



# Investigating the Influence of the Added Mass Effect to Marine Hydrokinetic Horizontal-Axis Turbines Using a General Dynamic Wake Wind Turbine Code

## Preprint

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# Investigating the Influence of the Added Mass Effect to Marine Hydrokinetic Horizontal-Axis Turbines Using a General Dynamic Wake Wind Turbine Code

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**Abstract**—This paper describes a recent study to investigate the applicability of a horizontal-axis wind turbine (HAWT) structural dynamics and unsteady aerodynamics analysis program (FAST and AeroDyn respectively) to modeling the forces on marine hydrokinetic (MHK) turbines. This paper summarizes the added mass model that has been added to AeroDyn. The added mass model only includes flow acceleration perpendicular to the rotor disc, and ignores added mass forces caused by blade deflection. A model of the National Renewable Energy Laboratory’s (NREL) Unsteady Aerodynamics Experiment (UAE) Phase VI wind turbine was analyzed using FAST and AeroDyn with sea water conditions and the new added mass model. The results of this analysis exhibited a 3.6% change in thrust for a rapid pitch case and a slight change in amplitude and phase of thrust for a case with 30° of yaw.

*Keywords*—added mass; unsteady; wind turbine; water turbine; marine hydrokinetic; AeroDyn; FAST

## I. INTRODUCTION

Marine hydrokinetic (MHK) turbines have the potential to capture the energy in waves, tides, ocean currents, rivers and streams. MHK turbine design and analysis requires accurate modeling of the fluid dynamic and structural dynamic forces on the rotor blades. The more accurately these forces can be modeled, the more efficiently MHK turbines can be designed. More efficient MHK turbine designs help to reduce the cost of marine energy and make it more competitive with other forms of energy production. Recent studies of MHK devices mainly utilize quasi-unsteady modeling techniques (Batten et al., 2008; Li and Calisal, 2010; Clarke et al., 2007; Corio et al., 2005). Accurately modeling the unsteady effect of the rotor fluid dynamics is still a challenge.

The National Renewable Energy Laboratory (NREL) is conducting several studies of MHK turbines using different methods. For example, Lawson et al. (2011) used RANS to model the unsteady effect of the turbine. Churchfield et al. (2011) used a hybrid approach combining the LES-actuator line to study an MHK turbine array. Although these approaches presented reasonable results, the computational costs were too high for practical design applications. NREL is currently investigating a more cost-effective way to simulate MHK turbines.

Because MHK turbines are very similar to wind turbines, researchers at NREL decided to modify the lab’s wind code, FAST, to study MHK turbines. FAST has been used to model the structural dynamics of wind turbines for many years. In FAST, AeroDyn is used to model unsteady aerodynamic effects via the General Dynamic Wake (GDW) theory. The two programs work together to provide full unsteady aerodynamic and structural dynamic analysis capabilities for wind turbines. To model the unsteady fluid dynamics of horizontal-axis MHK turbines, the research presented in this paper focuses on the implementation of an added mass model in AeroDyn.

An added mass force is created when the mass of fluid surrounding a body is accelerated. This force is not used for large bodies moving relatively quickly in air, such as a wind turbine. The added mass force needs to be added to the fluid dynamics models for MHK turbine blades because water is approximately 800 times denser than air. This paper includes descriptions of the added mass effect, the GDW theory used in AeroDyn, and how this theory was modified to model the added mass effect for MHK turbines. The paper also describes how the added mass effect influences MHK rotor blade forces.

## II. GENERAL DYNAMIC WAKE THEORY OVERVIEW

AeroDyn uses the GDW theory to model the time-dependent relationship between changes in the rotor forces and corresponding changes in the rotor induced velocity distribution. The GDW theory for wind turbines used in AeroDyn was developed by Suzuki and Hanson (1999). This theory was created by modifying the Pitt and Peters (1981) model, which was developed for helicopters in forward flight. Suzuki (2000) showed that empirically based correction factors could be applied to the GDW model for wind turbine applications. For the research presented in this paper, the GDW model was used to find the flow field accelerations necessary to compute the added mass forces. The GDW model does not account for added mass forces; hence the need for a separate added mass model. An overview of the theory and implementation of this model in AeroDyn can be found in the AeroDyn Theory Manual (Moriarty and Hansen, 2005).

