



# Stochastic Methods for Planning and Operating Power System with Large Amounts of Wind and Solar Power

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# Stochastic Methods for Planning and Operating Power Systems with Large Amounts of Wind and Solar Power

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**Abstract**—Wind and solar generators differ from conventional generators in their generation characteristics. The variable output and imperfect predictability of these generators face a stochastic approach to plan and operate the power system without fundamentally changing the operation and planning problems. This paper overviews stochastic modeling challenges in operations, generation planning, and transmission planning with references to current industry and academic work. Different stochastic problem formulations, including approximations, are also discussed.

**Keywords**—variable generation; probabilistic modeling; generation planning; transmission planning; long-term planning

## I. INTRODUCTION

Wind, solar, and other variable generators are fundamentally different than the thermal generators that dominate power systems today. The output of these generators is subject to natural phenomena that cannot be perfectly forecast. Although the characteristics of these generators are different, the problems a power system planner and operator must solve remain the same. Across timescales, the system must be balanced, operating in a reliable and efficient manner. Sufficient generation and transmission must be planned for and brought into service. However, by its variable and uncertain nature, wind and solar generation complicate the way the problems are solved. Unlike conventional thermal generators, where simplifying assumptions are easily made, using probabilistic methods is the only proper way to plan for the stochastic nature of variable generators.

The tools required for power system operators and planners are not mature. Research-scale models are being developed in academia and other laboratories, but industrial tools are lacking. This paper first outlines several probabilistic operational and planning issues. It then discusses different framings for stochastic problems. Finally, an overview of possible approximations and simplifications is given for when a stochastic problem is too difficult to be solved using conventional methods.

## II. PROBABILISTIC PROBLEMS IN POWER SYSTEM OPERATION AND PLANNING

The problems below are introduced from shortest to longest timescale. Although the list provides a representative sample of the types of probabilistic problems presented by the integration of variable generation, it is by no means exhaustive. Each problem is first described generally, followed by the stochastic variables, and finally a sample of research conducted on the specific problem.

### A. Operational Planning Issues

Operational planning issues range on the timescale of seconds to days and focus on how to operate existing generation on the system to meet demand. It has long been recognized that meeting demand with existing generation is an inherently stochastic problem. Generators may fail to be available as predicted and load cannot be predicted perfectly. To hedge against these uncertainties, reserve generation is built into current system operations.

Systems with high penetrations of solar and wind generation require the same hedging strategy using reserves. In these systems, however, determining the correct levels of reserves requires a more explicitly probabilistic approach because wind and solar generators rely on naturally occurring phenomena that are not perfectly predictable. This changes the probability of generator availability from a historic average to a forecasted capacity factor, which indicates if a wind/solar generator will be available and at what level of output. Two of the major problems are discussed below.

#### 1) Reserve Levels

The shortest timescale problems involve setting reserve levels and dispatching reserves to meet load levels. Reserve levels in systems with high penetrations of solar/wind generation depend on the quantity of variable generation dispatched, forecasted, and the current load. The work in [1] is an overview of current practice in the United States and Europe (Union for the Coordination of the Transmission of Electricity).

## 2) *Unit Commitment*

Large thermal generators cannot be dispatched without start-up times that range from several hours to a day or more. These plants may also have minimum run times before the generator can economically be turned off. The decision whether or not to turn plants on is the unit commitment problem.

Significant academic work has been done on stochastic unit commitment. [2] developed a scenario tree and Monte Carlo forecast error approach with the WILMAR model as part of the Irish All Island Study. This approach was later used in [3], a follow-on to the Eastern Wind and Solar Integration and Transmission Study.

## B. *Generation Capacity Planning*

Generation capacity planning problems focus on the addition of new generation capacity resources to the system rather than the operation of existing resources. New generation may be added to the system to meet increases in demand, meet reliability standards, or replace retiring generation. Planning the addition of new generation capacity is inherently a stochastic problem because of errors in demand prediction, retirement timelines for existing generators, and the introduction of new technology types.

Traditionally, generation has been divided into three types—base load, intermediate, and peak—based on economic capacity factors and physical plant characteristics. In a gross manner, differences in generator types were accounted for using a capacity planning model based on a load block/load duration curve model. These models implicitly assume that inter-hour and intra-hour dynamics are not necessary for planning. This simplifying assumption clearly does not hold for wind/solar plants, where the output varies both intra- and inter-hourly and differs on an hourly basis each year. Although the two types of uncertainty in generation expansion planning—quantity of new generation and types of new generation—are interlinked, they are addressed individually below for clarity.

