aero}, should be equal to the optimal aerodynamic torque at the current rotor speed, $k\omega^2$. The error between these can be used to drive the system toward the optimal value. This method was used by Conner and Leithead (1993).

A similar method was used by Cardenas-Dobsen et al. (1996). They used this relationship to derive a reference rotor speed from the estimated aerodynamic torque as

$$\omega_{ref} = \sqrt{\frac{T_{aero}}{k}} \tag{3}$$

However, if the error between ω and ω_{ref} is used to control generator torque to move the rotor speed toward the optimal value, the control system gain changes as a function of the rotor speed. That is the difference is a function of the rotor speed, with higher rotor speeds resulting in a larger difference at a given error in λ. At lower rotor speeds (lower wind speeds) the control effort and the turbine time constant are both reduced, which may lead to poor tracking. At high rotor speeds (higher wind speeds) the control effort may become too large. The same condition exists when using the torque error, $k\omega^2 - T_{aero}$. At higher rotor speeds the error is greater for a given error in λ.

To allow for similar tracking over the range of rotor speeds the torque error may be normalized as

$$T_e = \left(1 - \frac{T_{aero}}{k\omega^2}\right) \tag{4}$$

This is effectively gain scheduling of the torque error, and provides the same tracking torque for a given error in λ at any rotor speed. The tracking torque is given by equation 4 multiplied by a desired gain. This method is referred to as the normalized-torque (NT) method.

The method just described and tip-speed-ratio control methods attempt to drive the turbine toward λ_o. However, what we would like to control is the power loss of the turbine. For a turbine with a relatively flat C_p curve, there may not be much power loss due to operation slightly off λ_o. Also, as the time constant of the turbine increases with increasing wind speed, the turbine may track satisfactorily in higher winds. Therefore, a method based on the power loss may reduce power loss with less control effort.

As stated above, the time constant of a variable-speed turbine varies with wind speed. The time constant may be evaluated from the basic dynamic equations. Ignoring drive train dynamics and system losses we obtain

$$I\dot{\omega} = T_{aero} - T_{gen} \tag{5}$$

If the generator is controlled by the standard $k\omega^2$ law this produces

$$I\frac{d}{dt}\left(\frac{\lambda V}{R}\right) = \frac{1}{2}\rho\pi R^3 V^2 \lambda^2 \left(\frac{C_P(\lambda)}{\lambda^3} - \frac{C_{Po}}{\lambda_o^{\,3}}\right) \tag{6}$$

If we assume that V is a constant but the turbine is operating initially at an error in λ, we obtain

$$\dot{\lambda} = \frac{1}{2}\rho\pi\frac{R^4}{I}V\left(\frac{C_P(\lambda)}{\lambda} - \frac{C_{Po}}{\lambda_o^{\,3}}\lambda^2\right) \tag{7}$$

This may be expressed in terms of torque coefficients as well. The term in brackets is fairly linear, and may be approximated by a first order Taylor series about the point of operation. With this approximation and assuming a small initial error we obtain

$$\lambda(t) = \lambda_o + (\lambda(0) - \lambda_o)\exp\left(-\frac{3}{2}\rho\pi\frac{R^4}{I}\frac{C_{P0}}{\lambda_o^{\,2}}Vt\right) \tag{8}$$

This gives a good indication of the turbine time constant, which depends on several turbine parameters and the wind speed.

