



# Probability-Weighted LMP and RCP for Day-Ahead Energy Markets using Stochastic Security-Constrained Unit Commitment

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# Probability-Weighted LMP and RCP for Day-Ahead Energy Markets using Stochastic Security-Constrained Unit Commitment

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**Abstract**— Variable renewable generation resources are increasing their penetration on electric power grids. These resources have weather-driven fuel sources that vary on different time scales and are difficult to predict in advance. These characteristics create challenges for system operators managing the load balance on different timescales. Research is looking into new operational techniques and strategies that show great promise on facilitating greater integration of variable resources. Stochastic Security-Constrained Unit Commitment models are one strategy that has been discussed in literature and shows great benefit. However, it is rarely used outside the research community due to its computational limits and difficulties integrating with electricity markets. This paper discusses how it can be integrated into day-ahead energy markets and especially on what pricing schemes should be used to ensure an efficient and fair market.

**Keywords**- Electricity markets, power system operations, power system economics, unit commitment, wind generation)

## I. NOMENCLATURE

### Indices

$i$  = generator  
 $h$  = Hour  
 $k$  = piecewise linear cost block  
 $r$  = reserve category  
 $n$  = bus  
 $l$  = line (branch)  
 $s$  = scenario

### Sets

$H$  = Hours  
 $G$  = Generators  
 $K_i$  = Set of blocks for unit  $i$   
 $R$  = Reserve categories {SR = Spinning Reserve, NS = Non-Spinning Reserve, Rep = Replacement Reserve, Reg = Regulating Reserve}  
 $L$  = Lines (branches)  
 $N$  = Buses  
 $S$  = Scenarios  
 $G_{LSTART}$  = Generators with start times greater than one hour

### Variables

$P_g$  = Generation schedule (MW)  
 $P_p$  = Pumping schedule (MW)

RS = Reserve schedule (MW)

$u$  = Unit status {0,1} (no units)

$z$  = startup status {0,1} (no units)

$y$  = shutdown status {0,1} (no units)

LL = Lost (Shed) load (MW)

IR = Insufficient reserve (MW)

STL = Storage level (MWh)

### Matrices

[K] = bus connection matrix (with generation or with load)

### Parameters

IC = Incremental cost for generators (\$/MWh)

NLC = No-Load Cost (\$/h)

SUC = Startup Cost (\$)

RC = Reserve Cost (\$/MWh)

VOLL = Value of Lost Load (\$/MWh)

VOIR = Value of Insufficient Reserve (\$/MWh)

STV = Storage Value at end of day (\$/MWh)

$\pi$  = probability of occurrence (no units)

### Dual values

$\lambda$  = Dual of energy balance constraint

$\mu$  = Dual of transmission flow constraint

$\beta$  = Dual of reserve constraint

## II. INTRODUCTION

The power system deals with variability and uncertainty in its generation fleet, its load, and its network infrastructure. Power system operators have numerous techniques to deal with these characteristics. Operating reserve is held to accommodate generation or network failures [1]-[2]. Security-Constrained Unit Commitment (SCUC) and Security-Constrained Economic Dispatch (SCED) are used to efficiently commit and schedule generation resources to meet expected load and reserve demands [3]. These are adjusted when necessary to meet varying conditions. Some generation resources (as well as new emerging technologies and demand response) are being built with additional flexibility so that they can react to changing conditions with fast ramping ability, quick synchronization times, and inertial and frequency response capabilities [4]. Lastly, since restructuring of the electricity sector, efficient markets are put in place to incentivize the correct behavior to maximize reliability [5].

The SCUC is performed to ensure the most cost-effective selection of unit online status is made to reliably meet the system load [6]. SCUC ensures that reserve requirements are also met and that generation and system network constraints as well as selected contingency constraints are monitored. SCUC is also usually the foundation for the day-ahead energy market. The solution of SCUC determines the day-ahead energy schedules and day-ahead energy prices. This tie between SCUC and the day-ahead energy market is essential. The day-ahead energy market ensures an efficient operation whereby units receive day-ahead prices and schedules that best reflect what will occur in real-time, they are able to hedge against the volatility in real-time prices, and have guarantees from the Independent System Operator (ISO) that they will recover costs through side payments if the prices are not sufficient [7].

The current SCUC procedure is deterministic. A single prediction must be made for all conditions when solving SCUC. The discrepancy between real-time conditions and the SCUC solution is dealt with using operating reserves and real-time market incentives. It has been shown, however, that increasing amounts of variable generation (e.g., wind power) have unique uncertainty characteristics and that a deterministic SCUC may not be the most efficient procedure when determining what units should be online in advance. Research has shown that a stochastic SCUC (STSCUC) can provide great benefits reducing costs and increasing reliability [8]-[12]. A STSCUC determines a unit commitment solution that can meet multiple scenarios of expected outcomes. Its objective is to minimize the expected operation cost where each scenario is weighted by its associated probability of occurrence. The usual SCUC constraints are applied to all scenarios and the only condition that is satisfied which is common to all scenarios is the so-called first stage “now” constraints. In most cases, these constraints are the unit commitment decision for units that have long start times since their start-up decision must be made before the scenarios become realized. This creates a solution that can reliably meet multiple real-time outcomes and economically do so if they occur.