## III. ADDED MASS MODEL WITH GDW

This section describes the implementation of the added mass model in AeroDyn. The acceleration can be divided into two parts: the acceleration of the flow and the acceleration of the body (turbine blade for this application). Only the acceleration of the flow through the rotor disc was considered in this study, as the acceleration of the blade would need to be passed from the structural dynamic analysis program FAST to the fluid dynamic analysis program AeroDyn. Passing data on blade accelerations from FAST to AeroDyn was beyond the scope of this research, because such an operation would cause numerical stability issues resulting from the interaction between the two programs.

The acceleration of the flow was computed from the time rate of change of the axial induction factor, which was computed from the GDW solution found by AeroDyn. The body and flow accelerations were multiplied by the added mass of a blade element to compute the added mass force of that element. This study only examines the effect of the acceleration of the axial flow through the rotor disc.

The added mass force is distinct from the forces computed in the GDW or the Blade Element Momentum (BEM) theories. These theories are based on solving for the balance of force and pressure for GDW and momentum for BEM at a steady flow condition. The GDW theory has a term called apparent mass that is used to capture the time dependent change in force and pressure on a rotor caused by the dynamic response of the rotor wake. The apparent mass term in GDW theory does not account for the force required to accelerate the fluid from one time step to the next, which is the added mass force. As mentioned earlier, this force is typically very low for wind turbines, but can be significant for water turbines.

### *Derivation of Added Mass Force Equation*

The following derivation assumes an incompressible fluid with density  $\rho$ , no surface or wave interaction, and constant wind/water flow speed (although the flow through the rotor disc may change). It also assumes constant rotational rate of the turbine blades and ignores any changes in tangential induction factor, so that only flow acceleration perpendicular to the rotor disc plane needs to be account for.

The added mass force on a body is the same for a body accelerating through a flow field as for a stationary body with a flow field accelerating around it. Following the derivation found in the work by Brennen (1982), (1) represents the added mass force ( $F_a$ ) acting on a body moving with speed  $U$  in a flow of constant speed  $W$ . The added mass term ( $m_a$ ) represents the equivalent added mass of the entire flow field about the accelerating body.

$$F_a = -m_a * \frac{d}{dt}(U - W) \quad (1)$$

Since the wind speed is assumed constant, it can be removed from (1), because the time rate of change of  $W$  would go to zero. If the wind speed is accelerating, then a different solution is necessary, which can be found in Brennen (1982). An added mass model that accounts for accelerating wind speed will be examined in a future study. The resulting equation that assumes constant wind speed can be seen in (2).

$$F_a = -m_a * \frac{dU}{dt} \quad (2)$$

For the application of a horizontal-axis MHK turbine,  $U$  is the flow speed through the rotor disc, so it will be replaced by a term  $U_d$ . The wind speed ( $W$ ) can be replaced by  $U_\infty$ . The wind speed and flow through the rotor disc can be related by the axial induction factor ( $a$ ), as shown in (3). Combing (3) with (2) results in (4).

$$a = 1 - \frac{U_d}{U_\infty} \quad \text{or} \quad U_d = U_\infty(1 - a) \quad (3)$$

$$F_a = -m_a * \frac{dU_d}{dt} = m_a * U_\infty * \left(\frac{da}{dt}\right) \quad (4)$$

Where  $U_d$  is the flow through the rotor disc and is assumed to be constant across the disc. The flow-field acceleration was found by numerically differentiating the axial induction factor, resulting in (5).

$$\frac{da}{dt} = (a - a_{previousTimestep})/\Delta t \quad (5)$$

For the calculation of added mass ( $m_a$ ), it was assumed that the turbine blade airfoil sections could be approximated as a flat plate. Equation (6) gives the added mass ( $m_a$ ) per unit length of a flat plate with motion perpendicular to its chord length,  $c$  (Brennen, 1982).

$$m_a' = \pi * \frac{1}{4} * c^2 * \rho \quad (6)$$

Assuming that each blade section is aligned with the rotor disc plane removes the effects of blade pitch on the projected chord length of the airfoil, which results in the maximum possible added mass force. The pitch angles for the blade analyzed in this paper would cause less than a 1% difference in the projected chord length over 80% of the blade span, with a maximum 7% difference at the blade root. Ignoring the effect of blade section pitch and assuming that the chord of a blade section is aligned with the plane of the rotor disc, the added mass for a blade element is found by (7), where  $c$  is the local chord length of the blade element.

$$m_a = \pi * \frac{1}{4} * c^2 * \rho * element\_span \quad (7)$$

The added mass force ( $F_a$ ) is then found by applying (7) and (5) to equation (4). Then,  $F_a$  is added to the component of force normal to the rotor disc, because the flow acceleration ( $da/dt$ ) is assumed to be normal to the rotor disc and the rotation of the blade and flow are assumed constant.

