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Coordinated Optimization of Distributed Energy Resources and Smart Loads in Distribution Systems

Rui Yang, *Member, IEEE*, Yingchen Zhang, *Member, IEEE*

***Abstract*—Distributed energy resources (DERs) and smart loads have the potential to provide flexibility to the distribution system operation. A coordinated optimization approach is proposed in this paper to actively manage DERs and smart loads in distribution systems to achieve the optimal operation status. A three-phase unbalanced Optimal Power Flow (OPF) problem is developed to determine the output from DERs and smart loads with respect to the system operator's control objective. This paper focuses on coordinating PV systems and smart loads to improve the overall voltage profile in distribution systems. Simulations have been carried out in a 12-bus distribution feeder and results illustrate the superior control performance of the proposed approach.**

I. Introduction

The distribution grid has been undergoing dramatic transformation in recent years. On one hand, the penetration level of the distributed energy resources (DERs) such as photovoltaic (PV) systems keeps increasing. On the other hand, the electricity consumers become active players due to their capabilities of performing demand side management. These changes impose great challenges as well as opportunities on how to manage the distribution system.

With the increasing penetration of DERs, there is growing interest in how to incorporate these resources into the active operation of distribution systems. For instance, with the advanced inverter control functionalities, PV systems are capable of changing their active and reactive power for grid services. By optimally controlling the inverters, PV systems are able to provide additional services and values to the distribution system operation, such as voltage regulation [1], [2]. Hence, DERs will benefit the overall distribution system performance if controlled properly.

The other major change happening in the distribution grid is the implementation of the demand response, which allows loads to adjust their consumption according to either the direct control signals from the utility [3] or the price signals [4]. With loads being flexible, it is possible to reduce the peak demand in the distribution network and to decrease the total power drawn from the substation [5]. Furthermore, with the rapid development of smart appliances and home automation, in-house home energy management system is developed, operating the appliances based on the customer's own control objective [6]. With this management system, loads have the potential of being even more flexible without sacrificing customers' satisfaction. Therefore, smart loads become essential resources for the distribution system operation.

Both DERs and smart loads may aid operation flexibility in distribution systems, resulting in better system performance and lower energy bills for consumers. In this paper, the focus lies on how to incorporate these resources into the distribution system operation such that the overall system performance will be improved. A coordinated optimization approach for DERs and smart loads is proposed in this paper. A three-phase unbalanced Optimal Power Flow (OPF) problem is developed, which allows the optimal coordination of all the available resources in a distribution system to achieve the control objective defined by the distribution system operator. Specifically, in this paper the PV system is used as an example of DERs to demonstrate the proposed coordinated optimization approach with the objective to improve the voltage profile in a distribution system.

The rest of the paper is organized as follows: First, an overview over the proposed coordinated optimization approach for DERs and smart loads is given in Section II. In Section III, the models of DERs, smart loads and the distribution system are described, followed by the formulation of the OPF problem in Section IV. Simulation results in a 12-bus distribution feeder are given in Section V. Section VI concludes the paper.

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II. Coordinated Optimization

Conventionally, operating distribution systems doesn't require sophisticated optimization because of the limited number of resources. As pointed out in the introduction, that nature has changed in distribution systems because the increasing number of DERs and smart loads can be utilized as resources. This paper presents a novel approach to coordinate all the resources in a distribution system in an optimized fashion. An overview over the proposed approach is given in Fig. 1.

A utility scale PV system is depicted in Fig. 1 as an example of DERs. Other types of DERs, such as battery storage, distributed wind, can also be incorporated into the proposed coordinated optimization approach by modeling these resources properly. As shown in Fig. 1, each PV plant tells the system operator its available maximum active power while each smart load provides its desired power consumption and the flexibility range in which its consumption can be adjusted. An OPF problem is formulated by the system operator, which

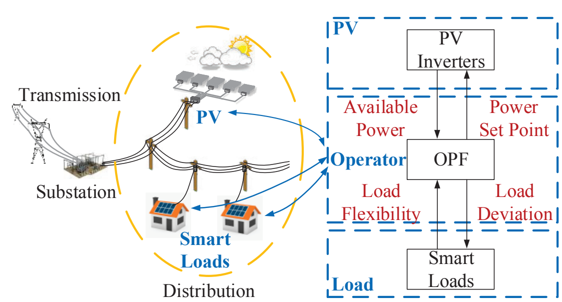


Fig. 1. Overview of the proposed approach

takes into account all the available resources as well as all the operational constraints in the system. By solving this OPF problem, the optimal active and reactive power set points for the PV plants and the optimal power consumption by the smart loads are determined and communicated back to each individual resource. With the proposed approach, system-level optimality with respect to the operator's control objective can be achieved.