### 1) *Resource Adequacy*

Generation and demand-side resources must be able to meet a changing demand profile with reserve margins at all times. Over time, existing generators retire and new generation must be built to replace retiring generators and potentially meet rising demand. Unlike generation from fossil generators, the potential generation from wind and solar generators fluctuates from year to year and seasonally within each year [4]. Like systems with large hydropower penetrations, systems with high wind and solar penetration require a probabilistic approach to determine the appropriate quantity of backup thermal power.

The effective load-carrying capability (ELCC) for these nature-driven generators can be determined using multiple years of weather data. [5] provided an overview of the ELCC calculation as well as approximations to the full ELCC calculation.

### 2) *Generation Mix*

In addition to planning for the gross quantity of generation required, the type of generation matters. Although thermal generators may be dispatched at output

levels determined by the system operator, the potential output of wind/solar generators fluctuates on an inter- and intra-hour basis. When added to the time-varying changes in demand, wind and solar introduce additional variability in the net load that the dispatchable generators must be able to accommodate. In systems with high penetrations of prioritized wind and solar, the uncertainty with regard to wind/solar forecasts can also lead to violation of minimum-load requirements for thermal generators. Accommodating the uncertainty with regard to wind/solar output and their inherent variation in output requires the flexibility to ramp up and down across multiple timescales, including quick-start capabilities.

Including the intra- and inter-hour characteristics requires abandoning the conventional load duration curve models and integrating additional operating characteristics. [6] demonstrated the difference between generation portfolios with and without unit commitment in different carbon price scenarios and reduced computational time using clustering techniques. [7] demonstrated the difference in value between portfolios planned with unit commitment and those that used only dispatch across three different power systems.

## C. *Transmission Planning*

Transmission planning is a decadal timescale problem to determine which new transmission lines should be added to the system. These lines may be added to reduce congestion, allow the dispatch of cheaper plants, interconnect new plants, or increase the reliability of the system. Because transmission lines are long-lived assets, investments should be robust to changes in the power system during a 40- to 50-year time horizon. During that time horizon, demand patterns will change, new generation will be brought online, old generation will be retired, and new technologies will be introduced into the transmission system. These variables all affect the value of a transmission plan, but none may be accurately predicted on a 40- to 50-year time span.

High penetrations of wind and solar increase the uncertainty regarding the location of generation. Unlike thermal generators—which to a first approximation may be located anywhere within the system—wind and solar generators are location-constrained. Economically, they can be located only where natural resources are the strongest, yet these locations are often remote from load. Installation of new wind/solar generators is also currently policy driven, leaving the quantity of new generators as a second uncertainty. Although the construction of a new transmission line takes place on a 5- to 15-year timescale, new generators can be added to the system in less than 3 years. When not centrally coordinated, the disconnect in timescales forces a transmission planner to predict where new generation will be sited. Planning for wind/solar complicates transmission planning because these resources are often located remotely from load and high penetrations can dramatically change the network structure and flows on the existing network.

Transmission planning has generally treated generation expansion as an exogenous factor because of the size of the transmission planning problem. [8] attempted to use

heuristic methods with a stochastic dynamic programming approach. [9] attempted to overcome this using Benders decomposition and parallelization to include many scenarios.

### III. ALTERNATIVE PROBLEM FORMULATIONS FOR UNCERTAINTY ANALYSIS

All of the stochastic problems in the power system described above require probabilistic methods. The specific goal of the system operator for each problem, however, varies greatly. For example, the goal of setting reserves may be to manage tail events by minimizing the impact of the worst possible outcome; whereas the goal of a resource adequacy problem may be to achieve, on average, the best possible economic impact. The desired output may also be different. To set reserve levels, the system operator may want a probability distribution of outages for multiple dispatch orders. On the other hand, the transmission planner may want a single coherent plan to present to stakeholders.

Common to all problems is the trade-off between reliability or adequacy and cost. In all cases, reliability issues can be mitigated through increased costs. This balance between cost and reliability or adequacy is unique to each power system entity; it is a subjective decision after all minimum required reliability standards have been met.