The percent power loss is given by

$$\%P_{Loss} = \left(1 - \frac{C_P(\lambda)}{C_{P0}}\right) \tag{9}$$

Estimation of λ requires a good estimate of the wind speed, which may be difficult to obtain. In the method described previously the estimated aerodynamic torque and rotor speed were used for control. A method based on these more robust estimates would be useful. If we evaluate

$$\left(1 - \frac{T_{aero}}{k\omega^2}\right)^2 = \left(1 - \frac{C_p(\lambda)/\lambda^3}{C_{p0}/\lambda_0^{\,3}}\right)^2 \tag{10}$$

Under what conditions is

$$\%P_{Loss} = \left(1 - \frac{C_P(\lambda)}{C_{P0}}\right) \cong \varepsilon\left(1 - \frac{C_p(\lambda)/\lambda^3}{C_{p0}/\lambda_0^{\,3}}\right)^2 \tag{11}$$

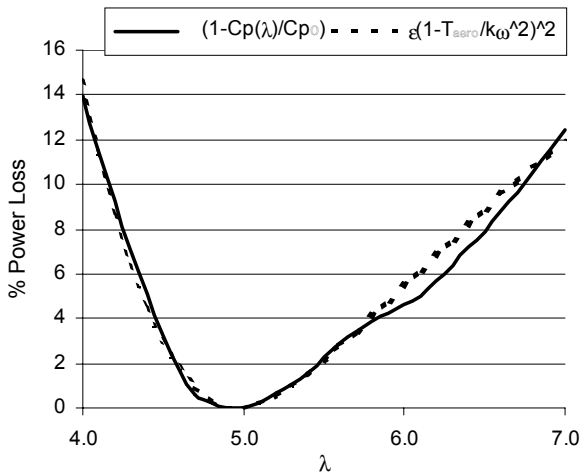

Figure 1. Percent power loss functions from eqn. 11

where ε is a constant or function of

$$T_e = \left(1 - \frac{T_{aero}}{k\omega^2}\right) \tag{12}$$

A plot of the two functions given in equation 5 is shown in Figure 1 for ε = .25 when T_e has positive values and ε = .38 for T_e negative. As can be seen the two functions are very similar. Other rotor C_p-λ curves were also found to provide good estimates of the percent power loss using the described method. Equation 10, multiplied by ε, then provides a reasonable method for estimation of the percent power loss. The tracking torque based on the percent power loss is given by

$$T_t = G\varepsilon\left(1 - \frac{T_{aero}}{k\omega^2}\right)\left|\left(1 - \frac{T_{aero}}{k\omega^2}\right)\right| \tag{13}$$

where G is the desired gain. This method is referred to as the percent power loss (PPL) method.

The equation for power loss is very nonlinear and the results obtained from using this method for turbine control are best evaluated by use of simulation. The Variable-Speed Test Bed Turbine ADAMS model is used for evaluation of the method. As mentioned previously this turbine has a direct-drive generator with a very stiff low speed shaft. This eliminates the need to include drive train dynamics for this turbine and makes initial evaluation of the method more straightforward.

To further improve energy capture it may be desirable to pitch the blades during operation significantly off of λ_o. C_p-λ curves for various pitch angles show increased C_p when the blades are pitched toward feather when λ is significantly below λ_o. Using small amounts of blade pitch at these times would improve energy capture and increase torque to drive the turbine to the desired operating point.

RESULTS

Four methods of variable-speed control are used to evaluate the effects of the control method on energy capture and drive train loads. The four methods are the percent power loss method, the normalized torque error method, the $k\omega^2$ method, and the percent power loss method including blade pitch.

The control system used is shown in Figure 2. Euler's method is used to form difference equations so that terms can be readily identified, however, any transform method could be used. Blade pitch is included in the observer to determine if further improvements in efficiency may be obtained by pitching the blades slightly for large errors in λ. A state is appended to the observer to estimate the effects of changing wind speed. The units for estimated aerodynamic torque and generator torque are kNm. This was sufficient scaling for the turbine model used in this study. For other turbines, additional scaling of the control system may be required. For the $k\omega^2$ method an observer was not used. The measured rotor speed was used directly to calculate the applied generator torque.