III. Modeling

A. PV

In this paper, the photovoltaic (PV) system is used as an example of DERs to demonstrate the proposed coordinated optimization approach. Specifically, stand-alone PV plants equipped with advanced inverters are considered. With advanced inverter control functionalities, the PV plant is capable to communicate with the distribution system operator and control its active and reactive power output in real-time. In order to fully explore the potential benefits to the distribution system performance by controlling the PV inverter, both the active and reactive power output from the PV inverter are allowed to be adjusted. The active power generation from the PV inverter is limited by the maximum active power available for a given solar irradiance while the reactive power output limited by the rated apparent power of the inverter as well as the active power generation. For a single-phase or three-phase PV inverter at bus k, let $\mathcal{P}_{S,k} \subseteq \{a_k, b_k, c_k\}$ denote the set of phases to which the PV inverter is connected, $\overline{P}_{S,k}^{\phi}$ ($\phi \in \mathcal{P}_{S,k}$) the maximum active power available and S_k^{ϕ} the rated apparent power for phase ϕ. The active $P_{S,k}^{\phi}$ and reactive power output $Q_{S,k}^{\phi}$ from the PV inverter in phase ϕ are limited by:

$$0 \leq P_{S,k}^{\phi} \leq \overline{P}_{S,k}^{\phi} \tag{1}$$

$$|Q_{S,k}^{\phi}| \leq \sqrt{S_k^2 - P_{S,k}^{\phi^2}} \tag{2}$$

For a three-phase PV inverter, it is assumed that the power output from each phase can be controlled independently.

B. Load

By participating in the demand side management, the loads are flexible to a certain extent, which allows adjusting the load consumption if needed. For every single load in the system, the most desirable power consumption and how flexible the consumption could be is determined by its own management system based on its needs. For example, a home management system could determine its consumption range at a certain time by minimizing or maximizing its controllable loads such as HVAC. Hence, for a single-phase or three-phase wye-connected load at bus k, the active $P_{L,k}^{\phi}$ and reactive power consumption $Q_{L,k}^{\phi}$ in phase ϕ ($\phi \in \mathcal{P}_{L,k}$) are modeled as:

$$P_{L,k}^{\phi,min} \leq P_{L,k}^{\phi} \leq P_{L,k}^{\phi,max} \tag{3}$$

$$Q_{L,k}^{\phi} = \sqrt{\frac{1}{PF_{L,k}^{\phi^2}} - 1} \cdot P_{L,k}^{\phi} \tag{4}$$

where $\mathcal{P}_{L,k}$ is the set of phases to which the load is connected and $P_{L,k}^{\phi,min}$ and $P_{L,k}^{\phi,max}$ represent the minimum and maximum active power consumption determined by the load management system. While the active power consumption of the load is flexible within a range, the reactive power consumption is determined by the active power consumption and the power factor of this load, as shown in (4). Again, for a three-phase wye-connected load, the power consumption in each phase can be adjusted independently.

C. System

Unlike the transmission system, the distribution system is inherently unbalanced, due to the existence of single-phase, two-phase and three-phase laterals, un-transposed lines and different configurations of unbalanced loads in the system. The single-phase equivalent models as used in the transmission system are not applicable in the distribution network. Hence, the distribution system is modeled using the three-phase AC power flow equations in this paper. In the following, the models for system components, i.e., distribution lines and transformers, are first described briefly and then the AC power flow equations are presented.

