Pursuing optimality in future distribution systems

Emiliano Dall'Anese

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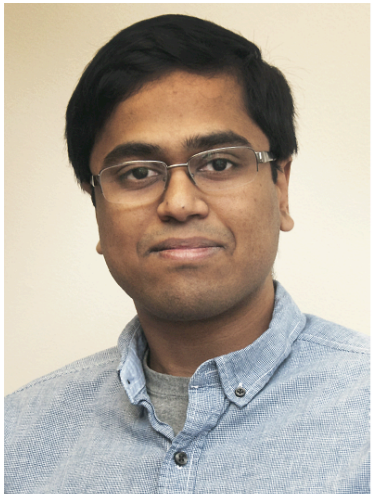
NREL is a national laboratory of the U.S. Department of Energy, Office of Energy Efficiency and Renewable Energy, operated by the Alliance for Sustainable Energy, LLC.

Collaborators

Andrea Simonetto

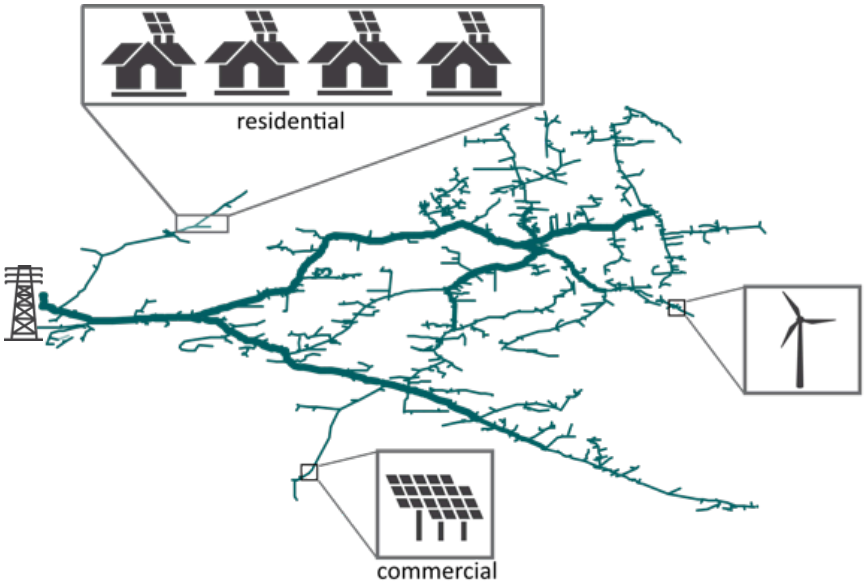


Sairaj Dhople

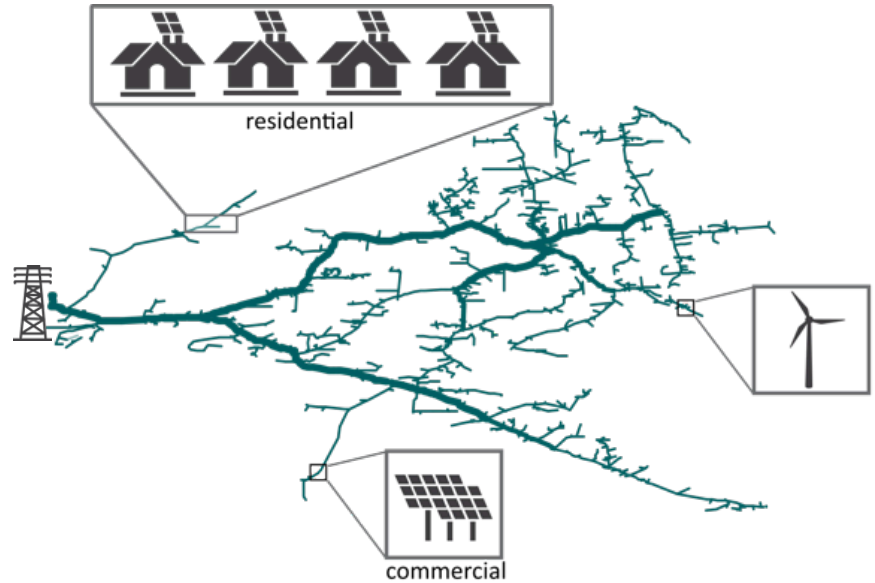
E. Dall'Anese and A. Simonetto, "Optimal Power Flow Pursuit," *IEEE Transactions on Smart Grid*, under review. Preprint available on ArXiv.

E. Dall'Anese, S. V. Dhople, and G. B. Giannakis, "Photovoltaic Inverter Controller Seeking AC Optimal Power Flow Solutions," *IEEE Trans. on Power Systems,* to appear.

Future distribution systems



Optimizing system operation



$$\{\mathbf{u}_n^{*,t}\}_{i\in\mathcal{G}} = \arg\min_{\{\mathbf{u}_i\}_{i\in\mathcal{G}}} \sum_{i\in\mathcal{G}} f_i^t(\mathbf{u}_i)$$

subject to

$$g_n^t(\{\mathbf{u}_i\}_{i\in\mathcal{G}}) \leq 0 \qquad \forall n \in \mathcal{N}$$

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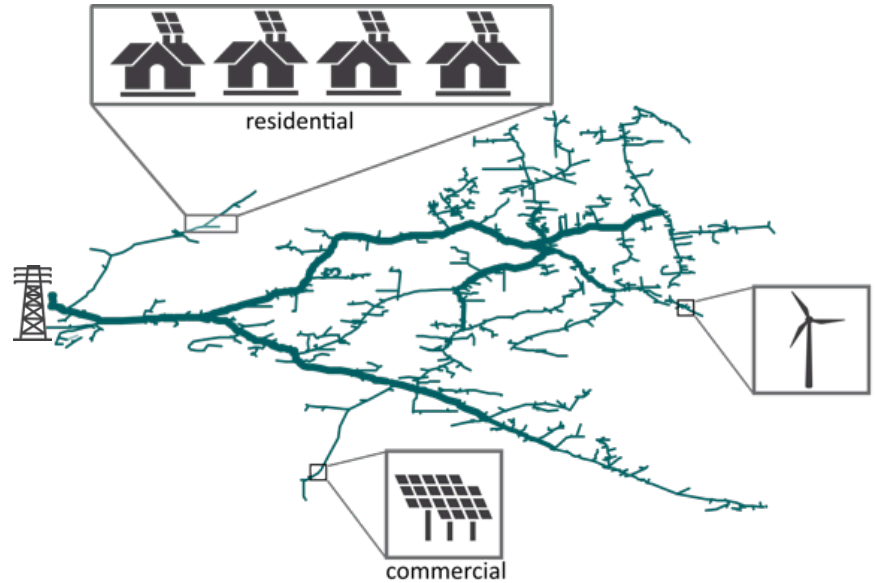

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Network-level problem:

- Well-defined objectives
- Voltage regulation
- Inverter limits
- ....

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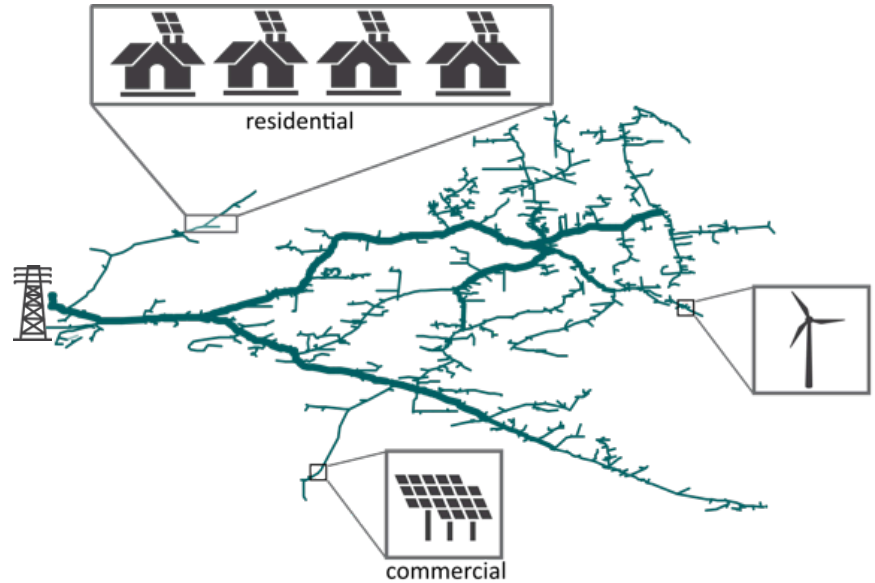

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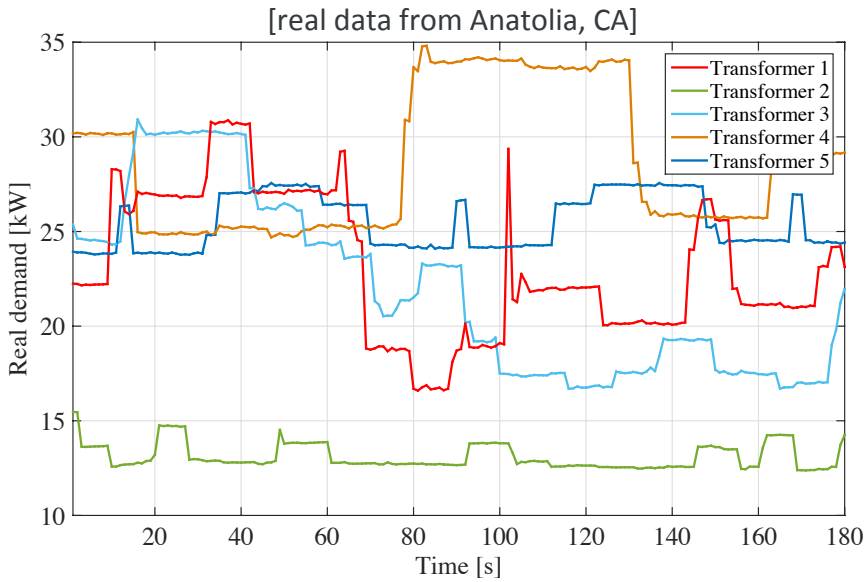
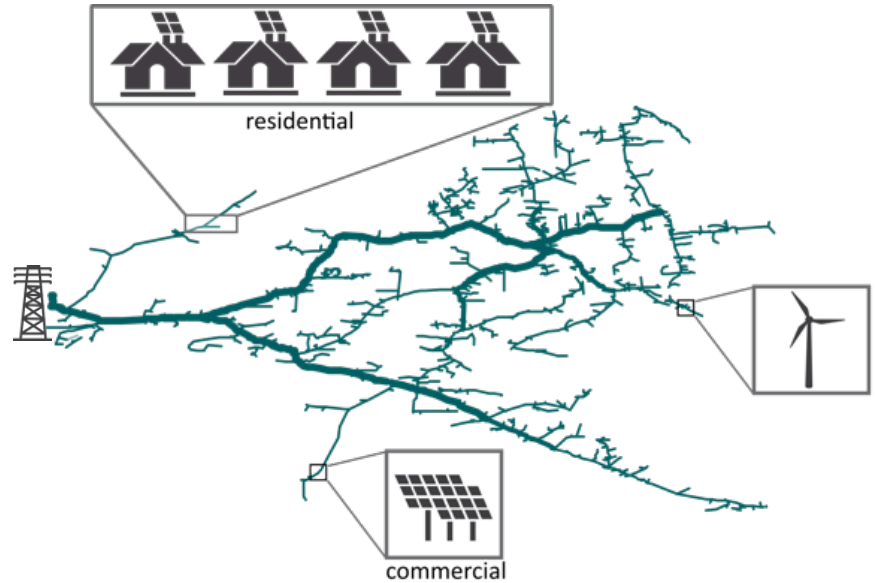
Optimization challenges





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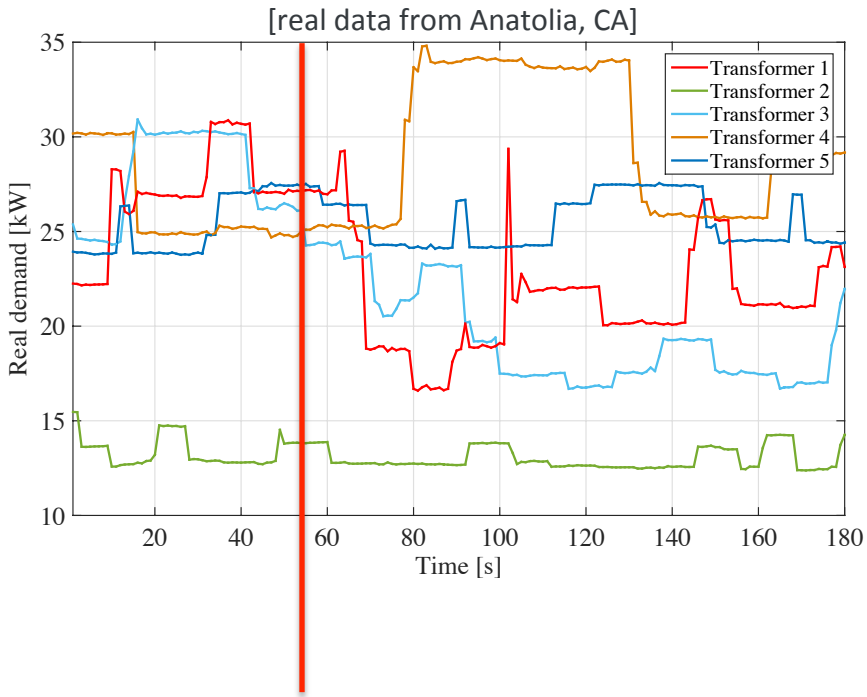