The force  $F_a$  was added to the blade force outside of the GDW solution, resulting in the added mass force not influencing the fluid dynamic solution, meaning that axial induction factor and angle of attack will not be affected by the addition of the added mass force  $F_a$ . This is a limitation of the method used to implement the added mass model in AeroDyn. The effect of this limitation was not quantified, although it is assumed to be small since the GDW solution depends on the time history of the rotor thrust which was only influenced by a few percent by the added mass model explored in this paper. The combination of only accounting for flow acceleration perpendicular to the rotor disc and not allowing the added mass model to influence the GDW solution resulted in the added mass model not influencing the rotor power, only the rotor thrust.

#### IV. WATER TURBINE MODEL DESCRIPTION

A model of the NREL unsteady aerodynamics experiment (UAE) phase VI wind turbine was analyzed using FAST and AeroDyn with sea water conditions and the new added mass model. The FAST Cert Test (Certification Test) is an automated procedure that runs a series of simulations using different turbine models that operate with various features enabled under a variety of wind conditions. This automated certification procedure can be used to compare the effects of alterations to FAST, including the effects of new fluid dynamic models (Jonkman and Buhl, 2005). The FAST input file used in the research for this paper is based on Test 10 of the FAST Cert Test cases, which is the UAE VI upwind turbine test case (Jonkman and Buhl, 2004; Jonkman, 2010). For more information about the versions of FAST and AeroDyn used in the research presented in this paper, see the Appendix.

The UAE wind turbine rotor was selected for analysis because there is already a significant amount of data available for this turbine. This test case does not represent an optimum water turbine, but rather a wind turbine operating in water. The original blade planform and airfoils of the UAE Test 10 case were used in the present model, which are labeled “Mod\_S809” in the Cert Test files.

The rotational speed of the water turbine test case was reduced from 71.9 RPM for the original wind turbine model to 21.9 rotations per minute (RPM). The rotational speed was reduced to account for the lower flow speed of the water turbine; this adjustment approximately maintains the range of tip speed ratios. The density and kinematic viscosity of sea water was used in the water turbine model, which is changed in the AeroDyn aerodynamic parameters input file.

Sea water conditions:

density: 1030.0 (kg/m<sup>3</sup>)  
kinematic viscosity: 1.17e-6 (m<sup>2</sup>/s)  
speed of sound: 1500 (m/s)

- Flow speed 1.6 m/s
- Axisymmetric inflow (aka equil. inflow)

- Rigid tower and blades
- No teeter
- No generator model
- No dynamic stall modeling

The unsteady aerodynamics (including dynamic stall) model was turned off because the model relies on empirically tuned data specific to each airfoil and its operating conditions, which was not available for the airfoil used on this turbine under sea water conditions. Applying the unsteady aerodynamics model would most likely decrease the peak values of forces caused by the added mass model; an exploration of this effect is planned as part of a future study.

## V. RESULTS

The water turbine was analyzed using the water turbine version of AeroDyn to illustrate the effect of the added mass model. The results from two cases are presented: a step pitch change and a rotor operating in yaw.

The first case is a step pitch change at 10 seconds from a blade root pitch of  $4.815^\circ$  to  $0.815^\circ$  over a time of 0.012 seconds (3 time steps).

Results for the step pitch change case are shown in figures 1 and 2. Figure 1 shows the time history of rotor thrust for this unsteady analysis. The general dynamic wake model was initiated at 1 second, resulting in initial transient forces that are not physical. These initial transient forces typically dissipate within 5 seconds for wind turbine models; however, this figure illustrates that 5 seconds is insufficient to reach a steady state solution for the water turbine analysis. A longer settling time of 10 seconds was used for all cases in this paper.