The π-model is used to model the distribution line [7]. For a distribution line connecting buses i and j, let $Z_{l,ij}$ and $Y_{s,ij}$ denote the series impedance and shunt admittance matrices for this line, respectively. The line currents from buses i and j are:

$$\begin{bmatrix} I_{ij} \\ I_{ji} \end{bmatrix} = \begin{bmatrix} Z_{l,ij}^{-1} + \frac{1}{2}Y_{s,ij} & -Z_{l,ij}^{-1} \\ -Z_{l,ij}^{-1} & Z_{l,ij}^{-1} + \frac{1}{2}Y_{s,ij} \end{bmatrix} \cdot \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{5}$$

where I_{ij} is the vector of line currents in all phases from bus i to j and I_{ji} the line current vector from j to i while V_i and V_j the vectors of voltages in all phases at buses i and j. Define the branch admittance matrix Y_{ij}^{BR} for line ij as [8]:

$$Y_{ij}^{BR} = \begin{bmatrix} Z_{l,ij}^{-1} + \frac{1}{2}Y_{s,ij} & -Z_{l,ij}^{-1} \\ -Z_{l,ij}^{-1} & Z_{l,ij}^{-1} + \frac{1}{2}Y_{s,ij} \end{bmatrix} = \begin{bmatrix} Y_{ij}^{ii} & Y_{ij}^{ij} \\ Y_{ij}^{ji} & Y_{ij}^{jj} \end{bmatrix} \tag{6}$$

Hence, the line current I_{ij} can be written as:

$$I_{ij} = Y_{ij}^{ii} \cdot V_i + Y_{ij}^{ij} \cdot V_j \tag{7}$$

Similarly, a branch admittance matrix Y_{ij}^{BR} is used to model the transformer connecting buses i and j, which is determined by the connection configuration along with primary and secondary parameters of the transformer [8]. The same equation (7) representing current I_{ij} can be used.

The current flowing out of bus i in phase ϕ is:

$$I_i^{\phi} = \sum_{j \in \mathcal{N}_i} \left(Y_{ij}^{ii} \cdot V_i + Y_{ij}^{ij} \cdot V_j \right)_{\{\phi\}} \tag{8}$$

where $\mathcal{N}_i$ is the set of buses which are connected with bus i. The complex power balance equation at bus i phase ϕ is:

$$V_i^{\phi} \cdot (I_i^{\phi})^* = P_{G,i}^{\phi} + P_{S,i}^{\phi} - P_{L,i}^{\phi} + j\left(Q_{G,i}^{\phi} + Q_{S,i}^{\phi} - Q_{L,i}^{\phi}\right) \tag{9}$$

where $P_{G,i}^{\phi}$, $Q_{G,i}^{\phi}$ are the power generation by the conventional generator, $P_{S,i}^{\phi}$, $Q_{S,i}^{\phi}$ the power generation by the PV plant and $P_{L,i}^{\phi}$, $Q_{L,i}^{\phi}$ the load consumption at bus i phase ϕ.

IV. OPF Formulation

In order to coordinate all the available resources in the distribution system to improve the overall system performance, an Optimal Power Flow (OPF) problem is developed to optimally manage all the resources in the system.

A. Control Variables

In this paper, the main focus is to optimally coordinate DERs and smart loads for regulating voltages in the distribution network. As the stand-alone PV plant is used as an example for DERs to demonstrate the proposed OPF problem, the control variables in this optimization problem include

- PV: active and reactive power output, $P_{S,k}^{\phi}$ and $Q_{S,k}^{\phi}$
- Smart loads: active power consumption, $P_{L,k}^{\phi}$

B. Constraints

The constraints of the OPF problem include the constraints of the system components, i.e., PV plants and flexible loads, and the three-phase AC power flow equations, defined by equations (1) - (9) in Section III. Furthermore, operational constraints on system voltages are also included in the OPF problem. Unlike the traditional approach of formulating the voltage constraints as hard constraints, which ensures the voltages are regulated within a certain range, we formulate the voltage constraints as soft constraints in this paper:

$$V_i^{\phi,min} - s_i^{\phi} \leq |V_i^{\phi}| \leq V_i^{\phi,max} + s_i^{\phi}, \quad s_i^{\phi} \geq 0 \quad (10)$$

where $|V_i^{\phi}|$ corresponds to the voltage magnitude at bus i phase ϕ, which is limited by the lower and upper bounds $V_i^{\phi,min}$ and $V_i^{\phi,max}$. The lower and upper bounds are determined by the system operator, which are usually 0.95 p.u. and 1.05 p.u., respectively. By introducing the nonnegative slack variable s_i^{ϕ} in (10), the operational constraints on voltage magnitudes may be violated if necessary. s_i^{ϕ} quantifies the violation of the voltage constraints and thereby provides a means to quantify how good the voltage profile is, which will be taken into account in the objective function.