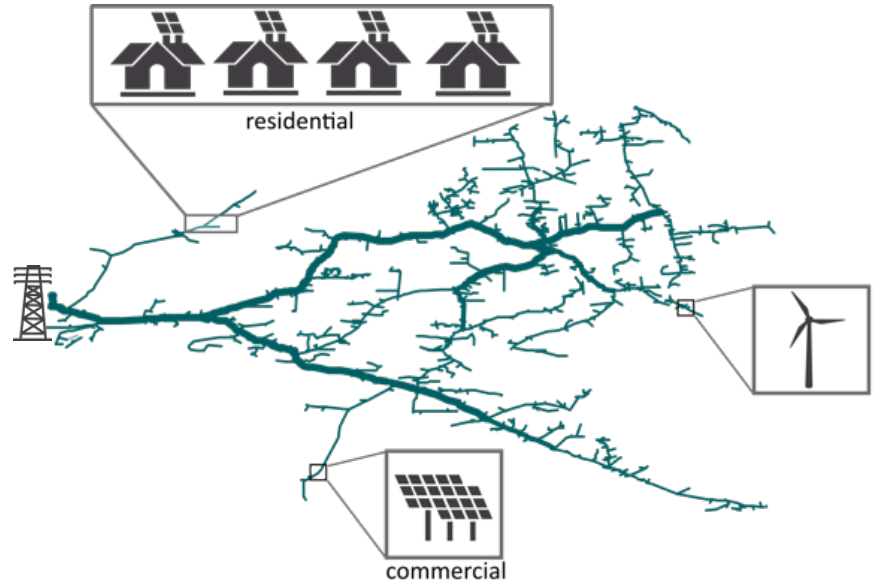

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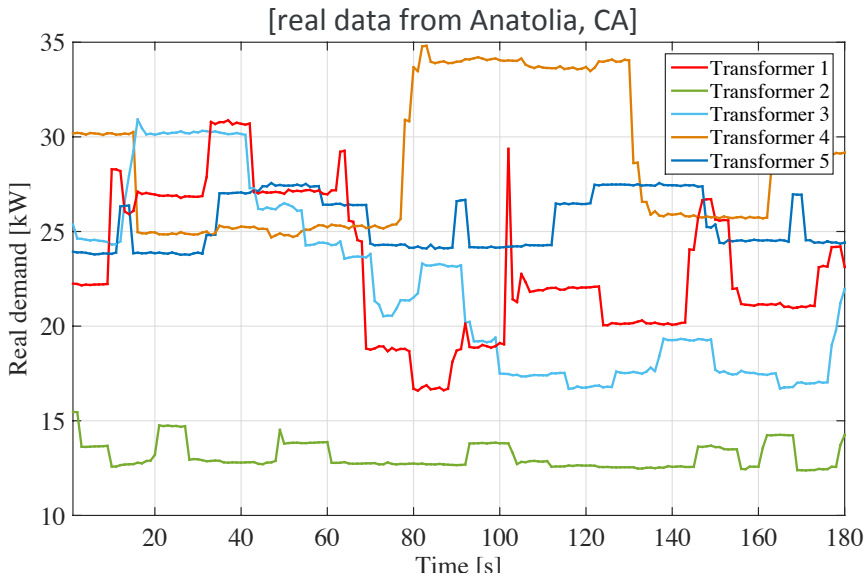
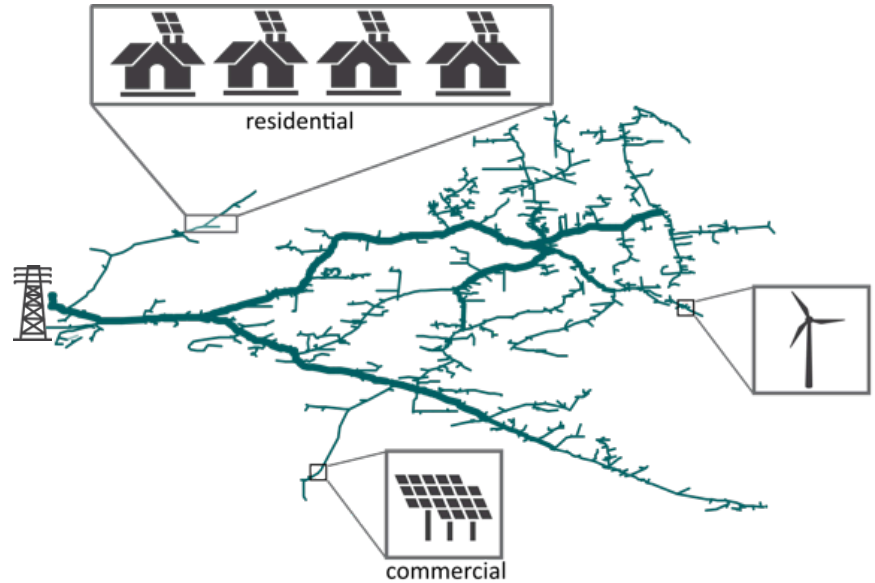
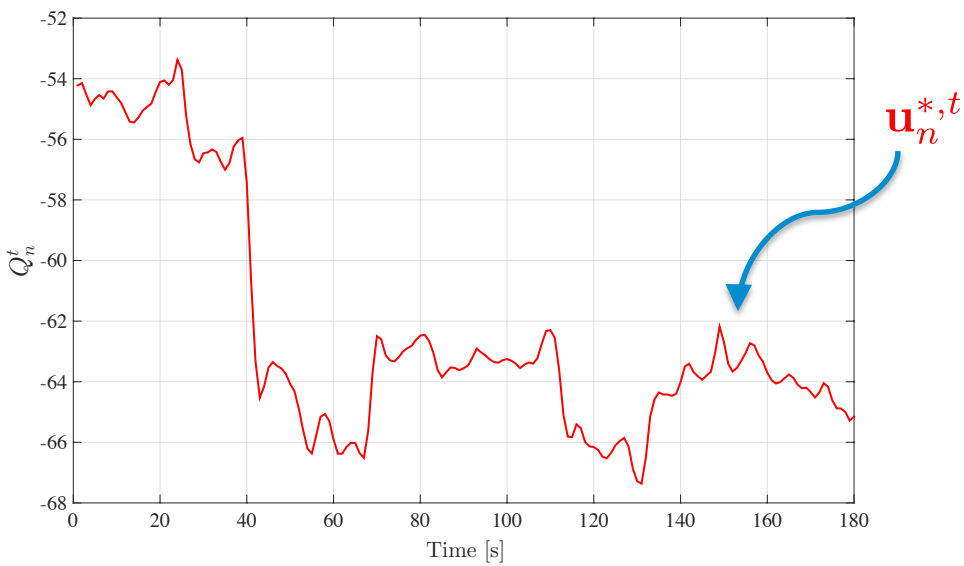
Time-varying network-level problem:

- Well-defined objectives
- Voltage regulation
- Inverter limits
- ....

Optimization challenges







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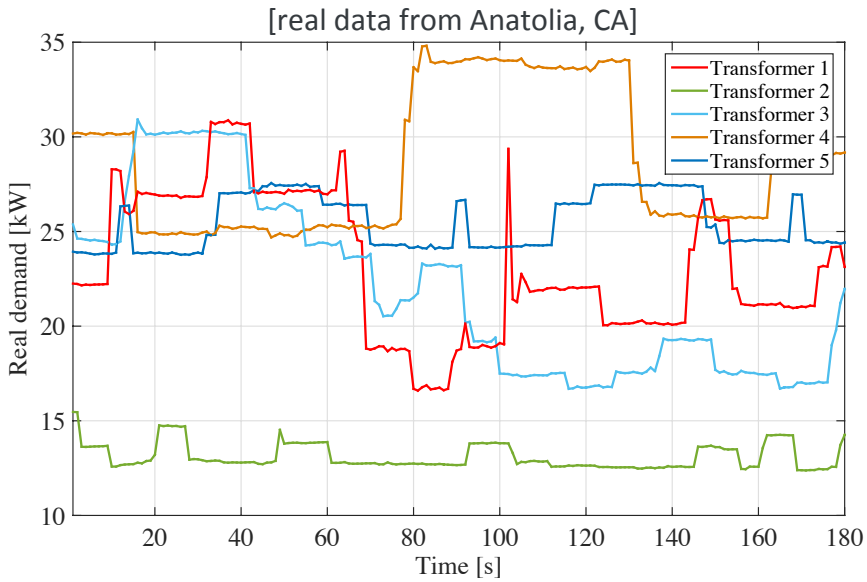

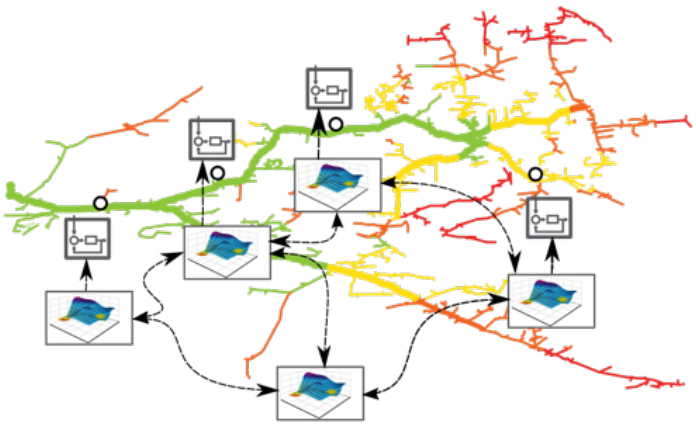

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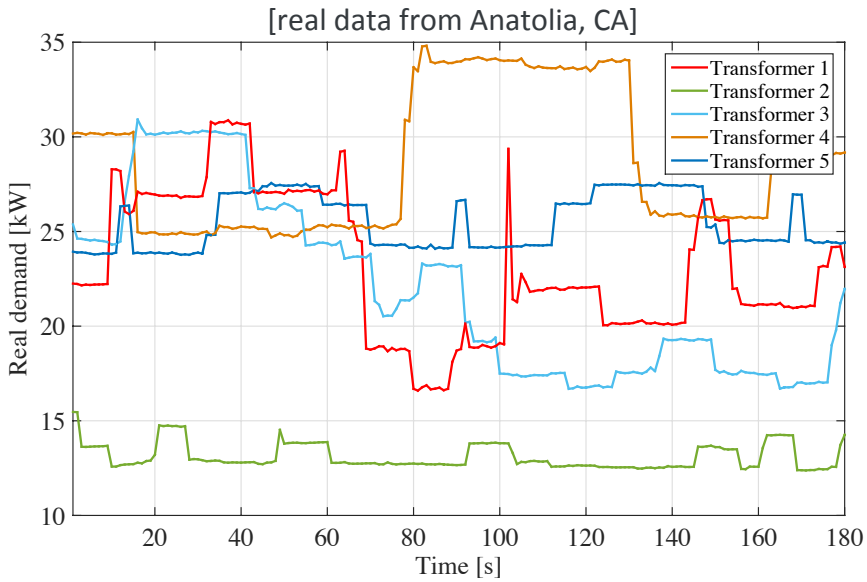
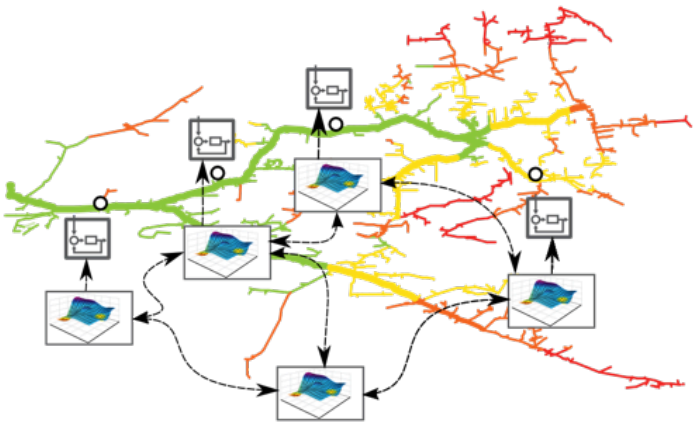
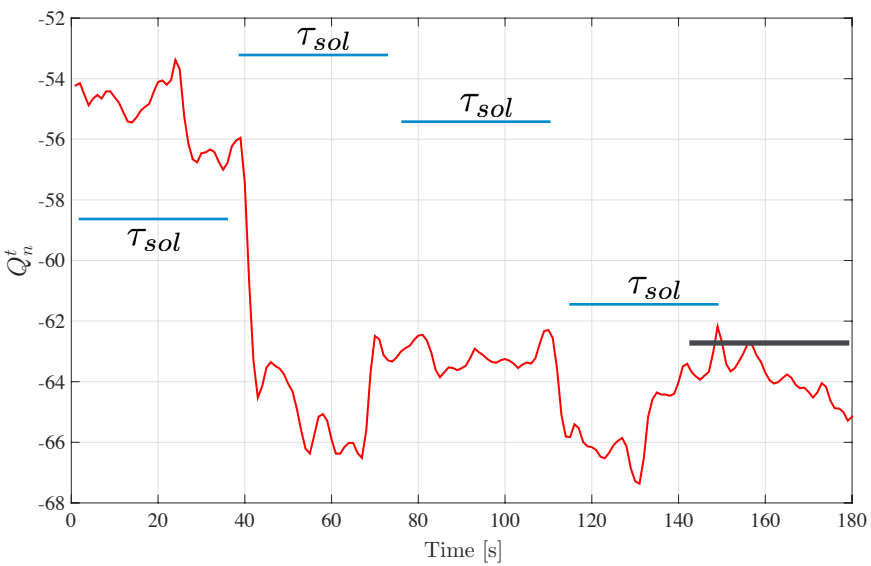
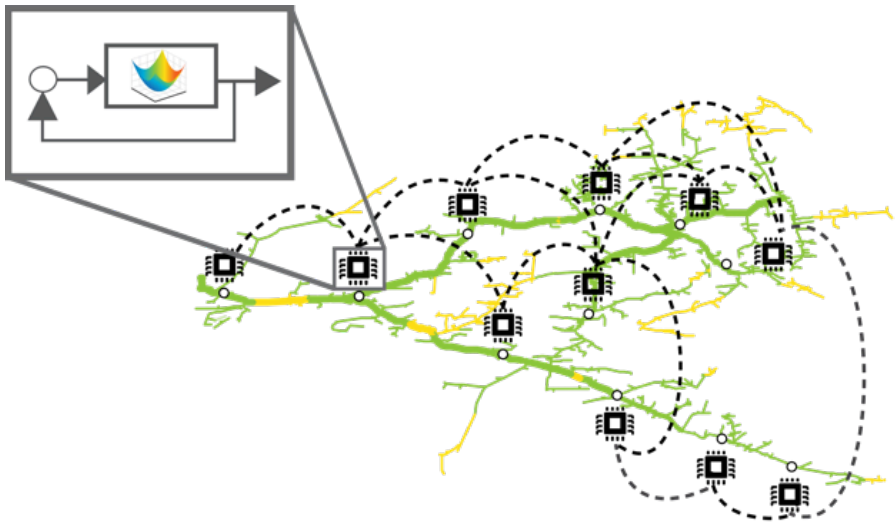






Takeaway:
fast dynamics require fast-acting algorithms

Objective: network-optimal distributed control



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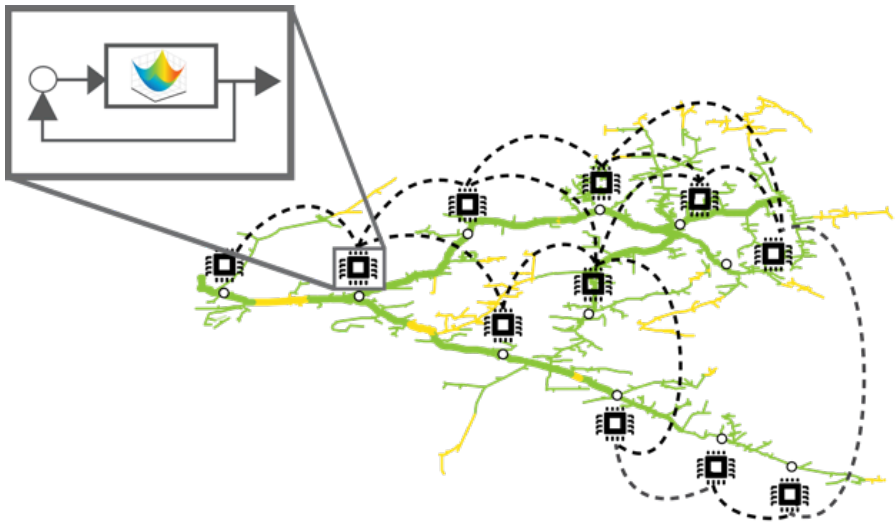

$$\mathbf{u}_n^t$$

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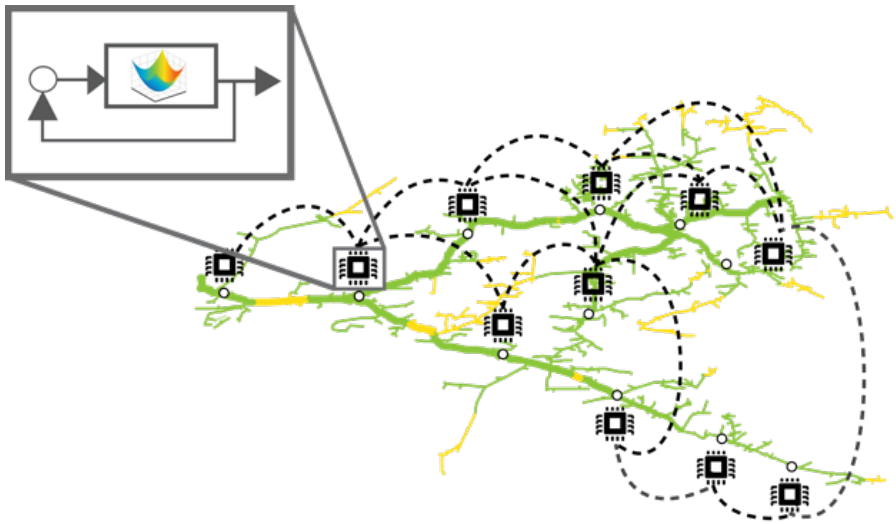

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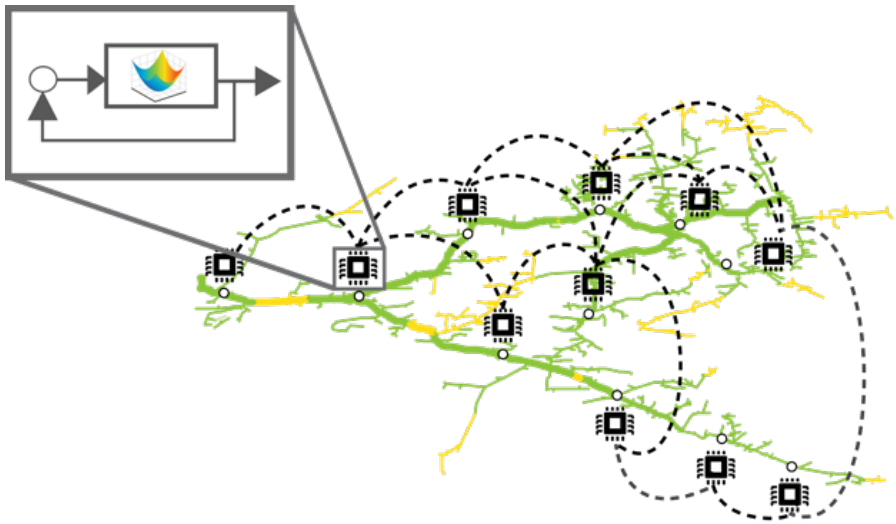