Although stochastic STSCUC is described extensively in the literature, it has generally not yet been applied in actual system operations. One reason for this is the great computational burden. The number of variables of the problem is about  $n$  times greater than those of a deterministic SCUC, with  $n$  being the number of scenarios. Therefore, the solution time on a practical ISO system with hundreds to tens of thousands of generators, buses, and branches is considerable and generally not acceptable for the current timelines the ISO procedure uses for the day-ahead energy market. Another complication of the STSCUC is that it may not be able to integrate seamlessly into the current ISO market structure. The SCUC and day-ahead market currently have a strong interaction that emphasizes reliability and efficiency. It is important for a paradigm where STSCUC is the means of clearing the day-ahead energy market for it to place the right incentives and administer rules that specifically apply to its unique qualities.

This paper proposes a probability-weighted locational marginal price (LMP) and probability-weighted reserve clearing price (RCP) that can be used to clear day-ahead energy markets that use a STSCUC. Section III describes the STSCUC model that is used. Section IV discusses the current and proposed pricing schemes. Section V then compares results of the two pricing schemes using a STSCUC model and a modified IEEE 118-bus test system. Section VI provides a conclusion on the proposal based on these results.

### III. STOCHASTIC SCUC MODEL

The STSCUC model used in this research is similar to [9] and [12]. A one day, 24-hour optimization is used to replicate the day-ahead market. Constraints are modeled towards typical constraints used in ISO SCUC programs. The one-hour start time is used as a limit for those units that do not need constant unit commitment status between scenarios. The model is developed in GAMS using CPLEX Mixed Integer Programming Solver [13] and called from Matlab [14] based on [15]. The formulation is briefly described in this section. Equation (1) explains the objective function used in the model.

$$\begin{aligned} \text{minimize } \sum_{s \in NS} \pi_s * \{ & \left\{ \sum_{h \in H} \left\{ \sum_{i \in G} SUC_i * z_{i,h} + NLC_i * \right. \right. \\ & u_{i,h} + \left\{ \sum_{k \in K_i} IC_{i,k} * Pg_{i,k,h} \right\} + \left\{ \sum_{r \in R} RC_{r,i,h} * RS_{r,i,h} \right\} \} + \\ & VOLL * LL_h + \left\{ \sum_{r \in R} VOIR_r * IR_{r,h} \right\} \} - \\ & \left. \sum_{i \in G_{storage}} STL_{i,24} * STV_i \right\} \end{aligned} \quad (1)$$

The objective function minimizes the expected costs based on all scenarios. For this model, a piecewise linear cost curve is used to represent fuel costs, which is included in the model along with reserve costs, penalty costs for violating the load and reserve balance, and maximal storage levels left at the end of the day based on the value of the associated storage resource.

The system constraints include the load balance, reserve balance, power flow equations, power flow limit constraints, and contingency constraints. There is allowance for phase angle regulator and HVDC branches. To keep linearity, the power flow is a DC approximation.

The generator constraints model the individual constraints on each generating unit based on its type. The constraints include capacity constraints, ramp rates, variable generation forecasts, minimum run times, minimum down times, and maximum starts per day.

The reserve constraints describe how individual units are able to supply each reserve category. The reserves used in this model are defined as spinning reserve, nonspinning reserve, regulation reserve, and replacement reserve. These reserves are generally based on NERC BAL standards and constrained by online status, quick start capability, AGC qualified status, and ramp rate limitations [16]. It should be noted that the reason for the reserve requirements are for system contingencies and for minute to minute frequency regulation. Contingencies are not modeled as a stochastic variable and since the time resolution of the model is one hour, minute-to-minute frequency regulation needs are assumed independent of the stochastic variables.

The contingency constraint equations model power flow immediately following a network contingency. These equations allow for post-contingency limitations for normal AC lines, phase angle regulators, and HVDC contingency conditions.

Equations are used to model the corresponding storage resources (e.g., pumped hydro storage). These constraints include efficiency losses, pumping capacity constraints, and storage reservoir limits.

Lastly, (2) shows the equation that represents the “now” decision that must be made. Units with long start times (greater than or equal to one hour) must be told to start during the first stage, and therefore, their unit commitment decision must be the same for all potential scenarios.

$$u_{i,h,s} = u_{i,h} \quad \forall i \in G_{LSTART}, \quad \forall h \in H, \quad \forall s \in S \quad (2)$$

The STSCUC program therefore selects a unit commitment set that can meet the scenarios while minimizing the expected cost. Unlikely scenarios may utilize the use of expensive combustion turbines during high net load scenarios or the use of wind curtailment during low net load scenarios rather than changing the commitment set for all scenarios. It is also possible that load is shed or reserves are insufficient for the very unlikely scenarios, based on evaluation of probabilities and the associated infeasibility penalty costs. However, in our analysis the number of scenarios is reduced to five and probabilities are generally still greater than or equal to 0.1; this does not occur frequently. For a two stage simulation, the unit status for units with startup times greater than or equal to one hour are kept constant for the real-time market, whereas unit status for units with start-up times less than one hour and the dispatch and reserve amounts for all units can be adjusted.