C. Objective Function

For a distribution system operator, various control objectives for managing the distribution network may be considered, such as minimizing the active power losses, minimizing the total power drawn from the substation, etc. In this paper, the main objective considered is to optimize the voltage profile by coordinating DERs and smart loads in the distribution network. Here, we formulate the objective functions as a weighted sum of multiple objectives, including:

- minimization of the violation of the voltage constraints
- minimization of the deviation of the loads from the desired consumption values

The mathematical formulation of the objective function is

$$f = \sum_{i \in \mathcal{N}} \sum_{\phi \in \mathcal{P}_i} s_i^{\phi} + \omega_L \cdot \sum_{k \in \mathcal{N}_\mathcal{L}} \sum_{\phi \in \mathcal{P}_{L,k}} \Big(\frac{P_{L,k}^{\phi} - P_{L,k}^{\phi,ref}}{P_{L,k}^{\phi,ref}} \Big)^2 \quad (11)$$

where $\mathcal{N}$ and $\mathcal{N}_L$ denote the set of all buses and the set of buses which have loads connected while $\mathcal{P}_i$ and $\mathcal{P}_{L,k}$ the set of phases at bus i and the set of phases to which the load at bus k are connected. $P_{L,k}^{\phi,ref}$ corresponds to the desired active power consumption for the load at bus k phase ϕ, which is determined by its own management system.

The first term in the objective function represents the total voltage violation in the system while the second term the summation of squared relative deviations of the loads from the desired consumption values. The weighting parameter ω_L represents the relative importance of one term with respect to the other in the objective function. By choosing the weighting parameter appropriately, the total voltage violation will be minimized without significantly adjusting the load consumption from the desired values.

Based on the needs of the distribution system operator, other control objectives can also be considered in the proposed OPF problem, just by formulating the appropriate objective function in the optimization problem.

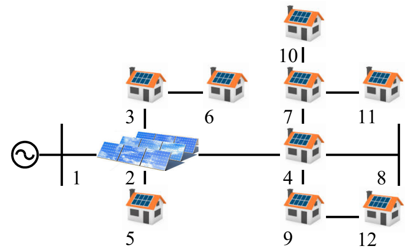


Fig. 2. A 12-bus distribution system

V. Simulation Results

A. Simulation Setup

Simulations have been carried out in a 12-bus distribution feeder shown in Fig. 2. A stand-alone PV plant with a three-phase inverter is at bus 2 and all the loads in the system are considered as smart loads.

The aggregated load data and the solar irradiance data from a real distribution feeder is used to create realistic total load and PV generation curves in 5-minute resolution for a whole day in this simulation. The peak demand for the three phases in total is around 4.2 MW while the rated capacity for the PV plant is 300 kVA. For each smart load, its desired active power consumption at every 5 minute is determined by the total demand in the system and its percentage with respect to the total demand. It is assumed that the active power consumption of every load can be adjusted within $\pm 20\%$ of its desired consumption. The lower and upper limits for the voltage magnitude are 0.95 p.u. and 1.05 p.u., respectively. The weighting parameter ω_L in the objective function is chosen to be 1. In the following, the 5-minute simulation results are first shown, followed by the 24-hour results.

B. 5-Minute Results

In this section, we use one scenario for load and PV to illustrate how the PV plant and smart loads are coordinated to optimize the voltage profile in the system. This scenario occurs at 11:05am and the total intended consumption in each phase is 1.1814 MW, 1.0275 MW and 1.2438 MW, respectively. Fig. 3 shows the voltage magnitudes of all buses in the system for the following two cases: 1) coordinating the PV plant and smart loads using the proposed approach ('opt') and 2) no PV plant and load not being flexible ('w/o c').