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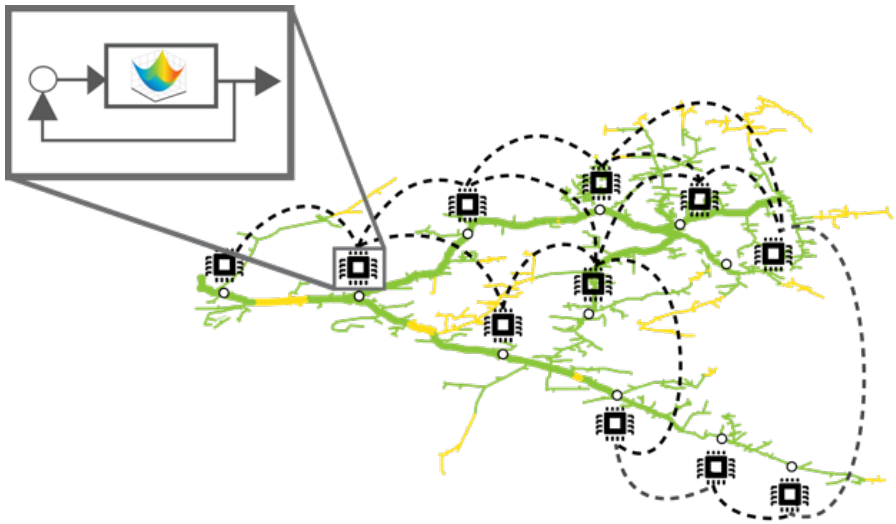

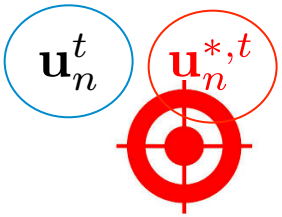

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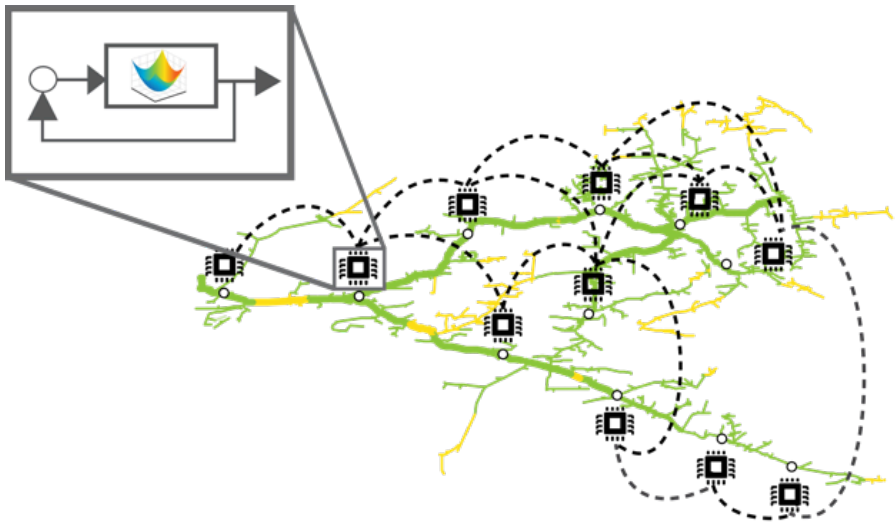

Desiderata: $\|\mathbf{u}_n^t \quad \mathbf{u}_n^{*,t}\|_2 \leq B(\alpha, \quad)$

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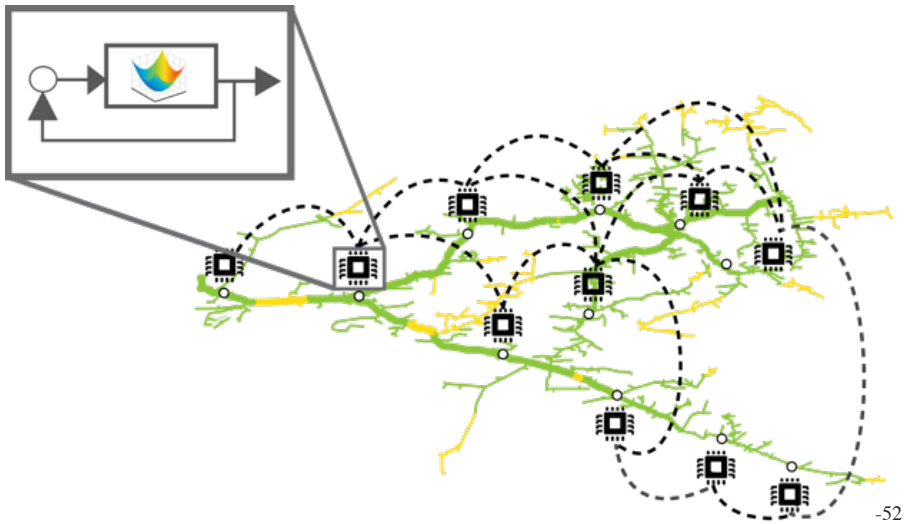

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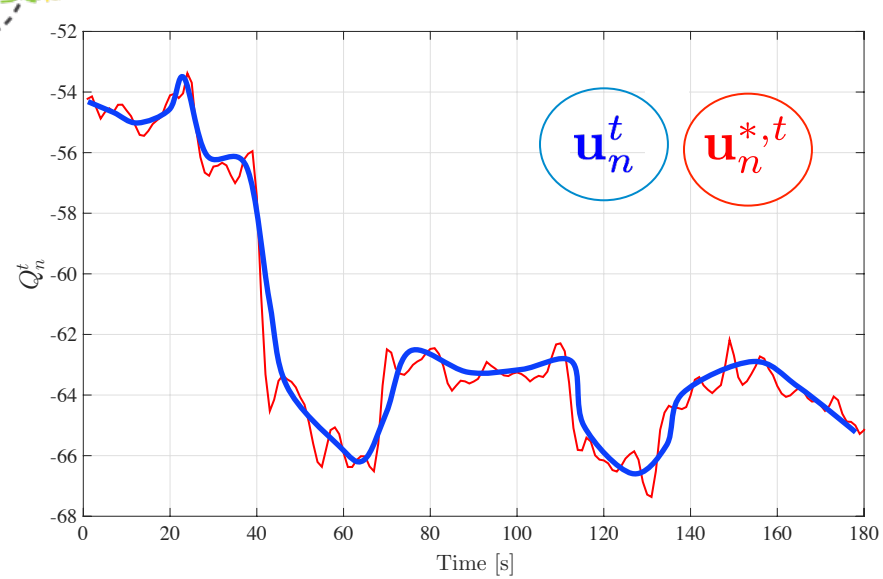
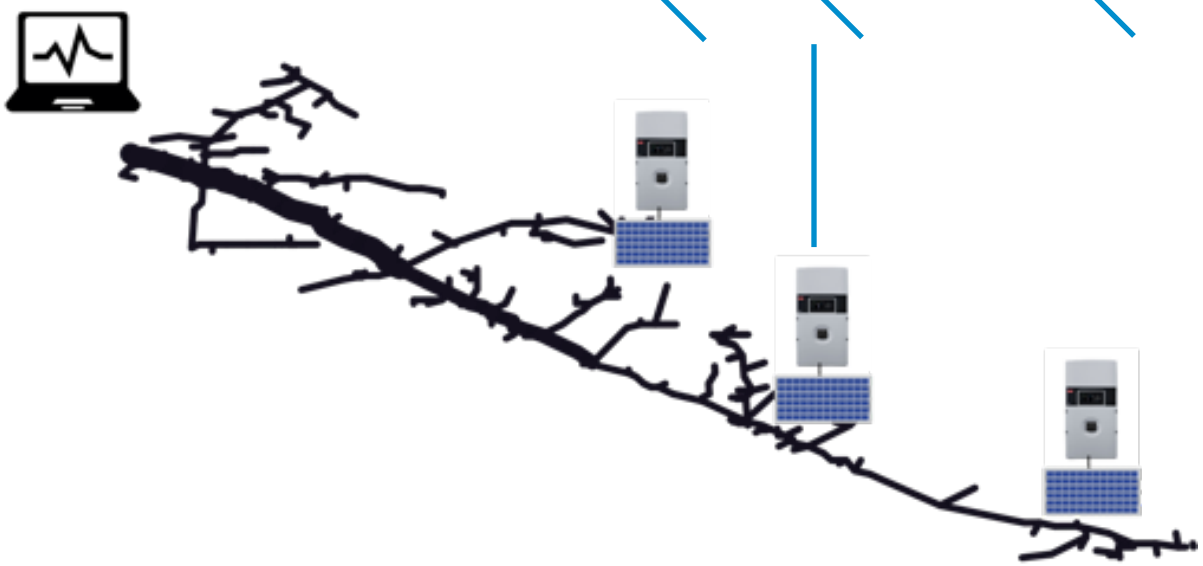


Traditional distributed optimization

$$\mathbf{u}_i^{t+1} = \mathrm{proj}_{\mathcal{Y}_i^t}\left\{\mathbf{u}_i^t - \alpha\nabla_{\mathbf{u}_i} L_{\nu,\epsilon}^t(\mathbf{u},\boldsymbol{\gamma},\boldsymbol{\mu})|_{\mathbf{u}_i^t,\boldsymbol{\gamma}^t,\mu^t}\right\}$$

$$\gamma_n^{t+1} = \mathrm{proj}_{\mathbb{R}_+}\left\{\gamma_n^t + \alpha(g_n^t(\{\mathbf{u}_i^t\}_{i\in\mathcal{G}}) - \epsilon\gamma_n^t)\right\}$$

$$\mu_n^{t+1} = \mathrm{proj}_{\mathbb{R}_+}\left\{\mu_n^t + \alpha(\bar{g}_n^t(\{\mathbf{u}_i^t\}_{i\in\mathcal{G}}) - \epsilon\mu_n^t)\right\}$$



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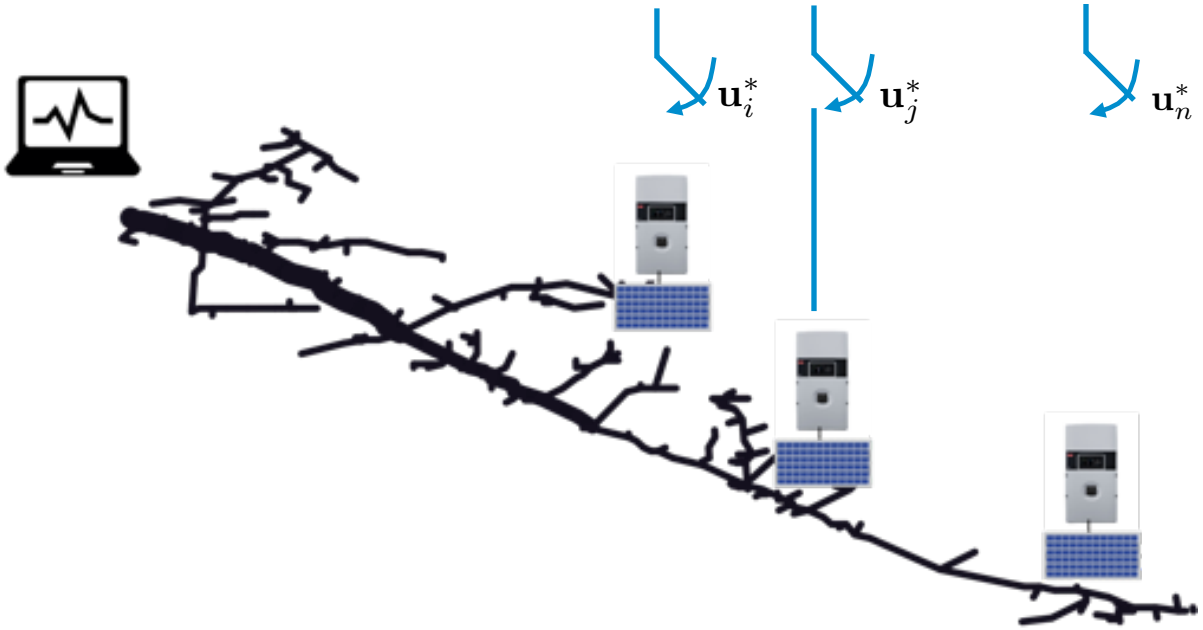

Objective: network-optimal distributed control

$$\gamma_n^{t+1} = \text{proj}_{\mathbb{R}_+}\left\{\gamma_n^t + \alpha(V^{\min} - |V_n(t)| - \epsilon\gamma_n^t)\right.$$
$$\mu_n^{t+1} = \text{proj}_{\mathbb{R}_+}\left\{\mu_n^t + \alpha(|V_n(t)| - V^{\max} - \epsilon\mu_n^t)\right.$$

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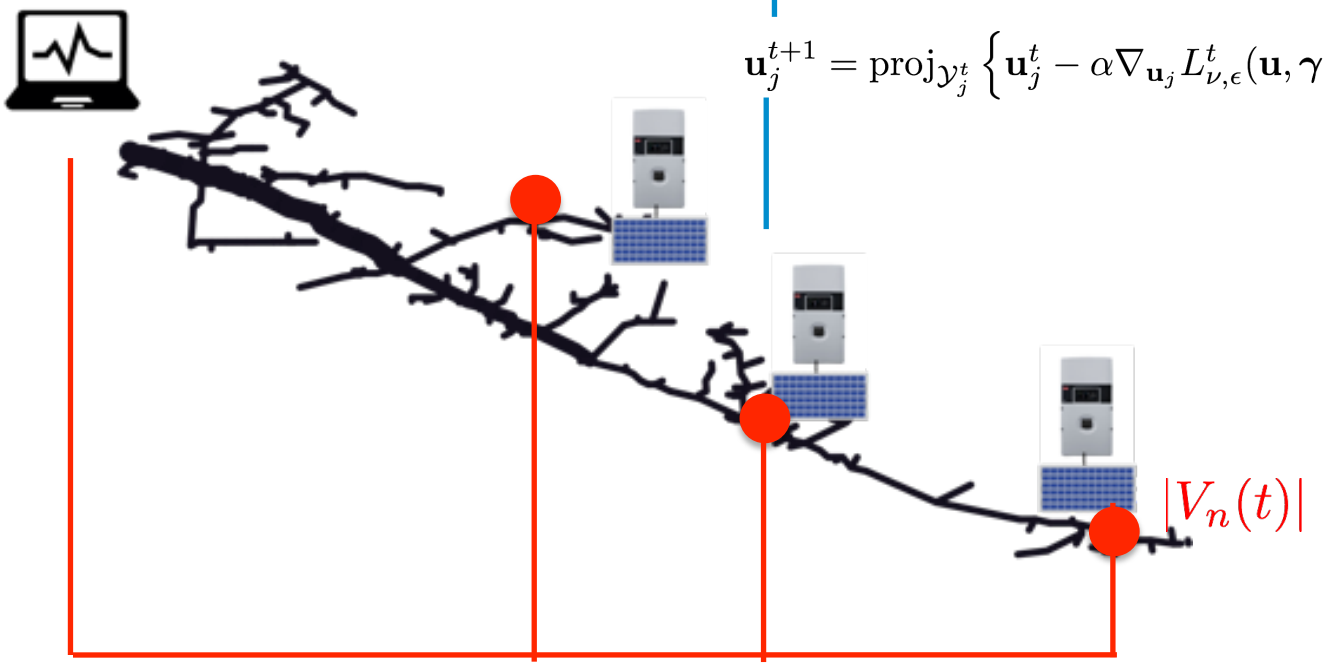

$$\|\mathbf{u}_n^t \quad \mathbf{u}_n^{*,t}\|_2 \leq B(\alpha, \quad)$$

Prototypical optimal power flow

$$
\begin{aligned}
(\mathrm{OPF}^t) \quad & \min_{\mathbf{v},\mathbf{i},\{\mathbf{u}_i\}_{i\in\mathcal{G}}} \sum_{i\in\mathcal{G}} f_i^t(\mathbf{u}_i) \\
& \text{subject to } \mathbf{i} = \mathbf{Y}\mathbf{v}, \text{and} \\
V_i I_i^* &= P_i - P_{\ell,i}^t + \mathrm{j}(Q_i - Q_{\ell,i}^t), \forall\, i \in \mathcal{G} \\
V_n I_n^* &= -P_{\ell,n}^t - \mathrm{j}Q_{\ell,n}^t, && \forall\, n \in \mathcal{N}\backslash\mathcal{G} \\
V^{\min} &\leq |V_i| \leq V^{\max} && \forall\, i \in \mathcal{N} \\
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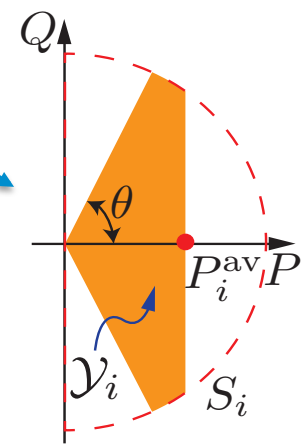