It is important to note that many problems in power systems are stochastic, multi-period, mixed-integer, and nonlinear; these are some of the most challenging problems to optimize. Thus, even though it may be possible to formulate a problem, today's state-of-the-art algorithms may not be able to solve the problem and approximations may be necessary.

#### A. Characterization of Uncertainty

Probability density functions and scenario techniques are the dominant representations of uncertainty. The selection of uncertainty representation should reflect the goal of the analysis, the level of underlying uncertainty and knowledge of the underlying uncertainty.

Probability density functions (PDFs)—usually referred to more succinctly as probability distributions—are the most granular form of uncertainty representation. The use of PDFs generally implies a solution method focused on sampling, such as Monte Carlo techniques. PDFs allow the greatest search space because, as discussed below, artificial correlations between variables are not required. Because PDFs do not include artificial correlations, unintuitive combinations of decision variables can be explored that may produce new best- or worst-case outcomes. Although the large search space produced by PDFs provides the most neutral analysis of a problem, it also increases the size of the optimization problem, and not all problems may converge.

Scenario analysis is the most common representation of uncertainty in stochastic power systems research. It is an intuitive way to reduce uncertainty and allows problems that may be otherwise untenable to be solved. It also allows bounding results by examining, for example, both the best and worst case. Reducing the uncertainty, however, implies correlations between variables that may be fictional. For

example, a high coal price is not necessarily correlated with high natural gas price even though a “high fossil cost” case may be constructed. It also has the potential to bias results because the highest probability cases may not be those modeled. In addition, scenarios may be treated as weighted, equivalent to a discrete probability density function, or unweighted. An unweighted approach implies that all scenarios are equally likely or that the modeler has no knowledge of the underlying stochastic phenomena. The most simplistic approach is the deterministic approach, which is scenario analysis with an assumed probability of one.

The use of scenarios and PDFs are not mutually exclusive. For example, it may be appropriate to have some variables with very low variability represented using scenarios and other variables with long-tailed distributions as PDFs.

#### B. Framing Stochastic Models

Stochastic models can be framed in a variety of different ways that produce distinct result types. The type of output—single action, probability distribution, action policy—should drive the analysis. The discussion here does not focus on the solution methodology—for example, Benders decomposition versus branch-and-bound—but rather on ways to conceptualize the problem. Five different conceptualizations are discussed below. Many of the approaches discussed below are not unique to stochastic problems but are used in both deterministic and stochastic frameworks.

##### 1) Scenario Trees

One of the most intuitive formulations is a multistage decision tree. Often, a predefined scenario tree is constructed with weighted probabilities for future states-of-the-world. The result of solving such a typical scenario tree is a single action generally with the highest expected value. Other formulations—such as stochastic dual dynamic programming, which relies on cut-based solution methods—produce a single action as the output of the optimization but also provide additional information in the form of an expected value function. This expected value function includes values for other solutions, including those near the optimum, as well as information about the sensitivity of the value to different inputs. If the value function is very flat, it indicates that the output solution is insensitive to inputs; whereas if the value function has steep curves, the output solution is sensitive to inputs. Depending on the solution methodology, this type of modeling goes by many names, including stochastic linear programming, stochastic dynamic programming, stochastic programming with recourse, and stochastic dual dynamic programming. All of these methods, however, are unified fundamentally as multi-period models with recourse.

##### 2) Optimal Policies

The optimal policy approach produces a general set of procedures rather than decisions for a single situation. For example, the decision tree approach above may produce a result that is to set reserves to 400 MW for a specific state of the power system and wind prediction. The optimal policy

approach, on the other hand, would produce a result such as to always set reserves to 20% of predicted wind power output. It is important to note that not all problems will have a stable policy solution.

### 3) Chance-Constrained Programming

The chance-constrained programming (CCP) formulation allows the decision maker to decide a specific risk level through probabilistic bounds. For example, a CCP formulation for reserves could specify that reserve amounts must be adequate so that load is met with a probability of 95%. The output from a CCP formulation is a set of optimal actions; however, if the problem is executed for varying probability levels, trade-offs between meeting constraints and cost can be developed.