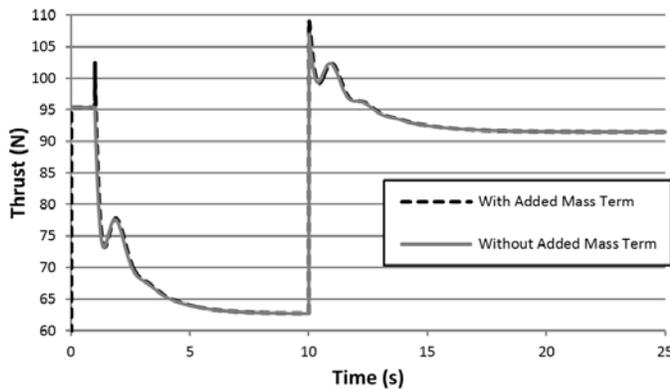


Fig. 1. Thrust force of the rotor versus time. At 10 seconds the rotor blade was rapidly pitched, causing a dynamic response. The initial transience before 5 seconds is due to initialization of the model and is not physical.

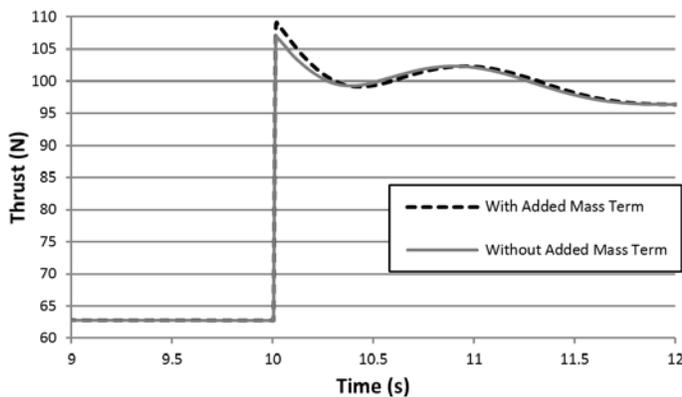


Fig. 2. Thrust force of the rotor versus time.

Figure 3 shows the percentage change in thrust due to the added mass model, relative to the final value of thrust 25 seconds into the simulation. This figure illustrates that the added mass model increases the load on the rotor blade by 4.1% for the case of a step change in blade root pitch. The increase in blade pitch results in a decrease of thrust on the rotor, hence the negative values

in Fig. 3 which indicate a relative increase in loads on the blade as compared to the analysis without the added mass model. The 4% change in blade thrust load due to the added mass model could have a noteworthy impact on MHK rotor blade structural design, as this load increment will impact the fatigue load on the blade, which determines the operational lifetime of the blade.

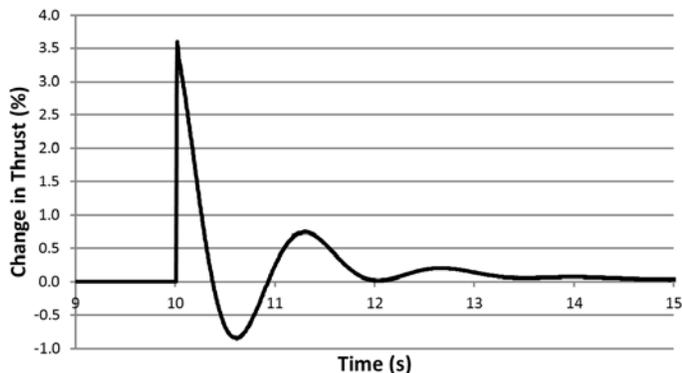


Fig. 3. Percentage change in thrust due to the added mass model. Thrust values were divided by the final thrust value at 25 seconds. The spike at 10 seconds shows the maximum change in thrust due to the model for this rapid pitch change case.

Figure 4 shows the initial spike in angle of attack (AOA) caused by the rapid pitch change. The initial change in AOA equals the pitch change and then slowly decreases toward the steady-state solution. The initial spike in AOA is because the shed and trailing vorticity in the wake has not had time to respond to the change in circulation caused by the increase in AOA. Figure 5 shows the response of the axial induction factor to the step-change in blade root pitch. The time rate of change of the axial induction factor is also shown, which is used to compute the added mass force. Table 1 shows the results of the time-step with the maximum difference caused by added thrust.