Compact set

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- Non-convex (and NP-hard) quadratically-constrained quadratic program

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\end{aligned}
$$

- Non-convex (and NP-hard) quadratically-constrained quadratic program
- Heuristics and off-the-shelf solvers: do not ensure optimality and impede distributed solutions

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- Non-convex (and NP-hard) quadratically-constrained quadratic program
- Heuristics and off-the-shelf solvers: do not ensure optimality and impede distributed solutions
- Relaxation: Second-order cone [Jabr'06], [Farivar-Low'13]

  Semidefinite programming [Bai et al'08], [Lavaei-Low'12], [Dall'Anese-Zhu-Giannakis'13]
- Linearization/approximation [Turitsin et al'11], [Bolognani-Zampieri'12], [Dhople et al'15]

Prototypical optimal power flow

$$
\begin{aligned}
(\mathrm{OPF}^t) \quad & \min_{\mathbf{v},\mathbf{i},\{\mathbf{u}_i\}_{i\in\mathcal{G}}} \sum_{i\in\mathcal{G}} f_i^t(\mathbf{u}_i) \\
& \text{subject to } \mathbf{i} = \mathbf{Y}\mathbf{v}, \text{and} \\
& V_i I_i^* = P_i - P_{\ell,i}^t + \mathrm{j}(Q_i - Q_{\ell,i}^t), \forall\, i \in \mathcal{G} \\
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- **Linearization/approximation** [Turitsin et al'11], [Bolognani-Zampieri'12], **[Dhople et al'15]**

A linear approximation of the AC power flows

- Objective:

$$\mathbf{v} \approx \mathbf{H}\mathbf{p} + \mathbf{J}\mathbf{q} + \mathbf{b}$$
$$\boldsymbol{\rho} \approx \mathbf{R}\mathbf{p} + \mathbf{B}\mathbf{q} + \mathbf{a}$$

- Set $\mathbf{v} = \bar{\mathbf{v}} + \mathbf{e}$, and discard second-order terms to obtain $\mathrm{diag}\left(\bar{\mathbf{v}}^*\right)\mathbf{Y}\mathbf{e} = \mathbf{s}^*$

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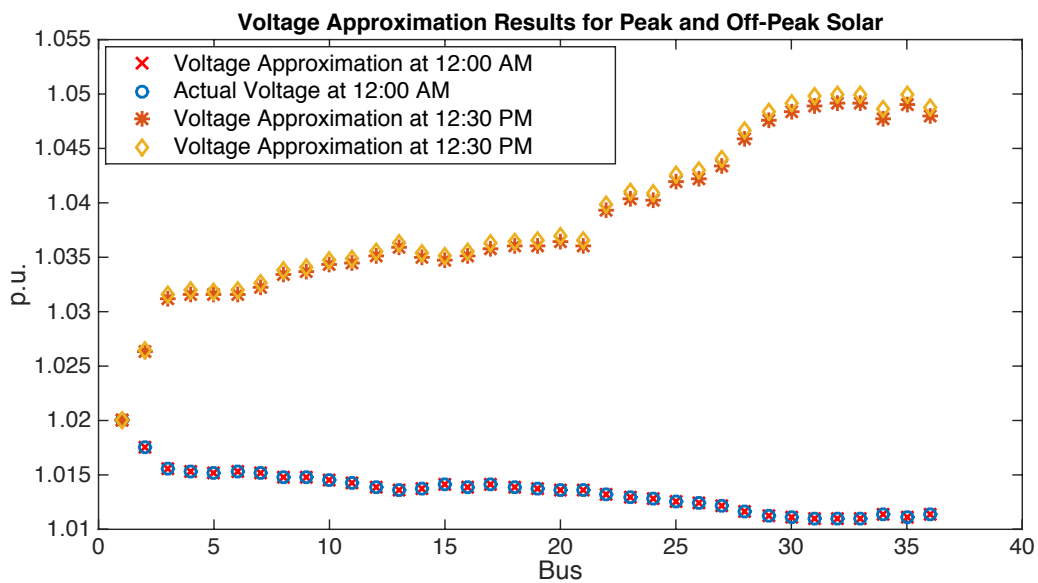
- Empirical evidences: error < 0.2%



S. Guggilam, E. Dall'Anese, Y. C. Chen, S. V. Dhople, and G. B. Giannakis, "Scalable Optimization Methods for Distribution Networks with High PV Integration," *IEEE Trans. on Smart Grid,* under review.

An approximate AC OPF

$$
\begin{aligned}
(\mathrm{P1}^t) \quad & \min_{\{\mathbf{u}_i\}_{i\in\mathcal{G}}} \sum_{i\in\mathcal{G}} f_i^t(\mathbf{u}_i) \\
& \text{subject to} \\
& \qquad g_n^t(\{\mathbf{u}_i\}_{i\in\mathcal{G}}) \le 0 && \forall n \in \mathcal{N} \\
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- $g_n^t(\{\mathbf{u}_i\}_{i\in\mathcal{G}}) := V^{\min} - c_n^t - \sum_{i\in\mathcal{G}}[r_{n,i}^t(P_i - P_{\ell,i}^t) + b_{n,i}^k(Q_i - Q_{\ell,i}^t)]$

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\end{aligned}
$$

- *(Ass. 1)* $f_i^t(\mathbf{u}_i)$ convex and continuously differentiable
- *(Ass. 2)* The map $\mathbf{g}^t(\mathbf{u}) := [\nabla_{\mathbf{u}_1}^\mathsf{T} f_1^t(\mathbf{u}_1), \ldots, \nabla_{\mathbf{u}_G}^\mathsf{T} f_G^t(\mathbf{u}_G)]^\mathsf{T}$ is Lipschitz continuous
- *(Ass. 3)* Slater's condition holds

An approximate AC OPF

$$
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\end{aligned}
$$

- *(Ass. 4)* $\|\mathbf{u}^{*,t+1} - \mathbf{u}^{*,t}\| \leq \sigma_{\mathbf{u}}$

- *(Ass. 5)* $|g_n^{t+1}(\mathbf{u}^{*,t+1}) - g_n^{t}(\mathbf{u}^{*,t})| \leq \sigma_d$ and $|\bar{g}_n^{t+1}(\mathbf{u}^{*,t+1}) - \bar{g}_n^{t}(\mathbf{u}^{*,t})| \leq \sigma_{\bar{d}}$

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$$

- $L^t_{\nu,\epsilon}(\mathbf{u}^t, \boldsymbol{\gamma}, \boldsymbol{\mu}) := L^t(\mathbf{u}^t, \boldsymbol{\gamma}, \boldsymbol{\mu}) + \nu\|\mathbf{u}^t\|_2^2 - \epsilon\|\boldsymbol{\gamma}^t\|_2^2 - \epsilon\|\boldsymbol{\mu}^t\|_2^2$

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$\longrightarrow$ $\max_{\boldsymbol{\gamma},\boldsymbol{\mu}} \min_{\mathbf{u}} L^t_{\nu,\epsilon}(\mathbf{u}^t, \boldsymbol{\gamma}, \boldsymbol{\mu})$

$\mathbf{u}^{\mathrm{opt},t}, \boldsymbol{\gamma}^{\mathrm{opt},t}, \boldsymbol{\mu}^{\mathrm{opt},t}$ unique primal-dual solutions

- Benefit: linear convergence without averaging (time invariant case) [Koshal et al'11]

Outset: online regularized primal-dual method

- At each time t:

$$\mathbf{u}_i^{t+1} = \mathrm{proj}_{\mathcal{Y}_i^t}\left\{\mathbf{u}_i^t - \alpha \nabla_{\mathbf{u}_i} L_{\nu,\epsilon}^t(\mathbf{u}, \boldsymbol{\gamma}, \boldsymbol{\mu})|_{\mathbf{u}_i^t, \boldsymbol{\gamma}^t, \mu^t}\right\}$$
$$\gamma_n^{t+1} = \mathrm{proj}_{\mathbb{R}_+}\left\{\gamma_n^t + \alpha(g_n^t(\{\mathbf{u}_i^t\}_{i\in\mathcal{G}}) - \epsilon\gamma_n^t)\right\}$$
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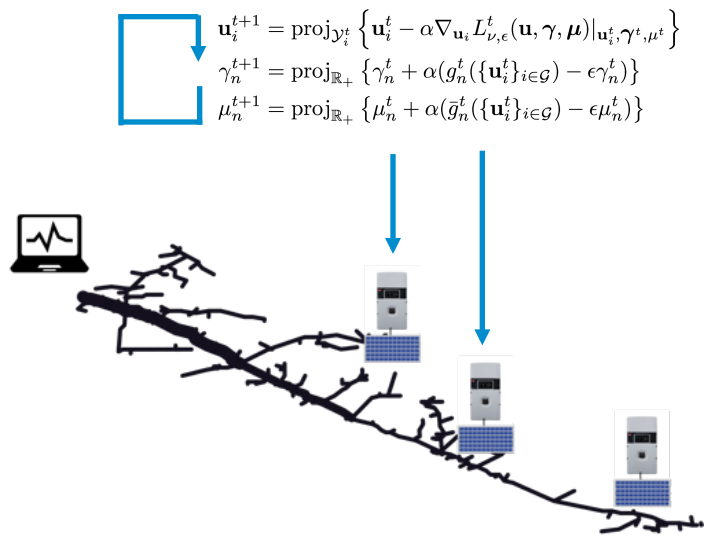
- Linear convergence within a ball [Simonetto et al'14]; time invariant in [Koshal et al'11]

- Where is the feedback? Open loop!



Network feedback control

- Open-loop optimization:

$$\mathbf{u}_i^{t+1} = \mathrm{proj}_{\mathcal{Y}_i^t}\left\{\mathbf{u}_i^t - \alpha \nabla_{\mathbf{u}_i} L_{\nu,\epsilon}^t(\mathbf{u}, \boldsymbol{\gamma}, \boldsymbol{\mu})|_{\mathbf{u}_i^t, \boldsymbol{\gamma}^t, \mu^t}\right\}$$
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- **Closed-loop strategy:**

$$\mathbf{u}_i^{t+1} = \text{proj}_{\mathcal{Y}_i^t}\left\{\mathbf{u}_i^t - \alpha \nabla_{\mathbf{u}_i} L_{\nu,\epsilon}^t(\mathbf{u}, \boldsymbol{\gamma}, \boldsymbol{\mu})|_{\mathbf{u}_i^t, \boldsymbol{\gamma}^t, \mu^t}\right\}$$
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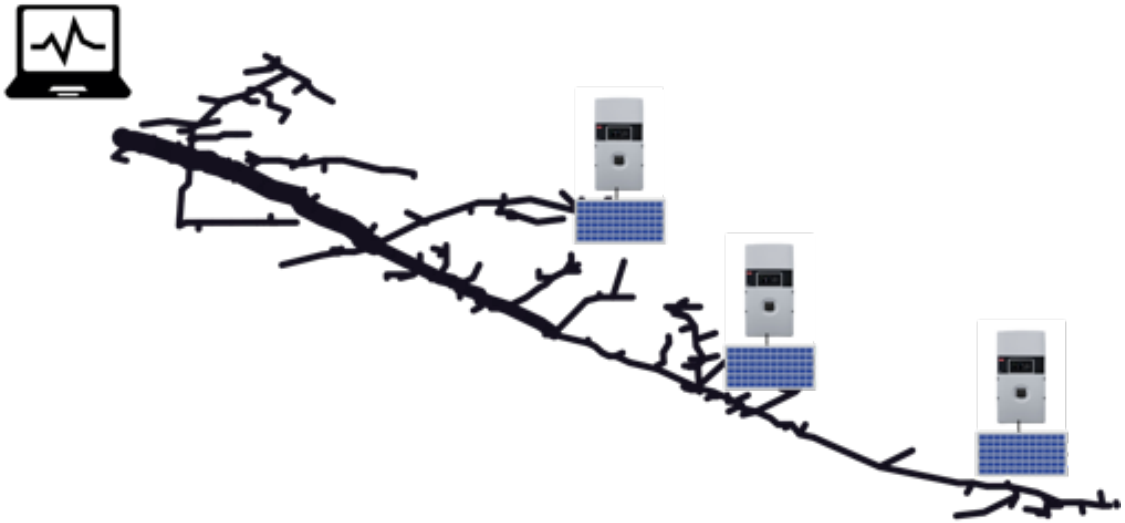
Network feedback control



Network feedback control

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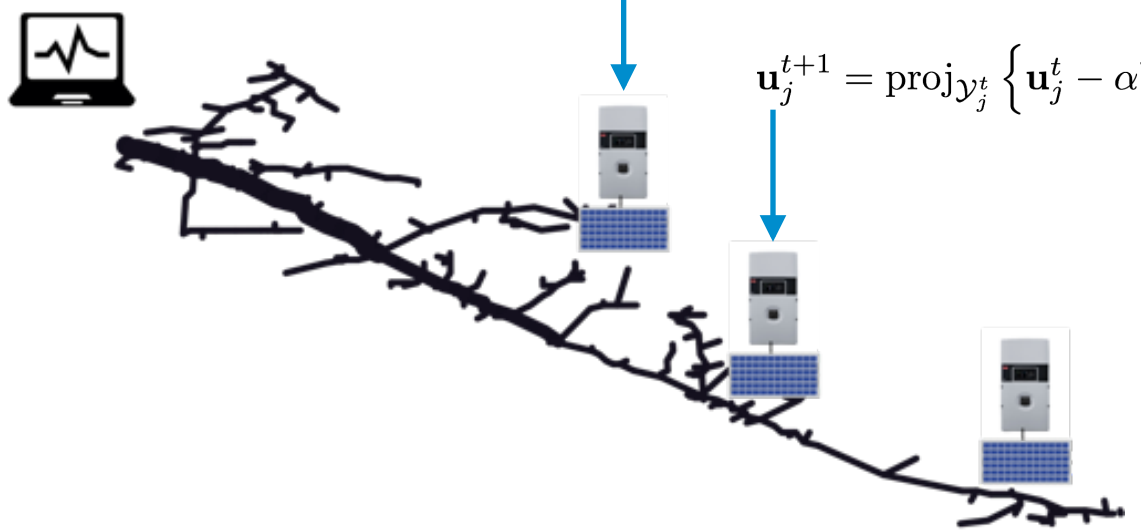