### 4) Simulation with Sampling

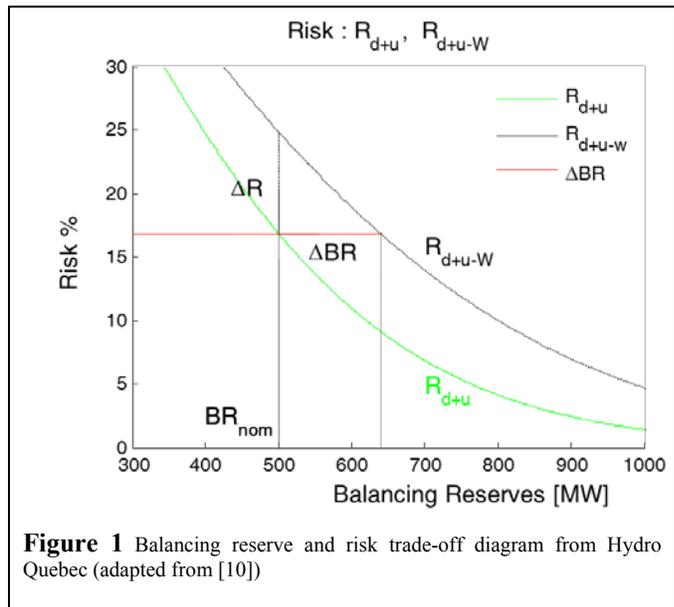
Decision trees, optimal policies and CCP approaches are explicitly optimization approaches with implicit risk valuations. Simulation and sampling approaches instead allow the modeler to build out risk and trade-off curves for either a single decision or multiple decisions. In the reserves problem, for example, a fixed level of reserves would be chosen and then multiple wind generation profiles would be sampled. For each sample, the non-served energy would then be calculated. The next level of reserves would be chosen and wind generation profiles would be sampled. In this way, it is possible to build a distribution that gives the probability of non-served energy based on the level of reserves set. This type of approach works well when the full optimization problem is difficult to solve but evaluation of a single solution is easy. This is true for problems such as transmission planning, where the full mixed-integer linear programming is computationally challenging to solve but an optimal power flow (OPF) for a given transmission configuration is computationally quick. Unlike the decision tree, policy, and CCP approaches, which give the decision maker a set of actions, the sampling and simulation approach simply provides information to the decision maker.

### 5) Pareto Curves

Pareto curves are constructed to demonstrate trade-offs in a multi-objective framework. Like the sample and simulate approaches, a Pareto curve does not give the decision maker a specific set of actions; instead, it provides the decision maker with additional information. In this case, it tells the decision maker the best solution that can be constructed for a set level of a single variable without sacrificing value from another variable. For example, a Pareto curve could be constructed from chance-constrained programming where cost is on one axis and probability of non-served energy is on the other. The curve would tell the decision maker the lowest possible cost for given reliability levels or, alternatively, what reliability level can be achieved for each cost point.

This Pareto approach is used by Hydro Quebec to study balancing reserves with an additional 3,000 MW of wind power on the system. For this Hydro Quebec case, the trade-off is between increased balancing reserves and decreased risk of non-served load [10]. As shown in Figure 1, at a nominal amount of balancing reserves ( $BR_{nom}$ ), the system

exists at 17% risk level ( $R_{d+u}$  curve). When wind in a high generation scenario is added to the system,  $R_{d+u+w}$ , the same level of balancing reserves produces a 25% risk of non-served energy. To return to the 17% risk rating, the balancing reserves must be increased by  $\Delta BR$  to 650 MW. Both with and without wind generation, the system operator is able to trade-off increased balancing reserves and decreased risk.



**Figure 1** Balancing reserve and risk trade-off diagram from Hydro Quebec (adapted from [10])

## IV. MODELS AS DECISION SUPPORT

The fundamental goal of incorporating stochasticity into power system models is to improve the decision-making capacity of power system operators and planners. Considering the formulations above, two dominant areas of concern arise: risk metrics and model execution.

### A. Risk Formulation

Stochastic model formulations produce two competing information problems for a system operator. In some cases, the model formulation may hide risk metrics from the system operator and provide a single deterministic action. In others, the model provides the operator with many varying actions for different risk profiles but no guidance on which to choose.

The most common decision metric in the decision tree and optimal policy is expected value, which is inherently a risk-neutral decision metric. A high expected value may reflect a decision with heavy tail impacts: very high value in certain scenarios and very low values in others. An outcome of using the expected value decision metric is the possibility of high-impact low-probability (HILP) events. Other risk metrics may be used to negate HILP events, such as maximizing the minimum value (maxi-min) or least regret. These decision metrics are inherently more risk averse and may negate HILP outcomes but with additional cost.