Three 10-minute simulations were performed using IEC Class A Kaimal turbulence for each mean wind speed of 5, 7, and 9 m/s to evaluate the methods. Only variable-speed operation was simulated for the comparison. The generator

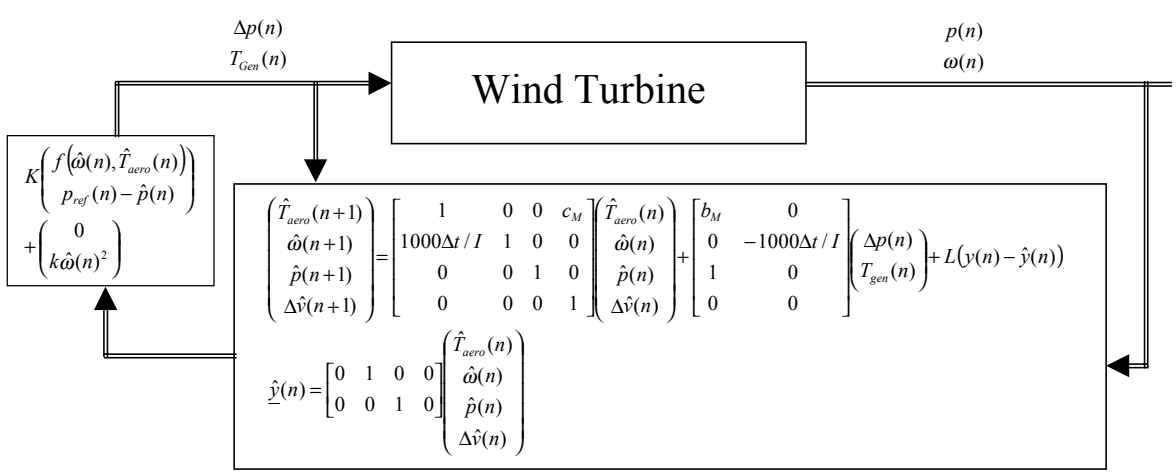

Figure 2. Control system used for evaluation of controlled variable-speed methods

tracking torque applied to the system for the normalized torque or percent power loss method can be quite large for large errors in λ. For this reason, the tracking torque was limited to approximately 15% of rated torque for simulation. This limited drive train loading without greatly effecting energy capture from the control system.

The simulation results are summarized in Table 1 and Figure 3. Table 1 gives the percent energy increase for the torque control methods at the given mean wind speeds compared to the $k\omega^2$ method. The increase in produced energy ranges from 1.26% to 3.75% depending on the mean wind speed and the control method used. The percent improvement increases with decreasing wind. This is due to the time constant being slow for low wind speeds resulting in poor tracking without control. Using blade pitch is seen to further increase energy capture.

Figure 3 shows the rainflow cycle counts of drive train torque for evaluation of shaft fatigue loads. The variable-speed control methods result in a larger number of cycles for the lower wind speeds, however, these are primarily at low amplitude and will contribute a lesser amount to the fatigue damage. For the 9 m/s case there is little difference regardless of the control method used. Histograms of the time at torque for the evaluation of gearbox loads, were one present, were also studied. The control system used seemed to have little effect on time at torque, especially at large torque values.

OTHER ISSUES

With larger or more flexible structures, other issues of concern during variable-speed operation are drive train dynamics and avoiding operation at system resonant frequencies. Also, system losses must be dealt with. These can be incorporated with modification to the control methodology.

In dealing with system losses, modifications to the observer and set points are needed. The variable-speed control methods described above use $k\omega^2$ as essentially the reference value for the estimated aerodynamic torque. This reference value can be modified as needed or a look-up table can be used. For example, it may be desirable to operate at a higher rotor speed