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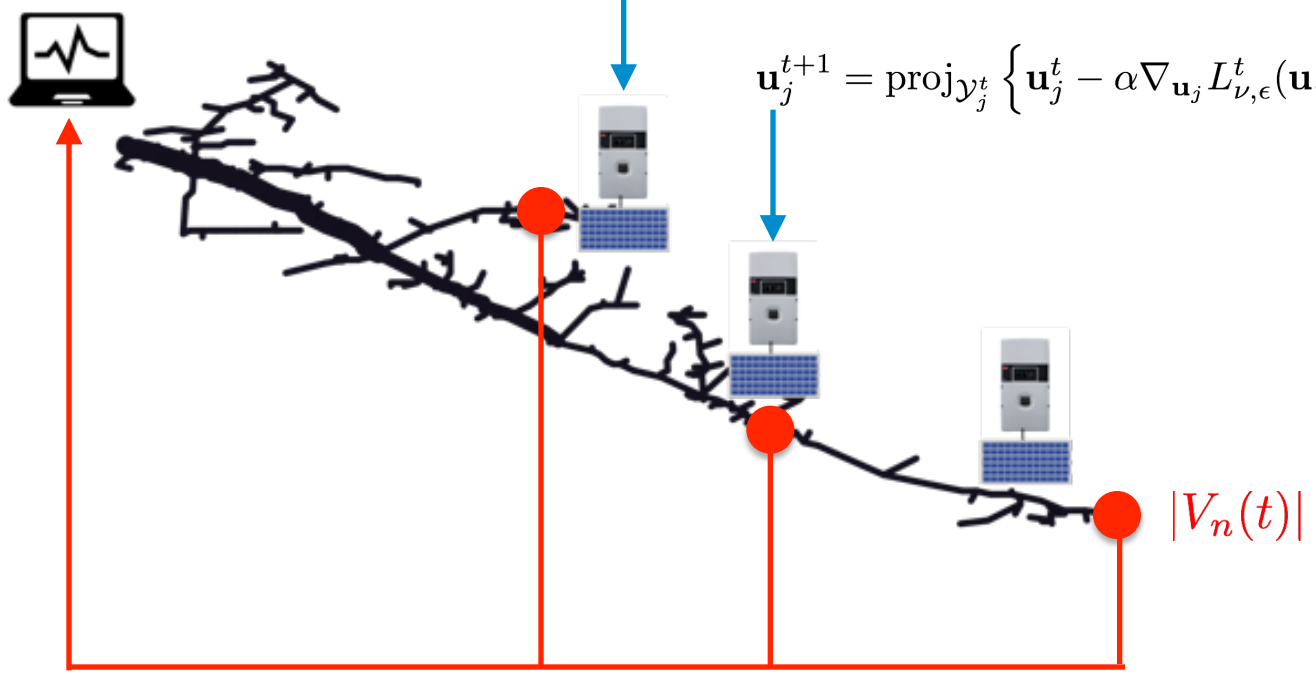

Network feedback control

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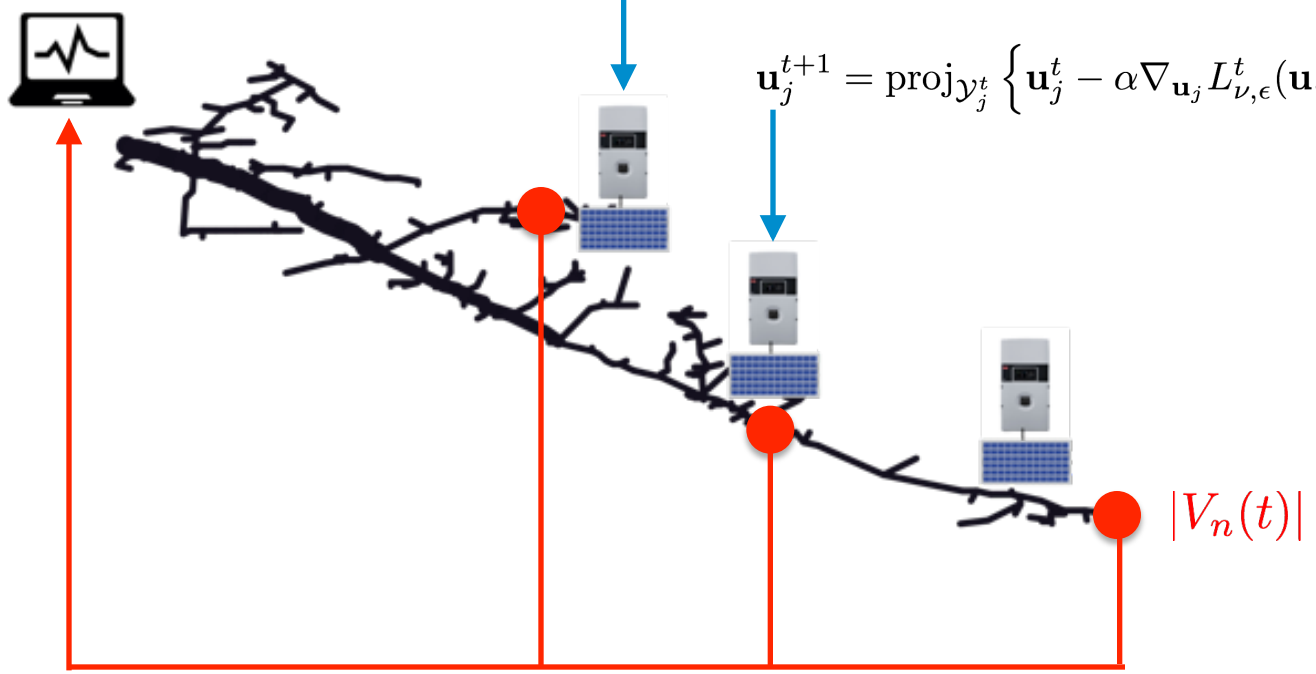

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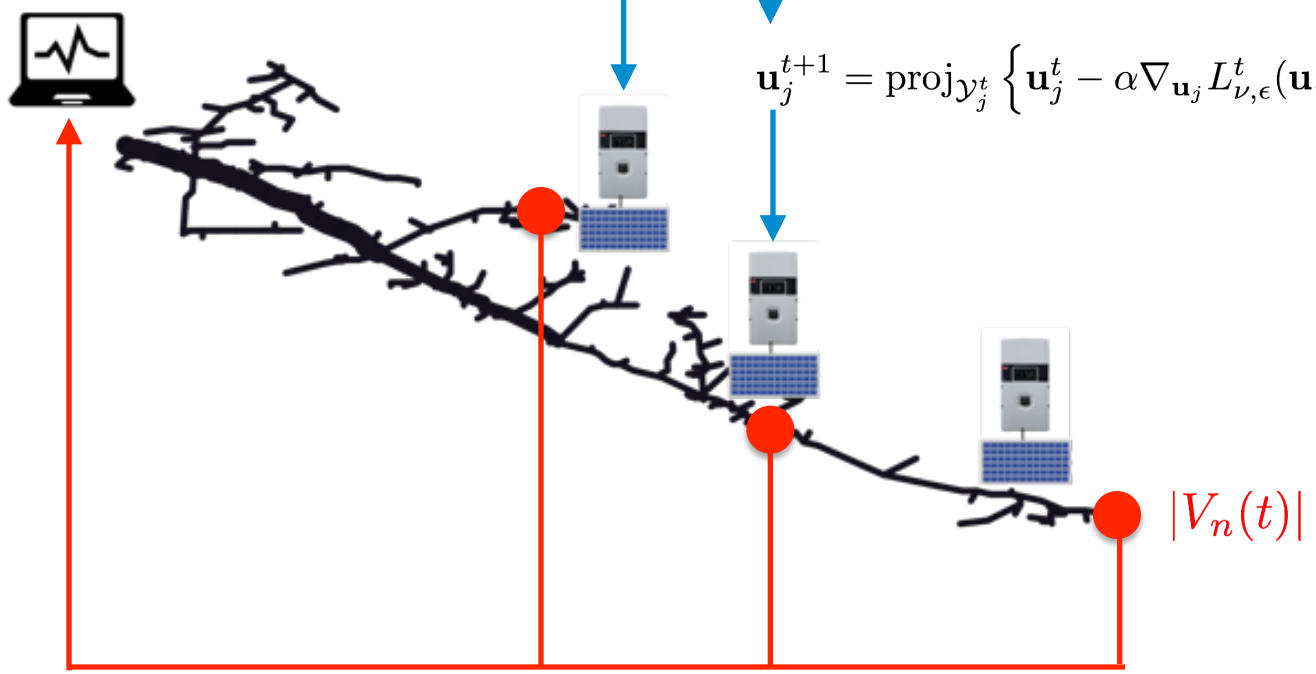

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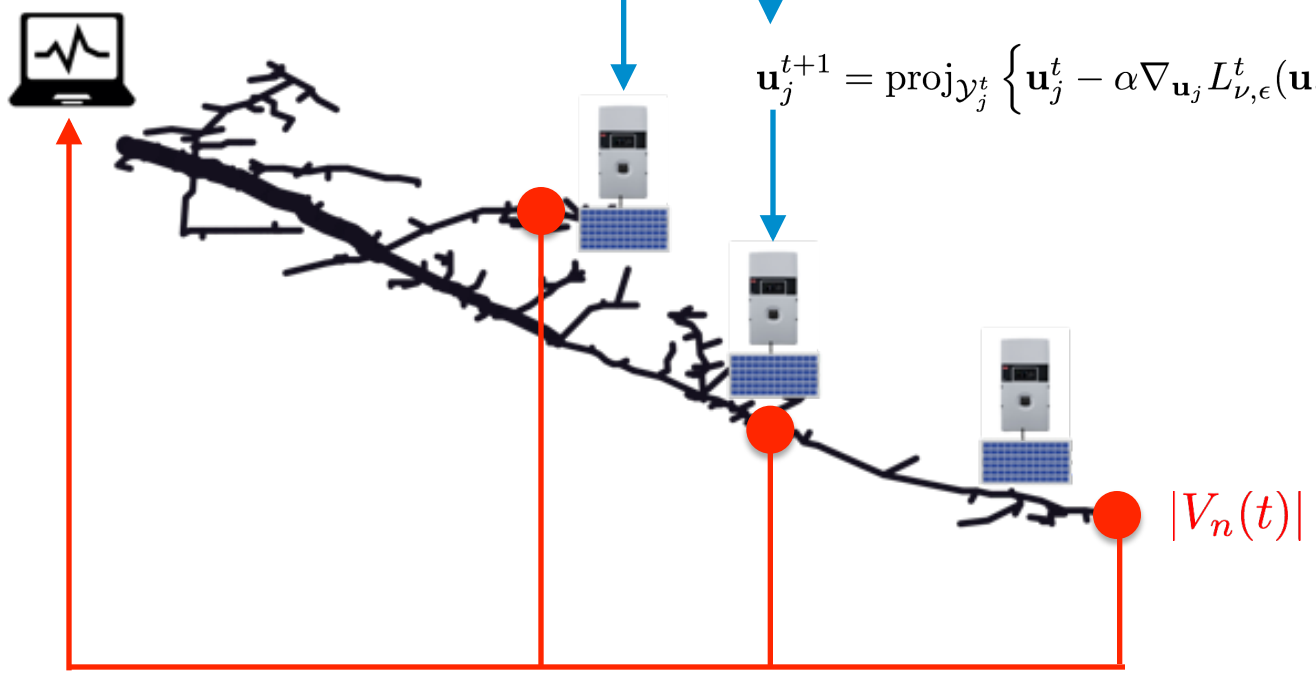

- **No** need to know loads at all nodes!

Network feedback control

- At each time t:

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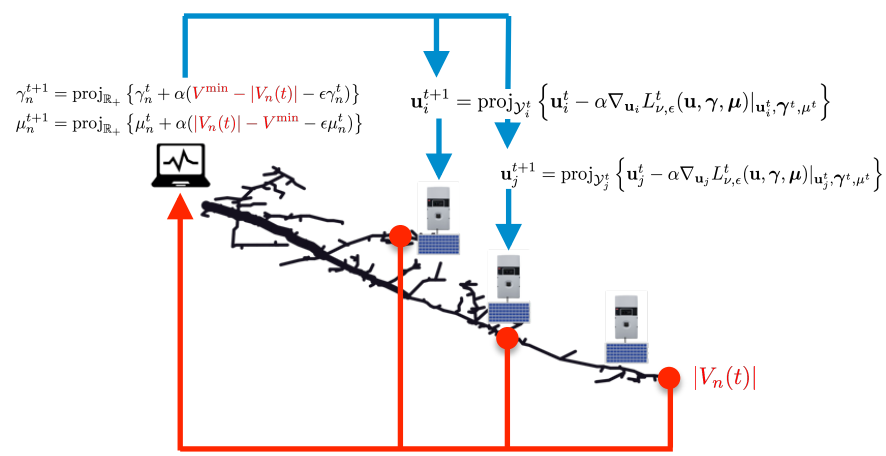
- **Convergence?**

- **Optimality?**



Incorporating voltage measurements

- $\mu_n^{t+1} = \mathrm{proj}_{\mathbb{R}_+} \left\{ \mu_n^t + \alpha(|V_n(t)| - V^{\min} - \epsilon \mu_n^t) \right\}$

Incorporating voltage measurements

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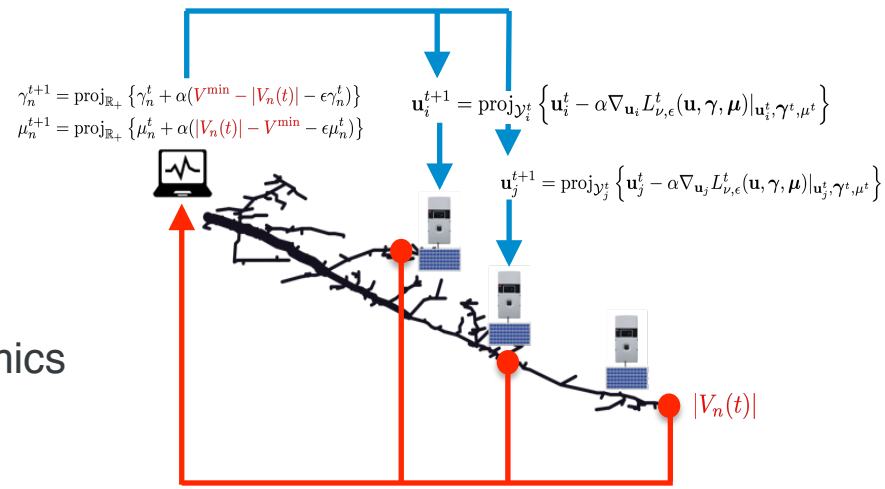
- Discrepancy:
  - Approximation errors (< 0.2%)
  - Measurement errors
  - Updates faster than underlying dynamics
  - Slow responding inverters



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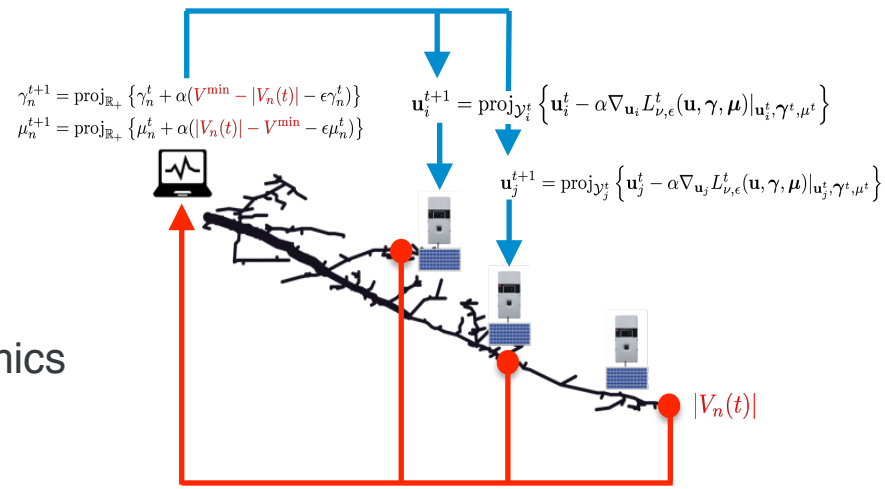
  - Updates faster than underlying dynamics
  - Slow responding inverters



- *(Ass. 6)* $\max\{\|\mathbf{e}_{\boldsymbol{\gamma}}^t\|_2, \|\mathbf{e}_{\boldsymbol{\mu}}^t\|_2\} \leq e\ , \quad \forall\, t \geq 0$

OPF point pursuit

$$\mathbf{u}_i^{t+1} = \mathrm{proj}_{\mathcal{Y}_i^t}\left\{\mathbf{u}_i^t - \alpha \nabla_{\mathbf{u}_i} L_{\nu,\epsilon}^t(\mathbf{u},\boldsymbol{\gamma},\boldsymbol{\mu})|_{\mathbf{u}_i^t,\boldsymbol{\gamma}^t,\mu^t}\right\}$$
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Theorem [Dall'Anese-Simonetto'16]. Under current modeling assumptions, if the stepsize is chosen such that:

$$1 - 2\eta\alpha + \alpha^2 L_{\nu,\epsilon}^2 < \frac{1}{(1+\beta_1)(1+\beta_2)}$$

then the following holds for the closed-loop system above:

$$\limsup_{t\to\infty} \|\mathbf{z}^t - \mathbf{z}^{\mathrm{opt},t}\|_2^2 = \frac{1}{1-\rho(\alpha)}\left[(1+\beta_2)\left(1+\frac{1}{\beta_1}\right)\alpha^2 e^2 + \left(1+\frac{1}{\beta_2}\right)\sigma_{\mathbf{z}}^2\right],$$