Since the total load in phase B is smaller than the load in the other two phases, in the case when there is no PV plant and smart loads, the voltage magnitudes in phase B are higher than those values in phases A and C and there is no voltage violation in phase B. As seen in Fig. 3, compared to the case without the PV plant and smart loads, the voltage magnitudes in phases A and C are increased for buses 2 to 12 if the PV plant and smart loads are coordinated using the

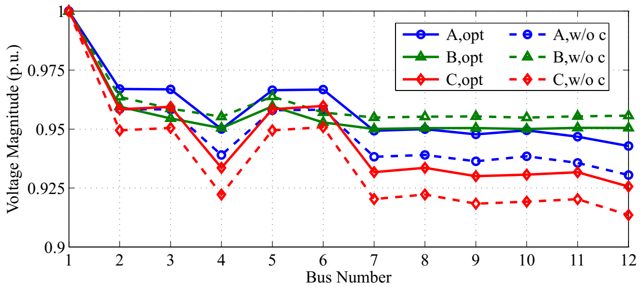


Fig. 3. Voltage magnitude of buses

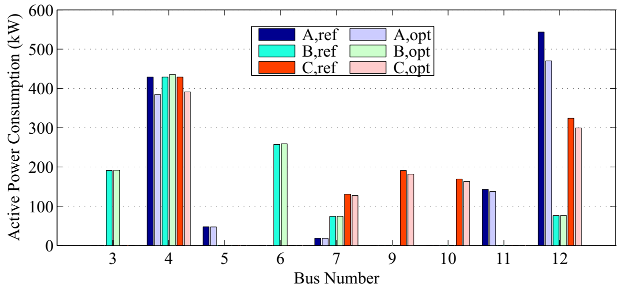


Fig. 4. Active power consumption of smart loads

TABLE I
PV OUTPUT

Phase ϕ	$\overline{P}_S^\phi$ (kW)	$\hat{Q}_{S,Lim}^\phi$ $(kVar)$	P_S^ϕ (kW)	$Q_{S,Lim}^\phi$ $(kVar)$	Q_S^ϕ $(kVar)$
A	94.6709	±32.2090	94.6709	±32.2090	32.2090
B	94.6709	±32.2090	0	±100	-100
C	94.6709	±32.2090	85.0634	±52.5759	52.5759

proposed approach. For phase B, the voltages become lower if the coordinated optimization is employed, however, all the voltage magnitudes are still higher than or equal to 0.95 p.u., i.e., no voltage violation is seen in phase B. Therefore, using the proposed optimization for the PV plant and smart loads the total voltage violation in the system is reduced significantly. Moreover, the three-phase voltages are more balanced without optimizing with respect to this objective specifically.

Fig. 4 depicts the desired ('ref') and optimal ('opt') active power consumption of smart loads. In Fig. 4, the active power consumption for loads in phases A and C is curtailed while the consumption for loads in phase B increases. The largest deviation happens for the load at bus 12 phase A, whose consumption is curtailed by 13.47%.

Table I summarizes the active and reactive power output from the PV plant. $\overline{P}_S^\phi$ represents the maximum active power available and $\hat{Q}_{S,Lim}^\phi$ the limits for the reactive power if $\overline{P}_S^\phi$ is produced. P_S^ϕ corresponds to the optimal active power generation, $Q_{S,Lim}^\phi$ the limits for the reactive power associated with P_S^ϕ and Q_S^ϕ the optimal reactive generation/absorption. As shown in Table I, the active power generation in phase A stays as its maximum value while the active power generation in the other two phases is curtailed so that more reactive power can be absorbed or generated. Especially, no active power is produced in phase B, resulting in zero power factor in this phase. Since operating the PV inverter at low power factors may not be desirable for the system operator, additional constraints limiting the power factor of PV inverters may be added in the proposed OPF problem.

In summary, the overall voltage profile in the system is improved with the optimal coordination of the PV plant and smart loads using the proposed optimization approach.