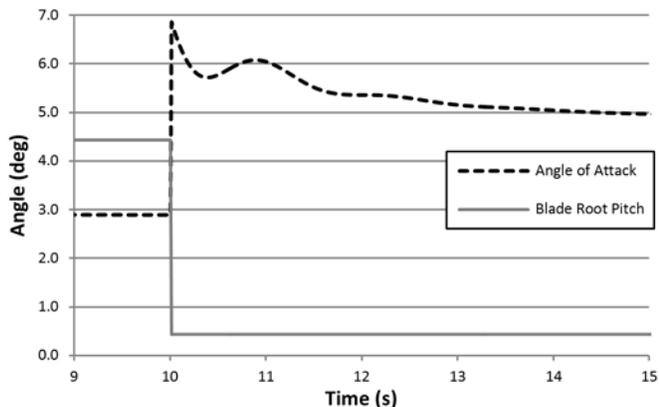


Fig. 4. Angle of attack response to a rapid pitch change. At 10 seconds the rotor blade was rapidly pitched by  $-4^\circ$  over 0.012 seconds.

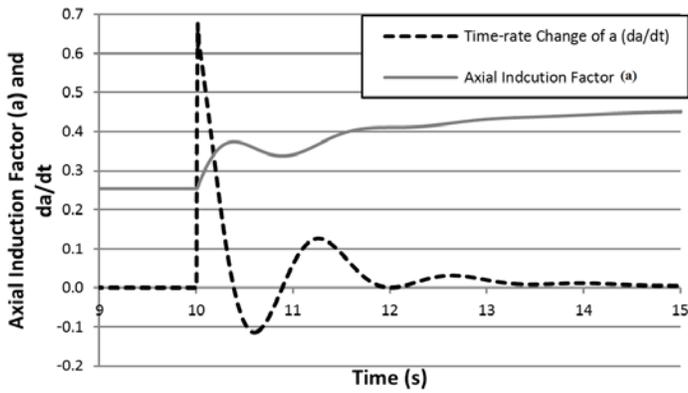


Fig. 5. Axial induction factor and the time rate of change of axial induction factor verse time for the pitch change case.

TABLE I. EFFECT OF ADDED MASS MODEL ON PITCH CASE

Time (s)	Thrust no_AM (N)	Thrust w/AM (N)	Delta Thrust (N)	Delta Thrust %
10.02	106.89	109.19	2.3	3.59

The second case that was analyzed was the model water turbine operating with a yaw of  $30^\circ$  and with constant velocity and blade root pitch. Figure 6 shows the response of the angle of attack to the changing inflow angle due to the rotor yaw angle. Note that the values before 5 seconds are due to the GDW model initialization.

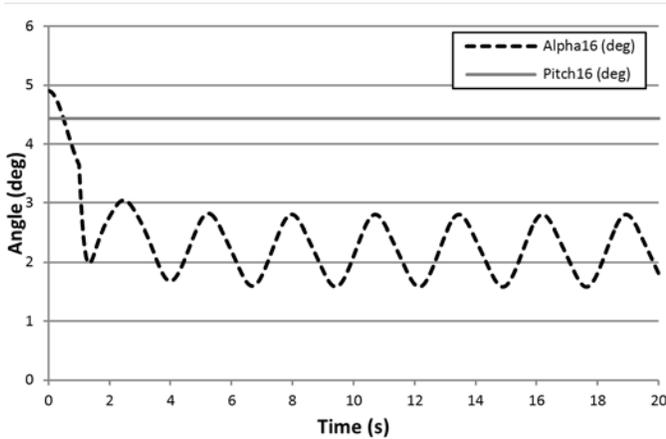


Fig. 6. Angle of attack response of a yawed rotor

Figure 7 shows the change in axial induction factor with time. As with the angle of attack, the oscillatory response is due to the changing local flow angle caused by the yaw angle. The time rate of change of the axial induction factor is also shown in Fig. 7. The peaks in the time rate of change of axial induction factor will cause the highest values in the added mass force.