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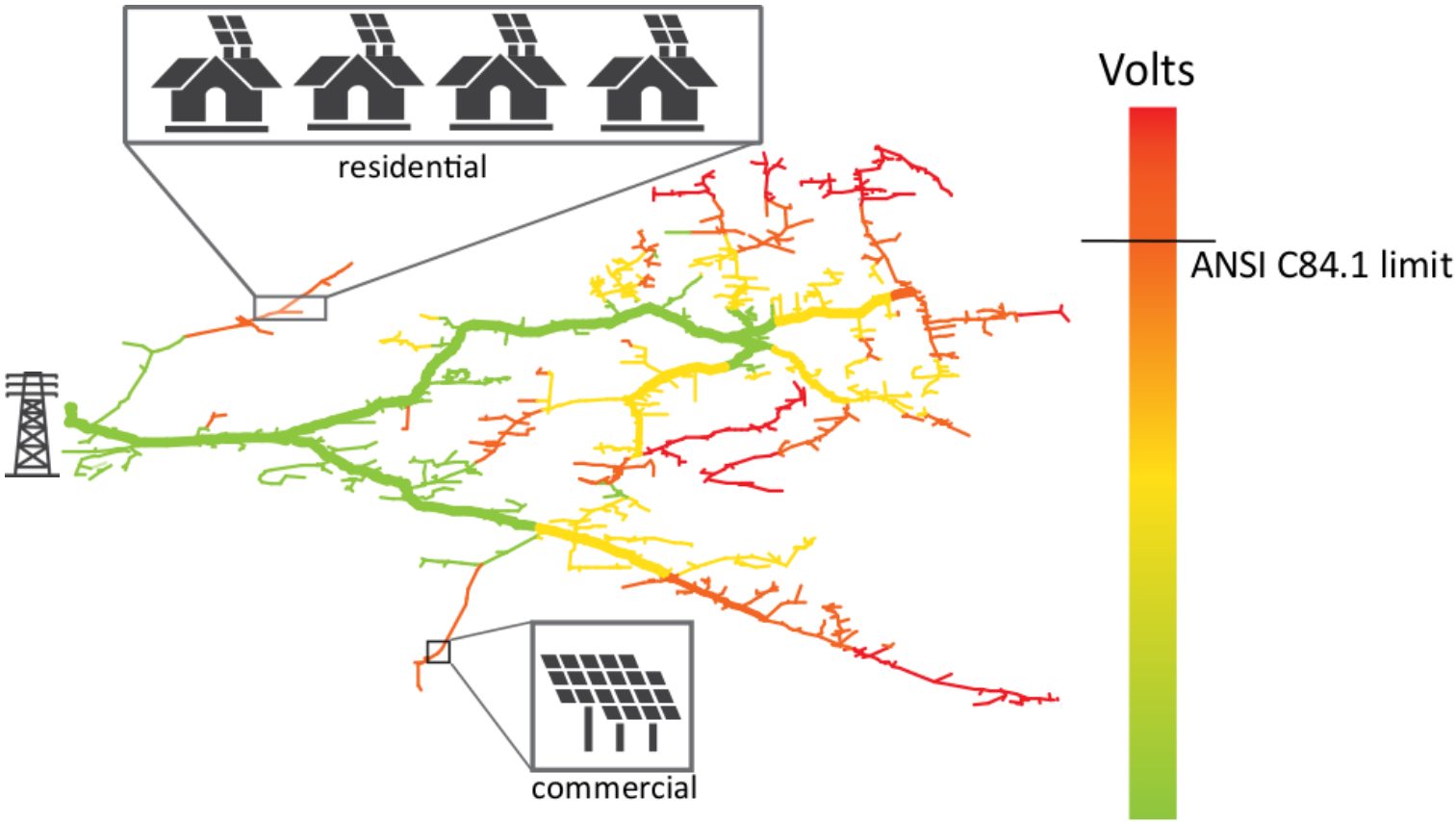
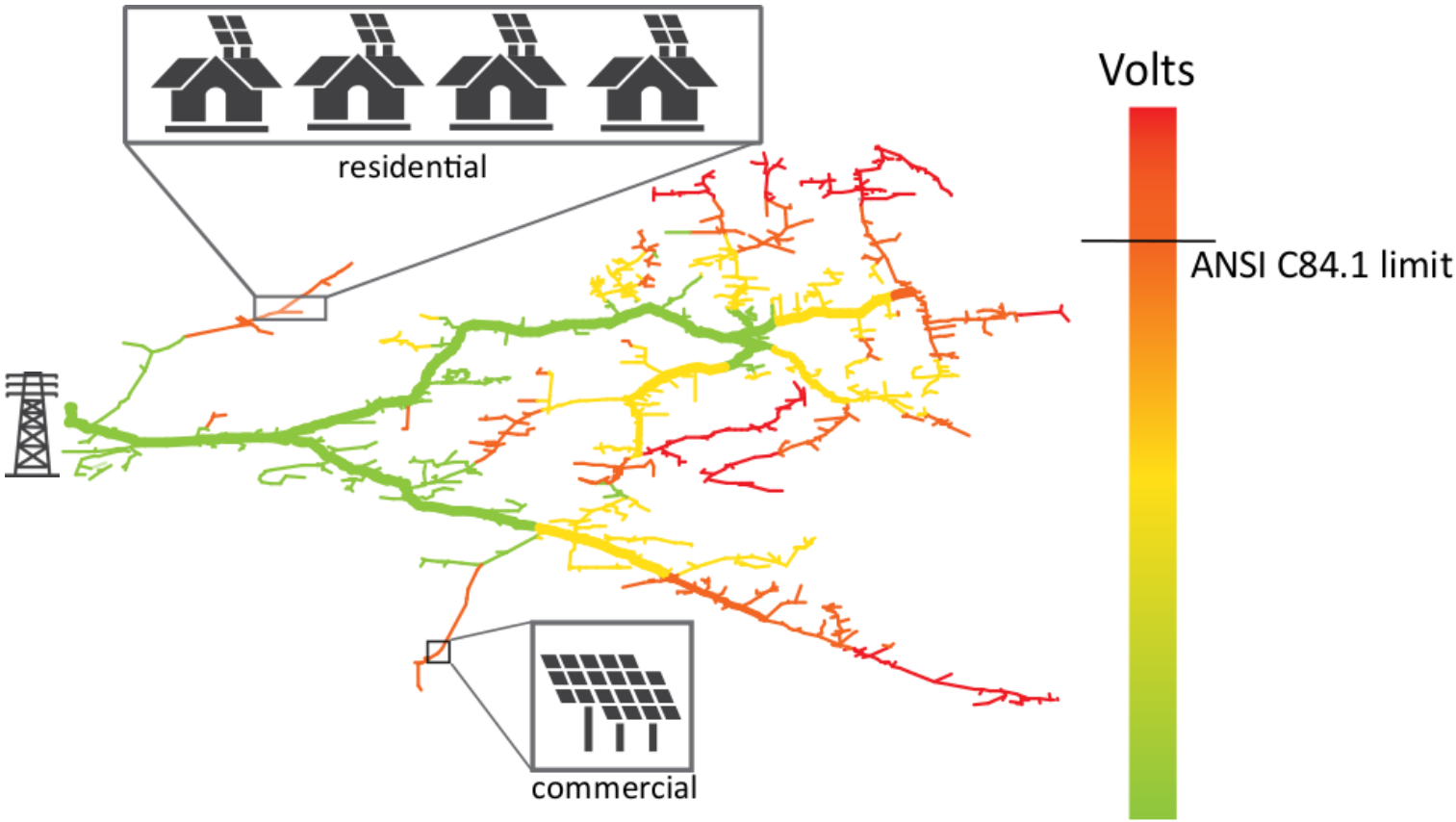
$$\frac{\eta - \sqrt{\eta^2 - L_{\nu,\epsilon}^2 E}}{L_{\nu,\epsilon}^2} < \alpha < \frac{\eta + \sqrt{\eta^2 - L_{\nu,\epsilon}^2 E}}{L_{\nu,\epsilon}^2}, \quad E := 1 - \frac{1}{(1+\beta_1)(1+\beta_2)}$$

Case study

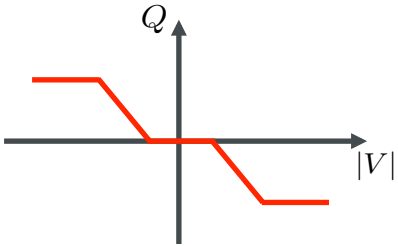


- Increased likelihood of overvoltage conditions due to “reverse power flow” [Liu-Bebic’08]

Case study

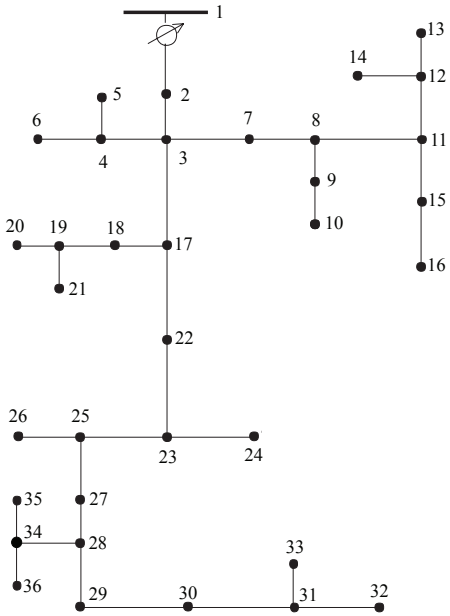


- Currently explored:
  - Volt/VAr
  - Real power curtailment



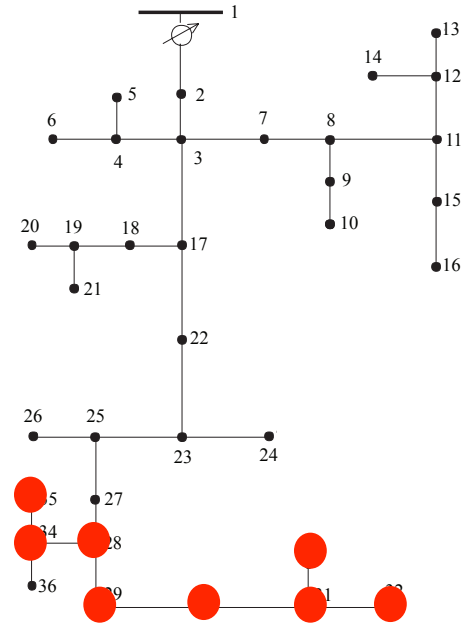
Case study

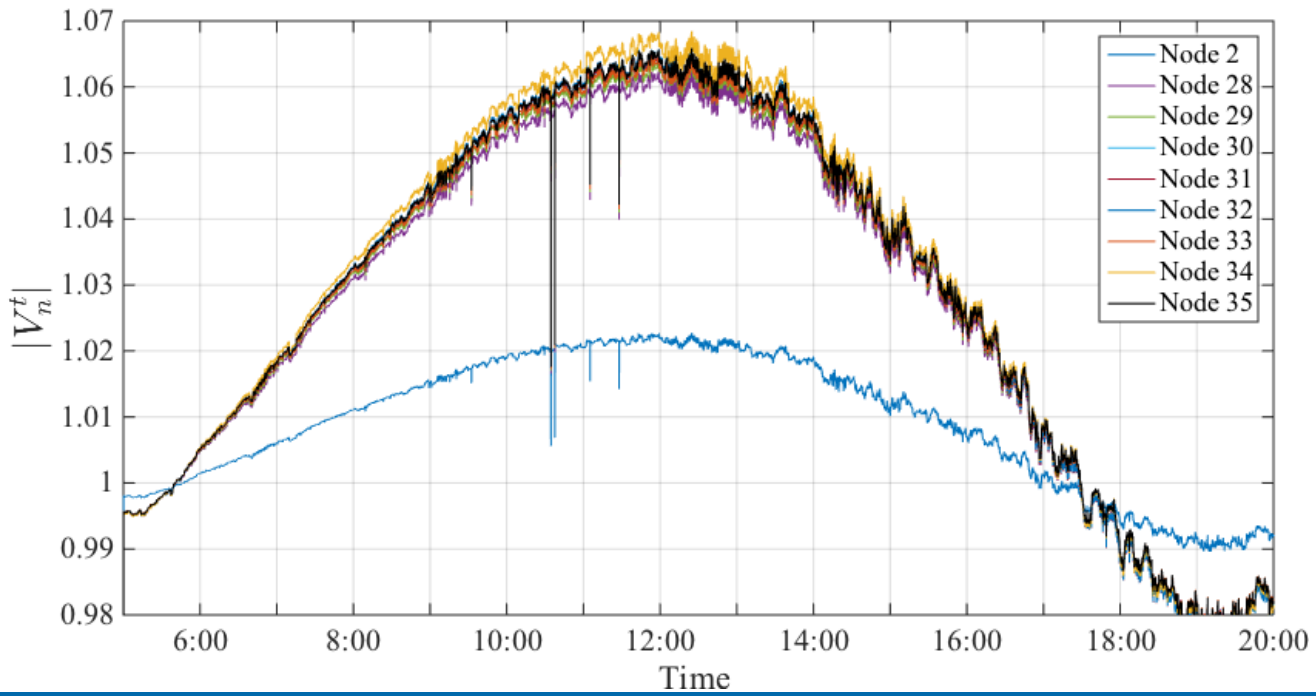
- IEEE 37-node test feeder
- Real load and solar irradiance data from Anatolia, CA
- First-order-type response of PV inverters



Case study

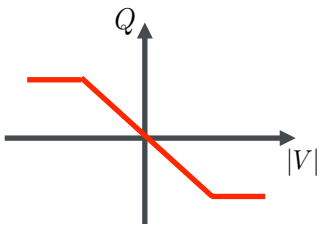
- IEEE 37-node test feeder
- Real load and solar irradiance data from Anatolia, CA
- PQ updated every 300ms
- $\sum_{i \in \mathcal{G}} c_q (Q_i^k)^2 + c_p (P_{\text{av},i}^k - P_i^k)^2$