C. 24-Hour Results

In the following, simulation results for the whole day are presented. Fig. 5 depicts the total desired active power consumption by smart loads in each phase. The total voltage violation for each phase is shown in Fig. 6. Fig. 7 shows the ratio of the total optimized demand in each phase with respect to the total desired consumption, i.e., $\frac{\sum_k P_{L,k}^\phi}{\sum_k P_{L,k}^{\phi,ref}}$. The active and reactive power from the PV plant are shown in Fig. 8.

Throughout the day, phase B has the smallest demand while phase C has the largest. As seen in Fig. 6, the total voltage violation in phase B is zero most of the day and the small violation only occurs during the highest demand period, i.e., around noon and from 6pm to 9pm, while the voltage violation in phase C is significantly larger than the violation in the other two phases. In Fig. 7, the ratio of the total optimized demand with respect to the total desired consumption in phase B is larger than 1 in most of the scenarios, which means the total consumption for the loads in phase B is increased while the increment is only around 1% most of the time. During the time period of the peak demand, the load consumption in phase B is also curtailed in order to reduce the voltage violations in the system and the maximum curtailment for the total demand in phase B is around 5%. For phases A and C, the load ratio in Fig. 7 is smaller than 1 all the time so that the total consumption in these two phases is reduced and the maximum curtailment is around 12%.

In Fig. 8(a), the PV plant's active power generation is shown and the maximum active power which can be generated by the

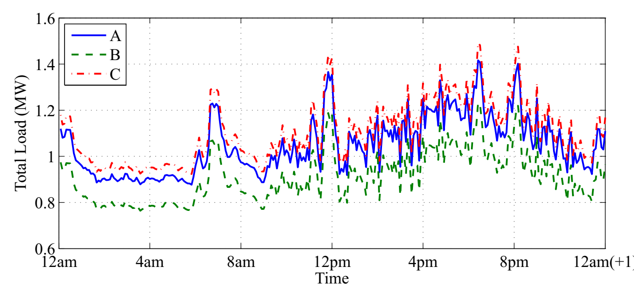


Fig. 5. Total load in the system

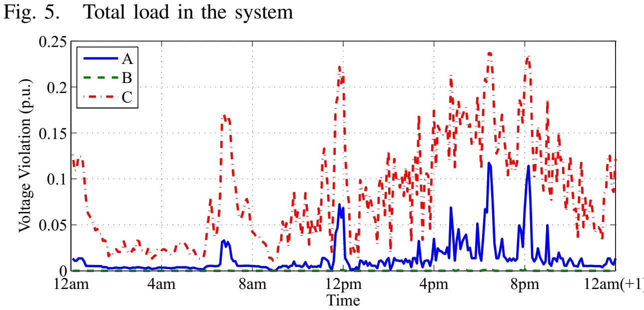


Fig. 6. Voltage violation in three phases

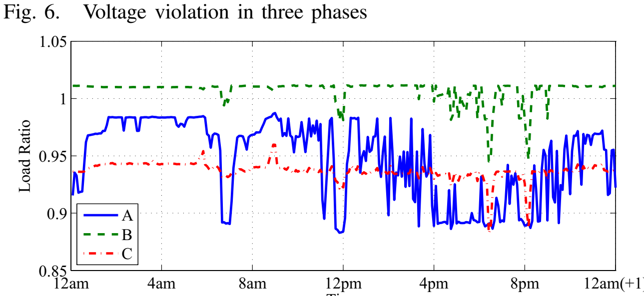
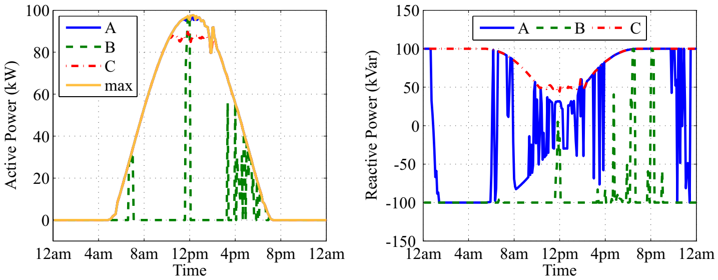