Table 1: Percent Power Increase for Controlled Variable-Speed Operation

Control Method	Normalized Torque	% Power Loss	% Power Loss + Pitch
5 m/s mean	2.77%	3.04%	3.75%
7 m/s mean	1.61%	1.88%	2.59%
9 m/s mean	1.26%	1.44%	2.49%

in low winds to improve reduced Reynolds numbers, or the overall maximum system efficiency may not occur at the condition for optimal rotor efficiency. System losses can be included in the observer in a form similar to

$$I\dot{\omega} = T_{aero} - T_{gen} - B\omega - H \tag{14}$$

Where B incorporates losses that depend on rotor speed and H incorporates other losses. H may be a constant or general function of any turbine parameter such as power output, torque, rotor speed, etc. Including these parameters requires only a small modification to the observer. The contribution of H not in the observer states is considered as an uncontrollable input. The nominal value of the generator torque as a function of rotor speed including losses is given by

$$T_{gen}^{ref}(\omega) = T_{aero}^{ref}(\omega) - B\omega - H \tag{15}$$

Alternatively the desired generator torque as a function of rotor speed could be given in a look-up table with the reference aerodynamic torque calculated from equation 15.

Avoidance of resonant frequencies can be obtained by modifying the reference value for the aerodynamic torque near the resonant rotor speed. A method that was found to perform well modifies the reference torque linearly near the resonance. If the rotor speed is within δ of the resonance rotor speed, ω_{res}, the reference torque is adjusted by

$$F = -\frac{\varepsilon}{\delta}\left((\omega - \omega_{res})\operatorname{sgn}(\omega_{res} - \omega) + \delta\right) \tag{16}$$

where ε is the maximum value that the reference torque is changed. This has the effect of decreasing the generator torque near the resonance so that the rotor may move rapidly through the resonant frequencies. This works well when the rotor speed

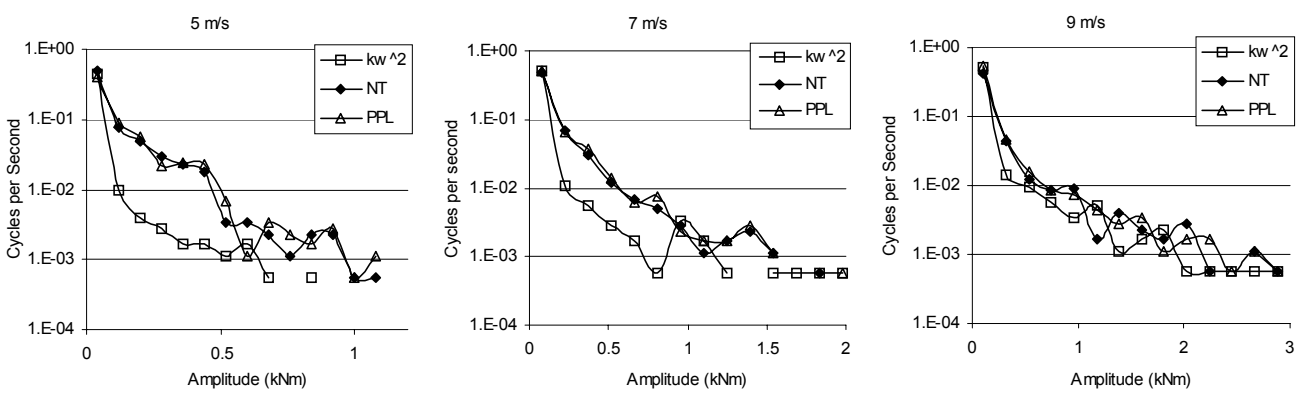

Figure 3. Rainflow cycle counts for the variable-speed control methods

is increasing, however, when the rotor speed is decreasing slowly through the resonance this has the effect of prolonging operation near the resonance. A better use of the reference torque adjustment would be to change the sign of the adjustment when the change in rotor speed is within some negative range to rapidly decrease the speed. Use the method as defined when the change in rotor speed is in the range of slightly negative to some positive value. And do nothing if the rotor speed is moving rapidly through the resonance following the reference torque value.