On the other hand, CCP, simulation and sampling, and Pareto formulations force operators and planners to make

explicit their risk preferences. Traditional deterministic approaches include risk levels that may not be explicitly defined. Although these risk preferences have always been embedded implicitly in the decisions made, they must be decided explicitly in the case where the reliability and cost trade-offs are presented. In the CCP formulation, a single risk level must be chosen as an input to the solution. A simulation and sampling or Pareto curve approach will provide the operator or planner with that explicit trade-off; the specific solution chosen will be a combination of a risk level and a cost level.

### B. Model Execution

Once an operator/planner has decided on appropriate formulation and risk metric, the problem remains to be solved. As mentioned before, many of the problems in power systems represent the most difficult problems from an optimization perspective. These problems are multi-period, stochastic, integer, and often nonlinear. Even if a problem may be formulated in a commercial solver, a solution may not be found on an appropriate timescale given the complexity and size of the problem. This is especially true when actual industrial, rather than academic-scale, problems are attempted. As laid out in Section II, stochastic problems in power systems concern decisions made on sub-hourly to decadal scales. Thus, the weeklong solution time for transmission planning may be acceptable; whereas a weeklong solution time for a unit commitment problem is not useful for an operator.

The fact that the full stochastic problem cannot be solved does not mean that stochastic formulations should be abandoned. There are a variety of simplifications and off-line simulations that can be done beforehand to incorporate stochasticity without requiring the solution of the full problem.

#### 1) Off-Line Preprocessing

The first option when there is a mismatch between solution time and real timescales is off-line preprocessing. Off-line preprocessing can take multiple forms. In the first, multiple possible scenarios are completed before the required time. When the decision must be made, the results from the scenario closest to existing conditions are selected. For example, multiple dispatch solutions with varying levels of wind generation may be constructed hours before the plants must be notified. At the time of dispatch, the scenario with the closest level of wind generation is selected and the results from that scenario are used. This technique does not reduce the computation time required for the problem, but allows the operator more time to perform the optimizations. The second use of off-line preprocessing is reducing the number of potential decision variables (plants that may be committed, potential capacity investments, potential transmission investments, etc). Models in increasing complexity can be employed to eliminate decision variables for the full model. For example, hierarchical decomposition (moving from the least complex to most complex) has been used in transmission planning by applying transportation and hybrid transportation–direct current (DC) OPF models

as screening tools for the full transmission planning problem with DC load flow [10].

#### 2) Simplifications

Two main classes of simplifications are available: to reduce the size of the problem and to relax the constraints within the problem. Reducing the size of the problem can be achieved by switching from a full PDF representation of uncertainty to scenarios or reducing the number of scenarios considered. To relax the constraints within an optimization problem, the transmission network can be neglected in the unit commitment problem and integer investment variables can be relaxed to linear variables.

In each case where simplifications are made, the solution to the relaxed problem must be simulated. This is because solutions to the relaxed problem may not be feasible for the fully constrained model.

#### 3) Meta-Heuristics

Unlike traditional optimization techniques, meta-heuristic solution algorithms are neutral to the type of problem considered. This neutrality allows them to be applied to stochastic integer, non-linear, multi-period, and other traditionally difficult problems. Meta-heuristics include genetic algorithms, simulated annealing, Tabu search, ant colony, and many other biologically inspired search algorithms. The advantage of these algorithms is that they can produce solutions quickly and often provide solutions to problems where traditional solvers cannot. The inherent disadvantage, however, is that there is no guarantee to the quality of the solution produced. Convergence for meta-heuristics is based on a set number of candidate solutions or reduction of difference in solution value of subsequent samples to a set level rather than provable characteristics.

## V. CONCLUSIONS

Traditional power systems operational and planning models were designed for conventional thermal generators. With the introduction of large penetrations of stochastic wind and solar generation, deterministic models are no longer sufficient. With a move to probabilistic models, decisions regarding risk levels become an explicit decision rather than an implicit assumption. Because of the challenging nature of stochastic power system problems, provably optimal solutions may not be achievable. Instead, robust heuristics and approximations will need to be developed.

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