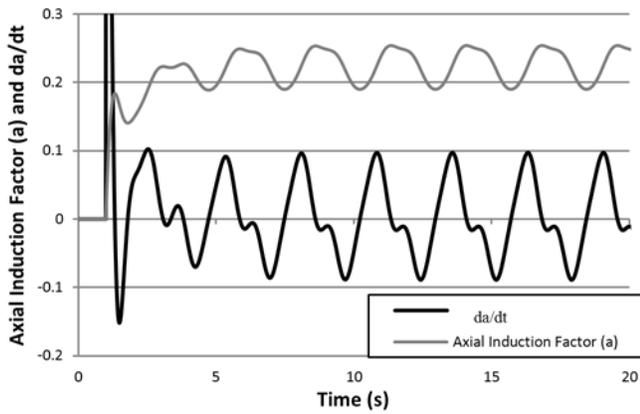


Fig. 7. Axial induction factor and the time rate of change of axial induction factor verse time for the yaw case.

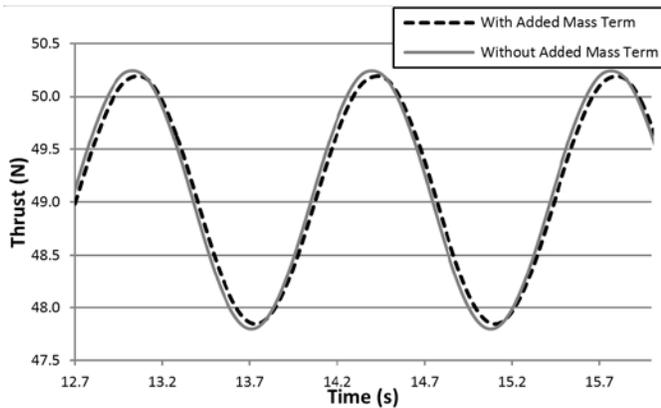


Fig. 8. Thrust force vs time for the yaw case.

Figure 8 shows the thrust force response with time, with and without the added mass model. With the added mass model turned on, the amplitude of the thrust force decreases by 3.9% and the phase to shift by  $6.37^\circ$ .

The rapid pitch change case showed an initial increase in thrust due to the added mass model while the yawed case showed a decrease in thrust amplitude due to the added mass model. The difference in thrust response between these two cases may be caused by the difference in time scales between them. The initial spike in the response of AOA,  $a$ , and thrust occurs over a very short time scale for the case of rapid root pitch change. The GDW model does not capture very short time scale aerodynamic response, such as with this case. Short time scale response is captured by the dynamic stall and unsteady aerodynamics model, which was turned off during this study. Future work will investigate the combined effects on blade response of the added mass model and the unsteady aerodynamics model.

## VI. CONCLUSION

The results indicate that the added mass model works as expected and has a noticeable influence on blade structural loads. The model could be improved by including the body accelerations of the turbine blade in the computation of the added mass force. This will require passing the body accelerations from FAST to AeroDyn, or moving the added mass computation routine into FAST.

Applicability of the GDW theory to water turbines is uncertain, because the current GDW model in AeroDyn is tuned for large wind turbines. Validation of the added mass model and the application of GDW theory to MHK turbines will require unsteady hydrodynamic experimental data for a MHK horizontal-axis turbine. Use of the unsteady aerodynamics model will also require unsteady experimental data of MHK specific airfoils in order to properly tune the model. Until such experimental data is available, models with fewer physical assumptions than the GDW theory can be used to calibrate, validate, or replace the GDW model in AeroDyn for MHK turbine analysis.

The next stage of this research will be to develop an unsteady free-wake vortex method for MHK HAWT analysis which will replace the AeroDyn GDW model in FAST. The investigation explained in this paper is the initial step toward a more comprehensive added mass model that will eventually be included in the under development unsteady FWVM-FAST code.

## APPENDIX VERSIONS OF THE CODE

The results in this paper were generated using FAST version “v7.00.00a-bjj, 31-Mar-2010”, which was not modified for this research. The water turbine version of AeroDyn used to generate the results presented in this paper was based on the latest released wind turbine version of AeroDyn at the beginning of the project: “v13.00.00a-bjj 31-Mar-2010 B. and J. Jonkman.” The latest version of the code designed for water turbines, including the added mass model described in this paper, is AeroDyn version “v13\_00\_01a-dcm 5-Aug-2011”; this version of AeroDyn is not publicly available.

Comparing results from versions of the code with and without the added mass model requires compiling a version of FAST and AeroDyn with the added mass model turned on and a separate model with the added mass term turned off. Turning the added mass model on and off requires setting the added mass force to zero within AeroDyn.