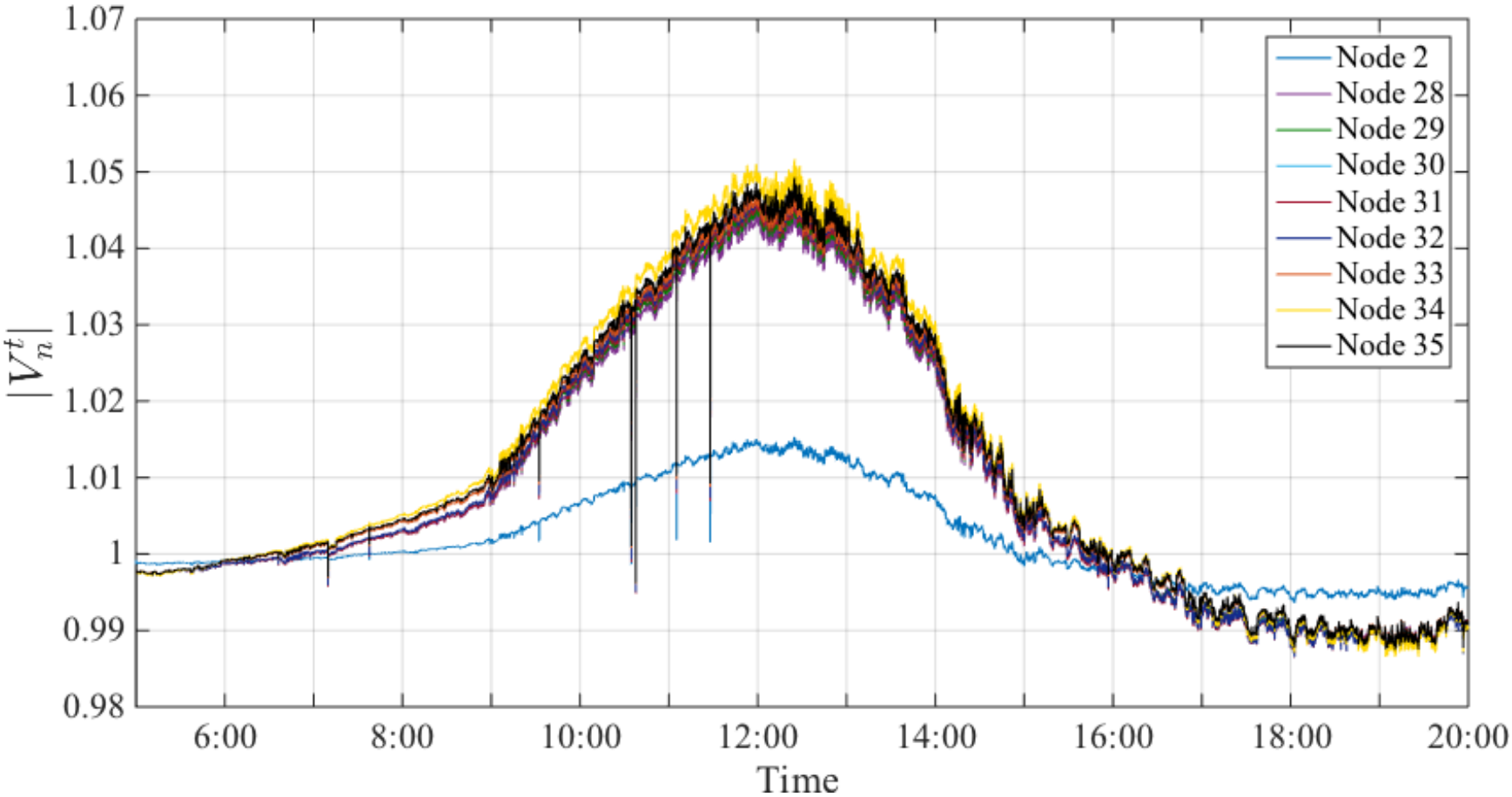




Case study

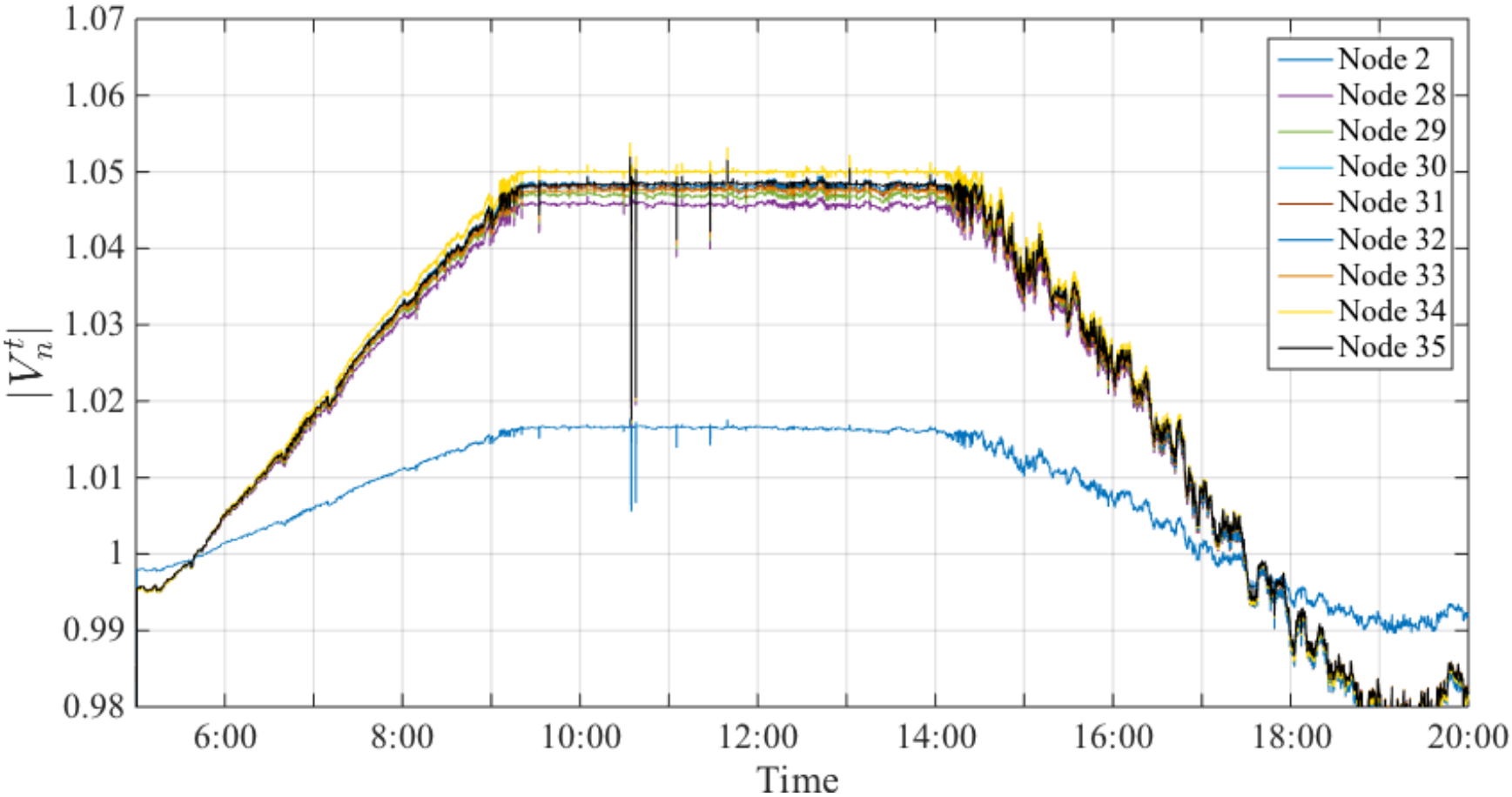
- Volt/VAr, no deadband





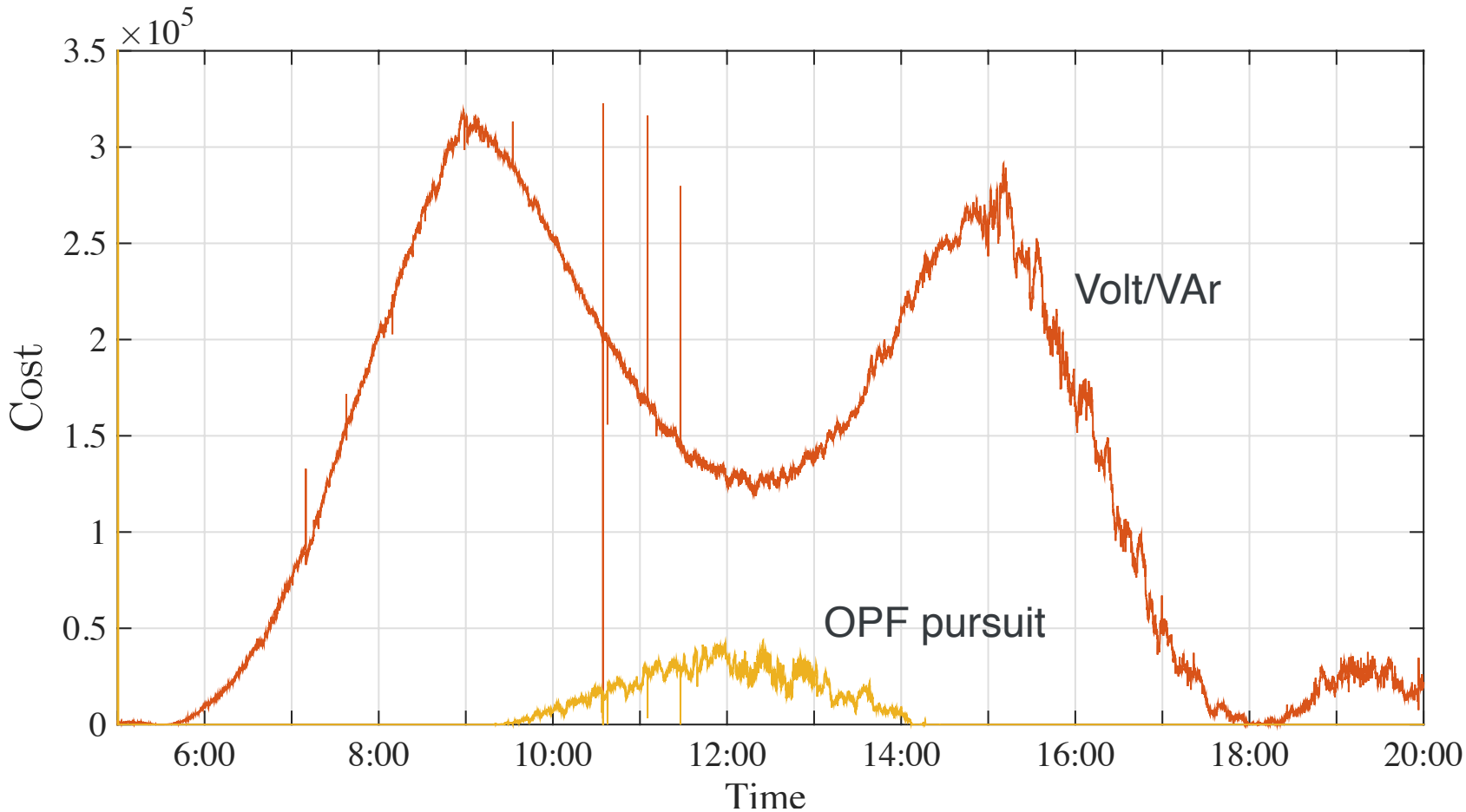
Case study

- Proposed controllers



Case study

- Cost of ancillary service provisioning



Concluding remarks

- Bridging the time scales of control and optimization
- Increase speed of power command computation to ensure adaptability and optimality
- Convergence to OPF solution
- Future efforts towards
  - Validation and implementation
  - Extension to account for various grid constraints and services

Thank you!

Comments?

Backup slides

OPF point pursuit

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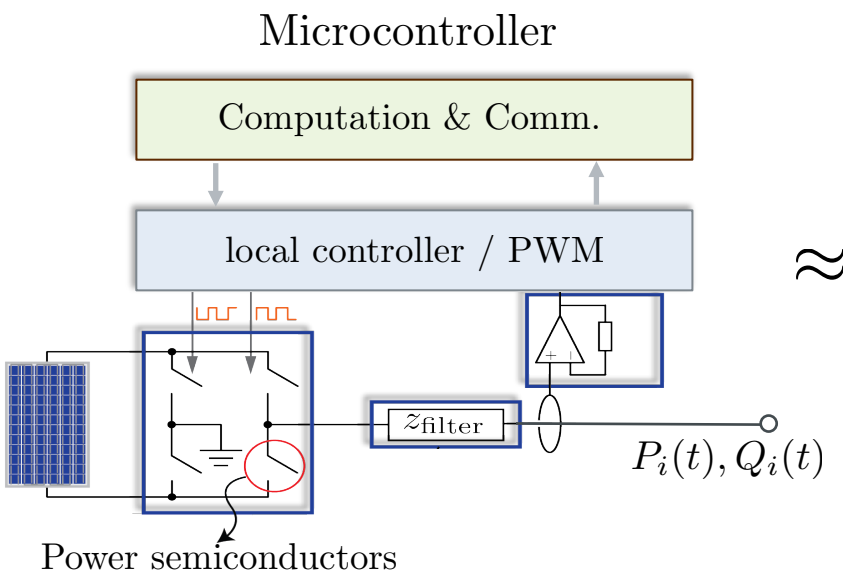
$$L_{\nu,\epsilon} \longleftrightarrow \boldsymbol{\Phi}^k : \{\mathbf{u}^k, \boldsymbol{\gamma}^k, \boldsymbol{\mu}^k\} \to \begin{bmatrix} \nabla_{\mathbf{u}_i}\mathcal{L}_{\nu,\epsilon}^k(\mathbf{u},\boldsymbol{\gamma},\boldsymbol{\mu})|_{\tilde{\mathbf{u}}_i^k,\tilde{\boldsymbol{\gamma}}^k,\tilde{\boldsymbol{\mu}}^k} \\ -(g_n^k(\{\tilde{\mathbf{u}}_i^k\}_{i\in\mathcal{G}}) - \epsilon\tilde{\gamma}_n^k) \\ -(\bar{g}_n^k(\{\tilde{\mathbf{u}}_i^k\}_{i\in\mathcal{G}}) - \epsilon\tilde{\mu}_n^k) \end{bmatrix}$$

Exploiting inverter flexibility



$\approx$

- $\dot{\mathbf{x}}_i(t) = \mathbf{f}_i\Big(\mathbf{x}_i(t), \mathbf{d}_i(t), \mathbf{u}_i\Big)$
- $\mathbf{x}_i(t) := \begin{bmatrix} P_i(t) \\ Q_i(t) \end{bmatrix}$ (output powers)
- $\mathbf{u}_i := \begin{bmatrix} P_i \\ Q_i \end{bmatrix}$ (commanded setpoint)

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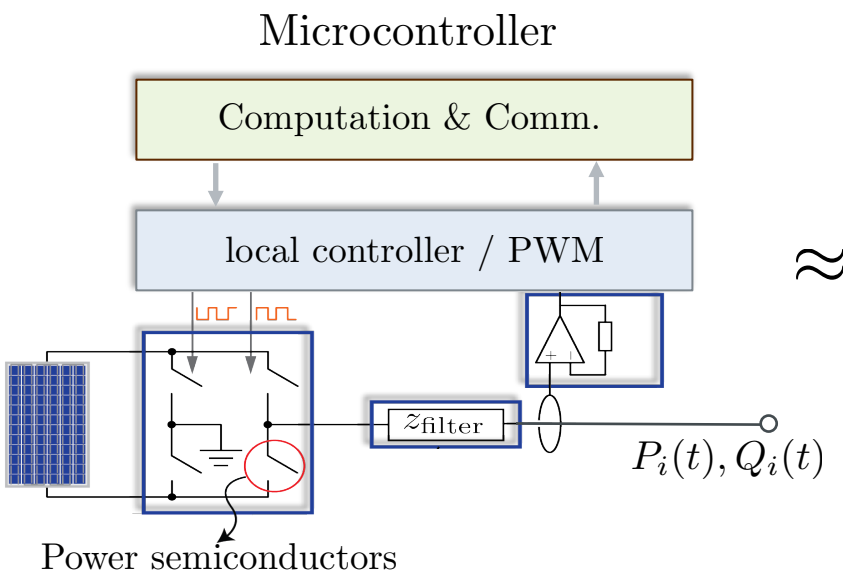

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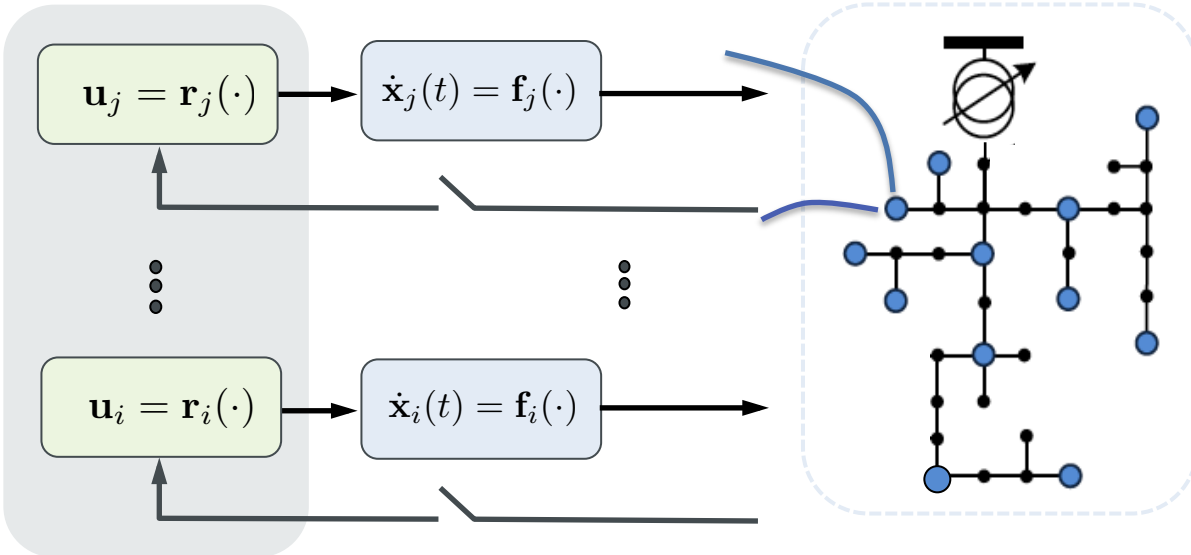
- For constant commanded inputs $\mathbf{u}_i \in \mathcal{Y}_i$, there exists an equilibrium point $\mathbf{x}_i \in \mathcal{Y}_i$ that satisfies:

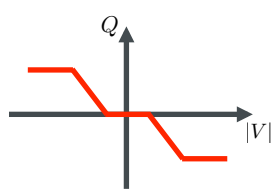
$$\mathbf{0} = \mathbf{f}_i\left(\mathbf{x}_i, \mathbf{d}_i, \mathbf{u}_i\right)$$
$$\mathbf{x}_i = \mathbf{u}_i$$

and the this point is asymptotically stable [Yazdani-Iravani '10], [Dorfler et al'14], [Johnson et al'15]