To investigate the effects of drive train flexibility, the drive shaft stiffness of the model was reduced to produce a natural frequency of 1.9 Hz. To deal with drive train dynamics from use of the variable-speed torque control methods some modifications to the control structure are needed. Using the drive train structure shown in Figure 4, the observer shown in Figure 5 can be used. The stabilizing controller, for variable-speed operation, is determined from the first three states shown in the observer using a linear quadratic design method. It is desired that the first two states have the same value, and that the change in spring torque not be too large. The cost function is then given by

$$J = \sum x(n)^T \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & C_1 \end{bmatrix} x(n) + C_2\left(\Delta T_{st}(n)\right)^2 \quad (17)$$

where

$$x(n) = \left[\dot{\omega}_r(n) \quad \dot{\omega}_g(n) \quad \dot{T}_s(n)\right]^T \quad (18)$$

The desired total generator torque using the percent power loss method is given by

$$\begin{aligned} T_{gen}(n) &= T_{gen}^{ref}(n) + T_{st}(n) \\ &+ G\varepsilon\left(1 - \frac{\hat{T}_{aero}(n)}{T_{aero}^{ref}(n)}\right)\left(1 - \frac{\hat{T}_{aero}(n)}{T_{aero}^{ref}(n)}\right) \end{aligned} \quad (19)$$

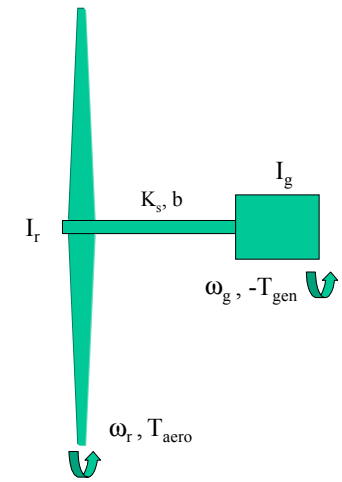

Figure 5. Turbine drive train structure

where

$$T_{st}(n) = CT_{st}(n-1) + \Delta T_{st}(n) \quad (20)$$

$\Delta T_{st}(n)$ is calculated from the stabilizing controller, and C is a constant less than 1 to prevent the sum from becoming large over time, which would result in $T_{gen}(n)$ not accurately following the reference value. $\Delta T_{gen}(n)$ input to the observer is the total change in generator torque in one time step. There are two methods to deal with the percent power loss tracking torque. We can let it be an input disturbance to the stabilizing control system or handle it explicitly. If we let it be an input disturbance, we must limit the change in tracking torque in one time step to allow the control system to deal with this disturbance. Limiting the change in torque is probably a good idea anyway since we do not want very rapid changes in torque. Using this method the resulting shaft torque values from the soft drive train are similar to those obtained from the very stiff system.

To handle the tracking torque explicitly, since the tracking torque is known, the stabilizing torque can be calculated

$$\begin{pmatrix} \hat{\dot{\omega}}_r(n+1) \\ \hat{\dot{\omega}}_g(n+1) \\ \hat{\dot{T}}_s(n+1) \\ \hat{\omega}_g(n+1) \\ \hat{T}_s(n+1) \\ \hat{p}(n+1) \\ \Delta\hat{v}(n+1) \end{pmatrix} = \begin{bmatrix} 1-\Delta tb/I_r & \Delta tb/I_r & -\Delta t & 0 & 0 & 0 & 1000c_M/I_r \\ \Delta tb/I_g & 1-\Delta tb/I_g & \Delta tI_r/I_g & 0 & 0 & 0 & 0 \\ \Delta tK_s/I_r & -\Delta tK_s/I_r & 1 & 0 & 0 & 0 & 0 \\ 0 & \Delta t & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \Delta t & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \hat{\dot{\omega}}_r(n) \\ \hat{\dot{\omega}}_g(n) \\ \hat{\dot{T}}_s(n) \\ \hat{\omega}_g(n) \\ \hat{T}_s(n) \\ \hat{p}(n) \\ \Delta\hat{v}(n) \end{pmatrix} + \begin{bmatrix} 1000b_M/I_r & 0 \\ 0 & -1000/I_g \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta p(n) \\ \Delta T_{gen}(n) \end{pmatrix} + L\left(y(n) - \hat{y}(n)\right)$$