Fig. 7. Load consumption ratio in three phases



(a) Active power

(b) Reactive power

Fig. 8. Power output from PV

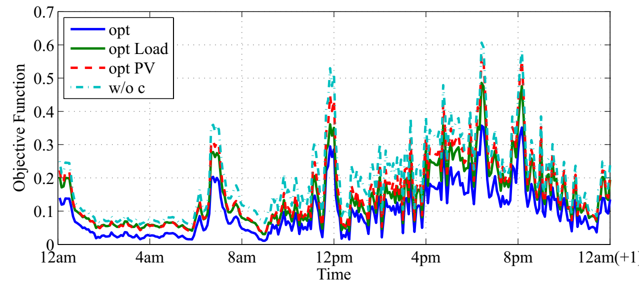


Fig. 9. Comparison of objective function values

PV plant for each phase given the solar irradiance data is also depicted in this figure ('max'). Compared to the maximum active power available, the PV's active power generation in phase A is only curtailed by 3% around noon while the active power generation in phase C gets curtailed more during PV's peak generation period, i.e., from 11am to 2pm. The maximum curtailment for the PV generation in phase C is 13%, which occurs around noon. By curtailing the active power generated from the PV in phase C, more reactive power can be generated. Since phase C has the highest demand and the total load consumption around noon is high, by injecting more reactive power into the system from the PV in phase C, the voltage magnitudes can be increased and consequently voltage violations reduced. For phase B, the active power generated by the PV plant is often curtailed to zero from 8am to 4pm so that the reactive power can be absorbed at the inverter's full capacity. Again, operating the PV inverter at low power factors may not be desirable for both the PV plant owner and the system operator. As mentioned earlier, additional constraints corresponding to the power factor of the PV output can be incorporated into the proposed coordinated optimization approach, rendering the PV plant operated at sufficiently high power factors.

Moreover, simulations have been carried out to illustrate the benefits of optimally coordinating the DERs and smart loads using the proposed approach. Here, we compare the results for the following four cases: 1) optimally coordinating the PV plant and smart loads ('opt'); 2) no control over the PV plant, which is operated at a constant power factor 0.9, only optimizing over the smart loads ('opt Load'); 3) no control over the loads, each of whose consumption is equal to their desired values, only optimizing over the PV plant ('opt PV') and 4) no control over the loads nor the PV plant, which is equivalent to no PV plant in the system and loads not being flexible ('w/o c'). The objective function defined as the summation of the total voltage violation and squared relative deviations of the loads from the desired consumption values is used to quantify how the overall system performance is. The objective function values for the aforementioned four cases in the considered one day are shown in Fig. 9.

In Fig. 9, the objective function values for the case of optimally managing the PV plant and smart loads are smaller than those in the other three cases for the whole day. For the two cases without control over the loads, the load deviation part in the objective function is zero and the objective function only corresponds to the total voltage violation. Compared to those two cases, the objective function values in the optimal case are much smaller, resulting in smaller voltage violations if the PV plant and smart loads are managed optimally. Hence, by allowing the loads to be flexible and managed coordinately with other resources, the voltage profile can be improved significantly. For the case without control over the PV plant, both the voltage violation and the load deviation contribute to the objective function. Compared to the optimal case, the objective function values in the case without control over PV are larger since without controlling the PV either the loads need to be adjusted more to reach the similar level of voltage violations or higher voltage violations occur with the similar amount of load consumption changed depending on system states. Hence, by controlling the active and reactive power output from the PV plant, the overall system performance is improved, i.e., voltage violations reduced without adjusting the load consumption too much.

VI. Conclusion

In this paper, a coordinated optimization approach is developed for optimally managing all the available resources, such as DERs and smart loads to achieve certain control objective determined by the system operator in a distribution system. A three-phase unbalanced OPF problem is formulated and solved to achieve this goal. This paper demonstrates how the PV systems and smart loads are coordinated using the proposed approach to improve the voltage profile in the system. As can be seen in the simulations carried out in a 12-bus distribution feeder, the overall voltage violation is reduced significantly using the proposed coordinated optimization approach. Future work includes incorporating other types of DERs in the proposed approach with different control objectives considered.

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