Local control paradigms



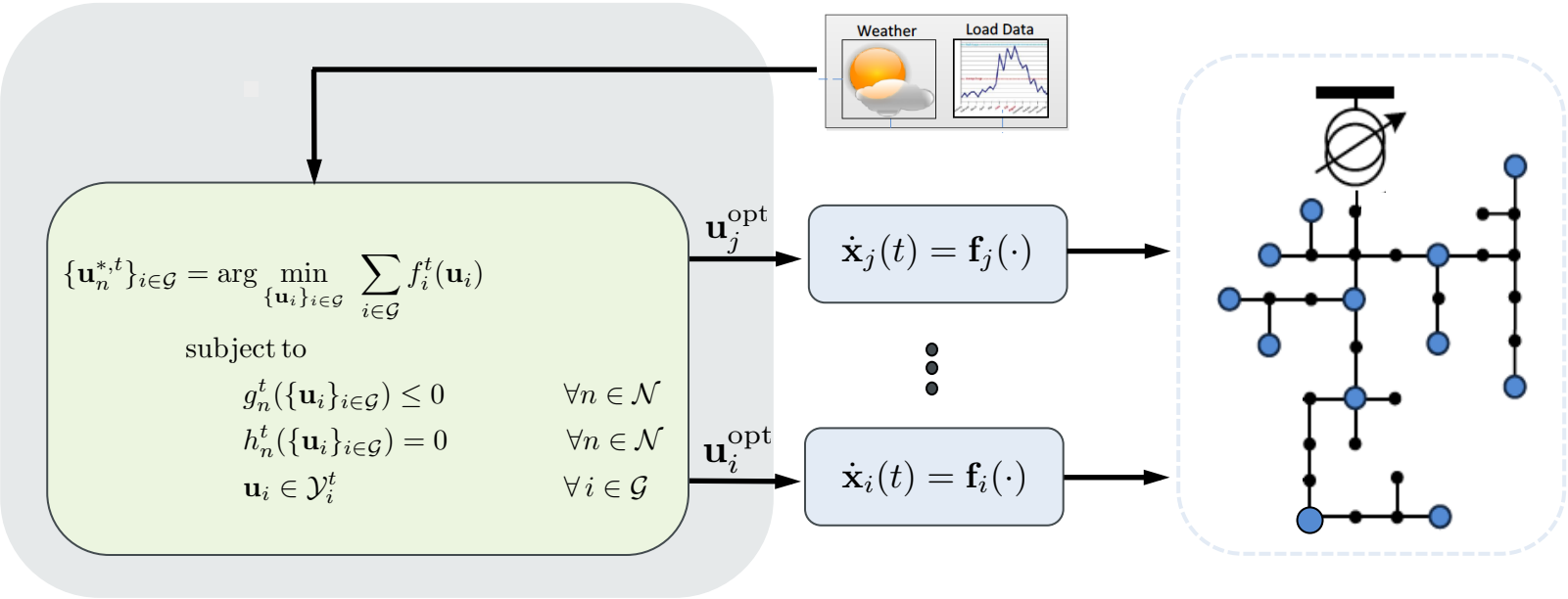
Local rules

- Volt/VAr control: 

- Active power curtailment [Tonkoski et al'12]: $P_i[t_k] = P_i^{\text{av}} - m(|V_i[t_k]| - |V^{\text{ref}}|)$

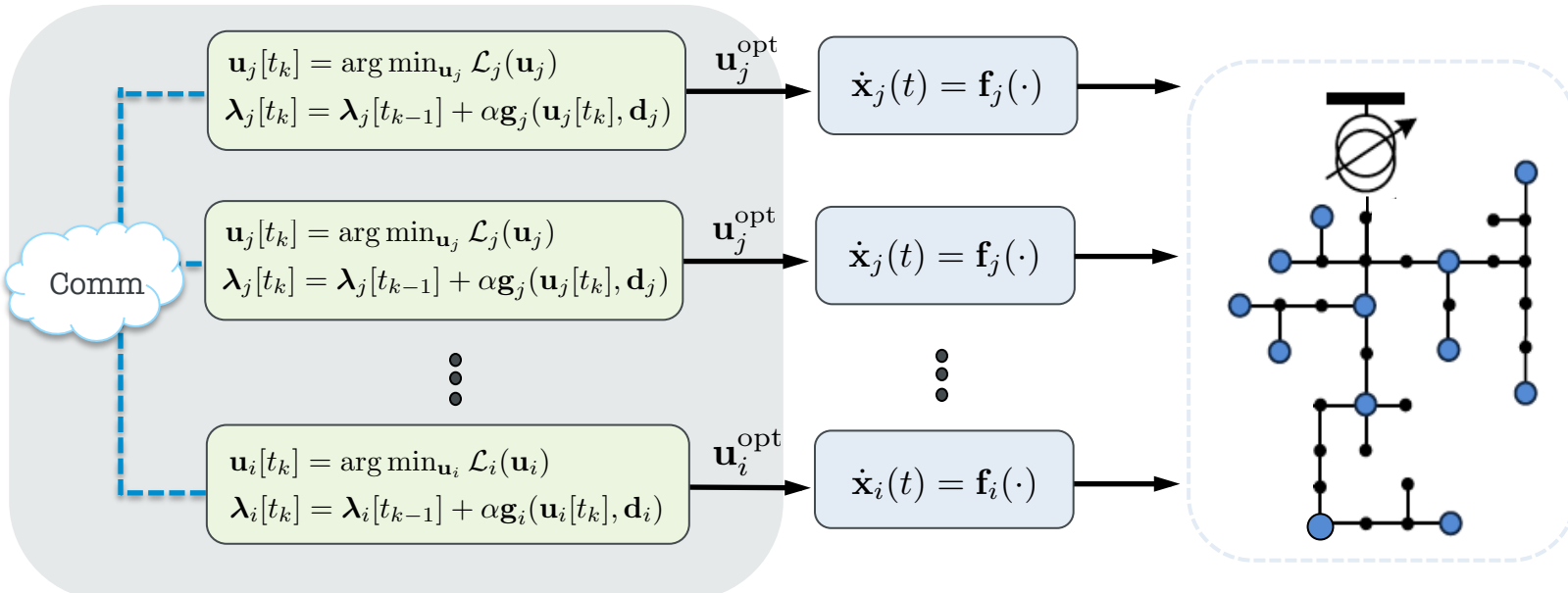
- System-level benefits? Stability?

Network optimization paradigms



- Optimal power flow (OPF) formulations
  - Centralized – via MINLP solvers [Khodr et al '07] or convex relaxations [Lavaei-Low'12]

Network optimization paradigms



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  - Centralized – via MINLP solvers [Khodr et al ’07] or convex relaxations [Lavaei-Low’12]
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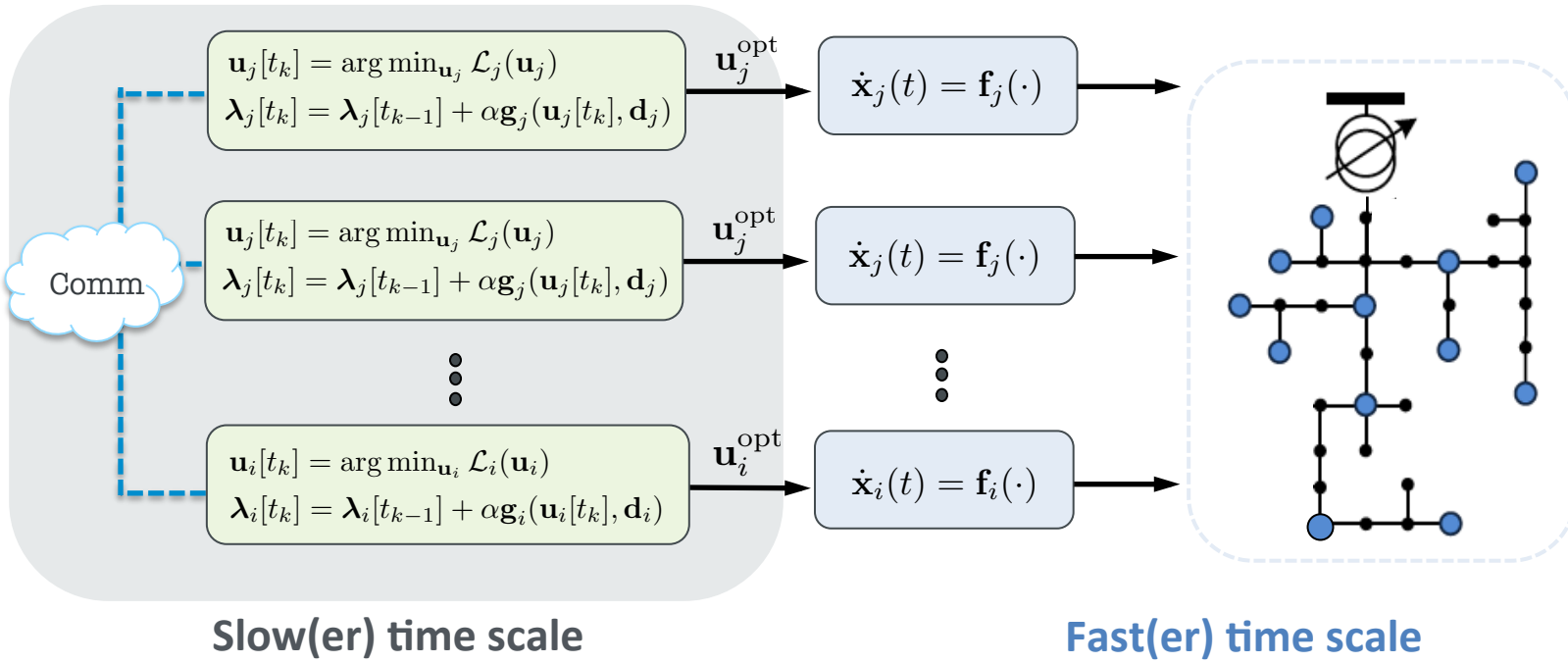
Network optimization paradigms



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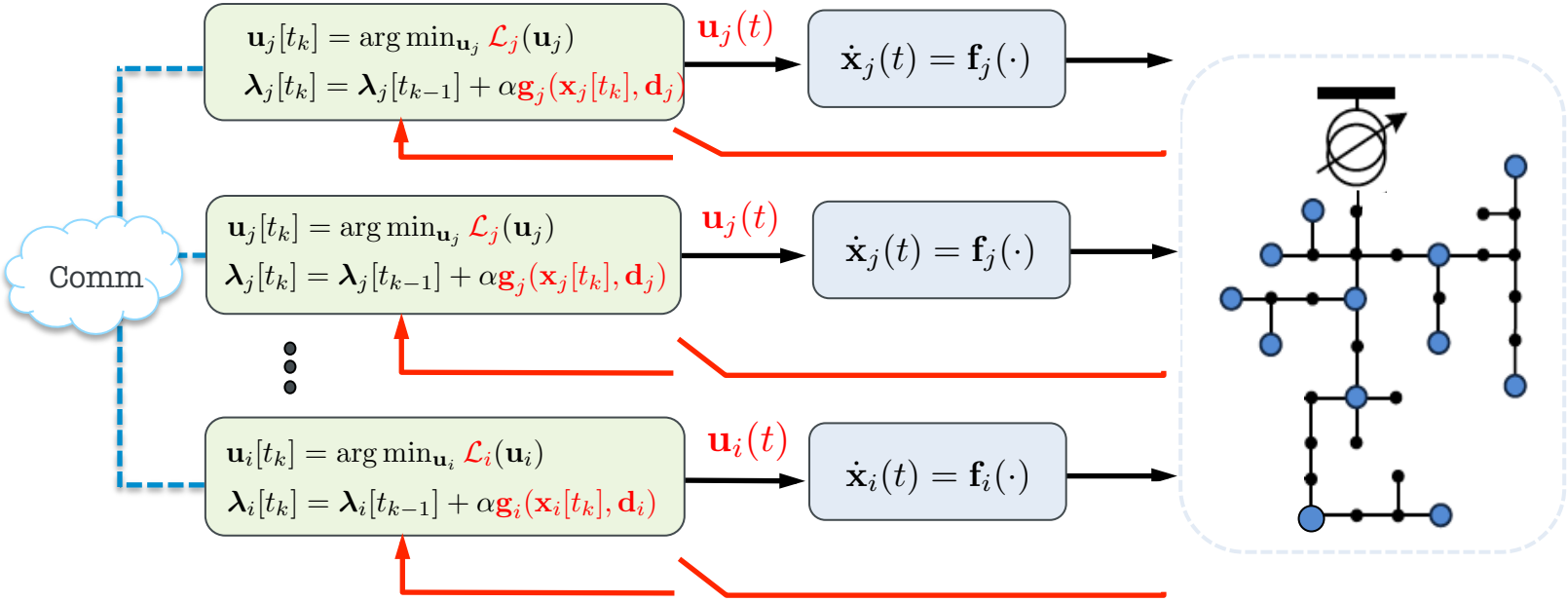
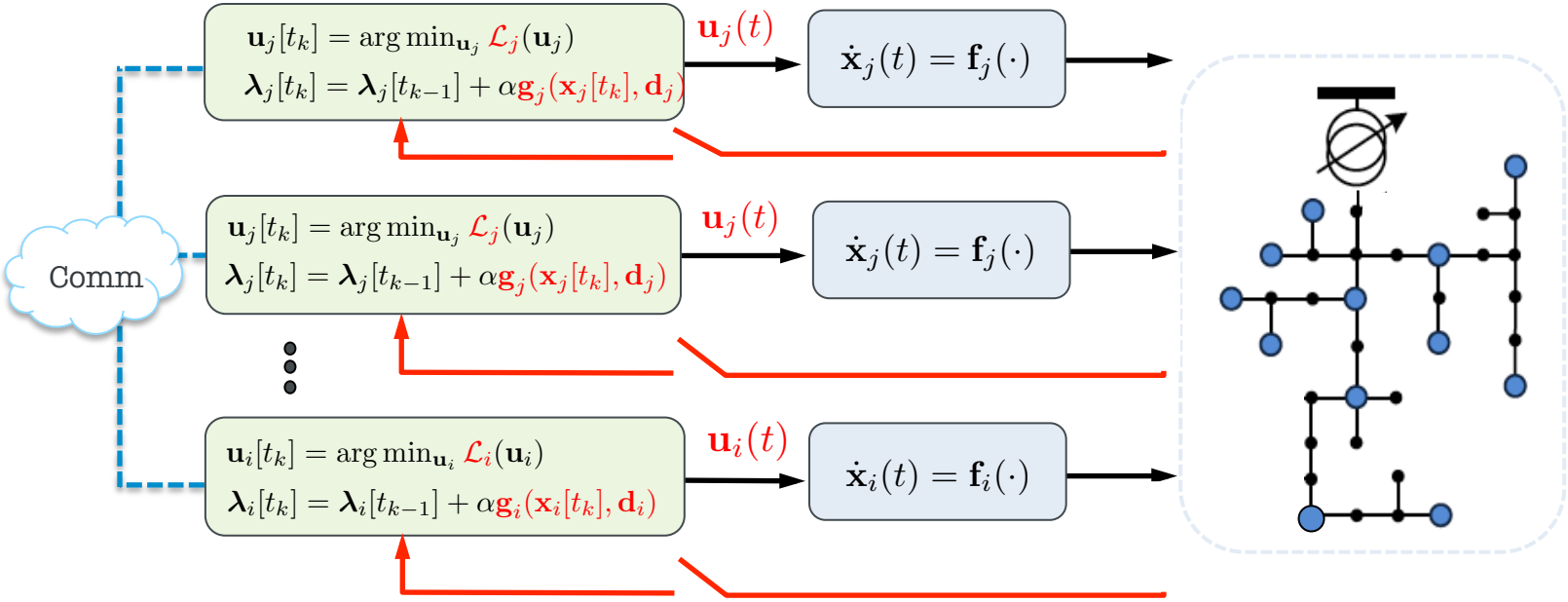
- Do not acknowledge grid-edge dynamics

Shrink time scales



- **Goal: synthesis of controllers that seek solutions of optimization problems**

Shrink time scales



- **Goal: synthesis of controllers that seek solutions of optimization problems**
- Benefits: adaptability to (fast-)changing conditions; operation efficiency
- Key questions: convergence? Communications and complexity constraints?