$$\begin{pmatrix} \hat{y}(n) \\ T_{aero}(n) \end{pmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -I_g & I_r b/K_s & 0 & I_r & 0 & 0 \\ I_r & 0 & I_r b/K_s & 0 & I_r & 0 & 0 \end{bmatrix} \begin{pmatrix} \hat{\dot{\omega}}_r(n) \\ \hat{\dot{\omega}}_g(n) \\ \hat{\dot{T}}_s(n) \\ \hat{\omega}_g(n) \\ \hat{T}_s(n) \\ \hat{p}(n) \\ \Delta\hat{v}(n) \end{pmatrix}$$

Figure 4. Observer structure for variable-speed control with drive train dynamics

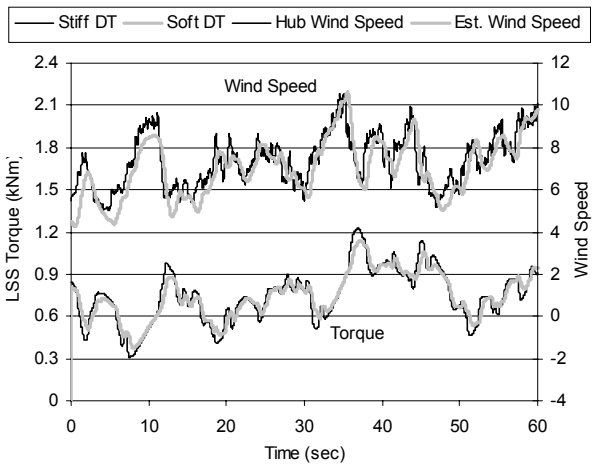

Figure 6. Torque and wind speed time series

including the effect of the tracking torque. The observed controller states are updated to include the effects of the tracking torque, then the stabilizing torque is calculated from the updated states. From the observer in Figure 6, the only state directly effected by an increment in torque is the generator acceleration. This state is then updated with the tracking torque input as

$$\dot{\omega}_g'(n+1) = \hat{\dot{\omega}}_g(n+1) - 1000\Delta T_t / I_g \tag{21}$$

The other controller states are unchanged. This method provides a better means of dealing with the tracking torque. With proper tuning, a good balance between tracking and damping could be obtained and the explicit handling of the tracking torque would be preferred. A one-minute time-series of shaft torque for the soft drive train, using the explicit method, and stiff drive train are shown in Figure 5. If the drive train dynamics are ignored, excessive oscillations result.

The wind speed may be estimated from observer states. The sum of the estimate of the change in wind speed from the observer, corrected by the wind speed derived from estimated aerodynamic power, produces a fairly good estimate of the wind speed. The method is given by

$$\begin{aligned} V_{est}(n) &= V_{est}(n-1) + \Delta\hat{v}(n) \\ V_P(n) &= \sqrt[3]{\left(\hat{T}_{aero}(n)\hat{\omega}_g(n)\right) / \left(1/2\rho\pi R^2 C_{po}\right)} \\ V_{est}(n) &= V_{est}(n) + C\left(V_p(n) - V_{est}(n)\right) \end{aligned} \tag{22}$$

The estimate of wind speed and the hub height wind speed are also shown in Figure 6.

CONCLUSIONS

The percent power loss method or the normalized torque method may be used to improve the energy capture of a variable-speed turbine. The percent power loss method has the advantage that if the turbine is performing well little control effort is applied. Also, the control effort for the percent power loss method is proportional to the percent power loss, which is the desired quantity to be reduced. Blade pitch can also be used to improve energy capture when the turbine is operating at large errors in λ. The improvement in energy capture from these methods depends on the turbine and operating environment.

Use of variable-speed control increases the fluctuation of output power and somewhat increases the shaft fatigue cycles. These issues must be weighed against the increase in power output obtained from use.

Drive train dynamics, system losses, and avoiding resonant frequencies can be incorporated using proper control system implementation.

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