### *Changes to the Code*

A subroutine in the previous version of AeroDyn fixed the speed of sound to the value for sea-level air. The subroutine (called “BEDDAT”) is part of the dynamic stall model in AeroDyn. Even though the dynamic stall model was turned off, it still had an effect on the speed of sound calculation in other parts of the code. This subroutine was modified in the added mass water turbine version of AeroDyn to use the speed of sound in sea water (1500 m/s, 4921 fps). The speed of sound is fixed in this part of the code, although future versions of AeroDyn will be available to read the speed of sound from the input file.

The GDW routine becomes unstable for high induction factors, which typically occur at low wind speeds. Therefore, at low wind speeds AeroDyn switches to BEMT. In the previous version of AeroDyn, this cut-off speed was set at 8 m/s, which is above the operating range for most water turbines. In the water turbine version of AeroDyn, the cut-off wind speed has been set as zero, to allow the GDW solution to always be used for water turbines. Future versions of AeroDyn should use the induction factor as a measure of when the GDW routine will be likely to be unstable, rather than wind speed.

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## REFERENCES

- Batten**, W.M.J., A.S. Bahaj, A. F. Molland, J.R. Chaplin. 2008. The prediction of the hydrodynamic performance of marine current turbine. *Renewable Energy*, 33(5), 1085-1096
- Brennen**, C. E. 1982. A Review of Added Mass and Fluid Inertial Forces. Naval Civil Engineering Laboratory, Port Hueneme, CA. CR 82.010
- Churchfield**, M., Y. Li, and P. Moriarty. 2011. A comparison between LES simulation of a large-scale tidal current turbine farm and a large scale wind turbine farm. In: *The 9th European Wave and Tidal Energy Conference*. Southampton, U.K.
- Clarke**, J., G. Connor, A. Grant, and C. Johnstone. 2007. Design and Testing of a Contra-rotating Tidal Current Turbine. In: *Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, 221. pp. 171-179.
- Corio**, D. P., F. Nicolosi, A. De Marco, S. Melone, and F. Montella. 2005. Dynamic Behavior of Novel Vertical Axis Tidal Current Turbine: Numerical and Experimental Investigations. In: *International Offshore and Polar Engineering Conference*. pp. 469-476. Seoul, South Korea.
- Jonkman**, J.M. and M.L. Buhl, Jr. 2004. New Developments for the NWTC's FAST Aeroelastic HAWT Simulator. In: *The 23rd ASME / 42nd AIAA Wind Energy Symposium*. Reno, NV:AIAA. NREL/CP-500-35077
- Jonkman**, J. and M. Buhl. 2005. NWTC Design Codes (FAST Verification by Jason Jonkman and Marshall Buhl). <http://wind.nrel.gov/designcodes/simulators/fast/verification/>. Last modified 19-May-2005; accessed June-2010.
- Jonkman**, J. 2010. NWTC Design Codes (FAST) <http://wind.nrel.gov/designcodes/simulators/fast/>. Last modified 05-November-2010; accessed June-2010
- Lawson**, M., Y. Li, and D. Sale. 2011. Development and Verification of a Computational Fluid Dynamics Model for Horizontal-Axis Tidal Turbines. In: *30th International Conference on Ocean, Offshore And Arctic Engineering*. Rotterdam, Netherland.
- Li**, Y. and S.M. Calisal. 2010. Three-dimensional effects and arm effects on modeling a vertical axis tidal current turbine. *Renewable energy*, 35(10): 2325-2334
- Moriarty**, P.J. and A.C. Hansen. 2005. AeroDyn Theory Manual, NREL/EL-500-36881, National Renewable Energy Laboratory, Golden, CO, December.
- Pitt**, D.M. and D.A. Peters. 1981. Theoretical Prediction of Dynamic Inflow Derivatives. *Vertica*, Vol.5(1):21-42
- Suzuki**, A. and A. Hanson. 1999. Generalized Dynamic Wake Model for Yawdyn. In: *The 1999 ASME Wind Energy Symposium*.
- Suzuki**, A. 2000. Application of Dynamic Inflow Theory to Wind Turbine Rotors. PhD Dissertation, The University of Utah.

## Figure Captions

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## Table Caption

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