#### IV. PROBABILITY-WEIGHTED LMP

In today’s energy markets, the LMP represents the marginal cost of energy at a specific location and is mathematically represented as the dual value of the nodal injection constraints. The RCP is calculated as the marginal cost of reserve at a location (in this model this is system-wide only) and is the dual value of the reserve balance constraint. This is shown in equation (3) and (4).  $\lambda_{ref}$  is the dual/lagrangian of the energy balance equation,  $\mu$  is the dual of the transmission constraint equation,  $SF_{n,l}$  is the shift factor of bus  $n$  contributing to branch  $l$ , and DF is the delivery factor of a bus with respect to the reference bus incorporating system losses.  $\beta$  is the dual of the reserve balance equation and must be positive since the equation is an inequality.

$$LMP_n = \lambda_{ref} - \sum_{l=1}^{Lcong} SF_{n,l} * \mu_l + (DF_n - 1) * \lambda_{ref} \quad (3)$$

$$RCP_r = \beta_r \quad (4)$$

In a STSCUC, there would be LMP and RCP values for every scenario that is being evaluated. In other words, the marginal cost of energy in one scenario is different than the marginal cost of energy in another scenario. Therefore, in day-

ahead energy markets which use STSCUC, numerous options may be taken when determining the energy and reserve settlement. One option may be to solve a deterministic SCED after the stochastic SCUC is solved using fixed commitments from the STSCUC. However, this would mean that one would have to select a single scenario from the many used in the STSCUC to be used in the SCED. Another option might be to use the most probable or median scenario marginal values from the STSCUC for calculation of LMP and RCP. However, this procedure assumes that there is a median or most probable scenario which with many stochastic variables may not be the case. For instance, a single scenario may include high load, low wind power in one area and high wind power in another area. This also complicates the option of creating an expected value for all stochastic variables in use of solving an additional SCED for calculation of LMP and RCP. The expected value calculation will ignore the correlations of the variables for each scenario. For instance, if one scenario represented a high wind, low-load event, and another scenario a low wind, high load event, the expected value would be medium wind and medium load scenario, unrepresentative of both likely scenarios.

This paper proposes a probability-weighted LMP and probability-weighted RCP as the means of integrating a stochastic SCUC with day-ahead energy markets. This proposal should properly price energy according to the objective function used, give day-ahead prices that are on average more closely aligned with real-time prices, incentivize proper flexibility in resources to be able to accommodate more variable resources, and reduce inefficient incentives and uplift. The proposal would calculate payments as the sum of the probability-weighted LMP times the energy schedule of each scenario and the sum of the probability-weighted RCP times the reserve schedule of each scenario. This can be seen in (5). Note that the marginal values used to calculate LMP and RCP from (3) and (4) that come from the equation according to the objective function in (1) would already be probability weighted and therefore, the equation in (5) assumes marginal values are normalized prior to the calculation. With the raw marginal values, the  $\pi_s$  would not be part of the equation and the calculation would simply be the sums of energy and reserve payments for each scenario.

$$Payment_i^{DA} = \sum_{s=1}^{NS} \pi_s * (Pg_{i,s}^{DA} * [K_{i,n}] * LMP_{n,s}^{DA} + \sum_{r=1}^R RS_{i,r,s}^{DA} * RCP_{r,s}^{DA}) \quad (5)$$

In an ISO, SCED is performed in real-time to correct schedules based on the real-time outcomes (although quick-start commitments may often be allowed as well). This SCED is aligned with the real-time market just as the SCUC is aligned with the day-ahead market. Currently, total settlements are a sum of payments in the day-ahead market and real-time market. When day-ahead markets are cleared with deterministic SCUC, the real-time payments are calculated as that energy scheduled by the real-time SCED above which was paid the day-ahead LMP multiplied by the real-time LMP, which can be a negative value if the real-time schedule is less

than the day-ahead. Equation (6) shows the total payments for the deterministic case for a unit during each hour.

$$Payment_{i,h} = Pg_{i,h}^{DA} * [K_{i,n}] * LMP_{n,h}^{DA} + (Pg_{i,h}^{RT} - Pg_{i,h}^{DA}) * [K_{i,n}] * LMP_{n,h}^{RT} + \sum_{r=1}^R RS_{i,h,r}^{DA} * RCP_{h,r}^{DA} + (RS_{i,h,r}^{RT} - RS_{i,h,r}^{DA}) * RCP_{h,r}^{RT} \quad (6)$$

This again becomes complicated for stochastic SCUC. Because real-time has less uncertainty without need of first and second stage decisions, and because real-time energy markets require simulations to be solved as quickly as every 5 minutes, it is likely to assume that the benefits of a stochastic SCED for the real-time market are not as great as they are for the day-ahead SCUC. Therefore we assume that a deterministic SCED is performed for the real-time market. The settlements must take into account the energy scheduled in the day-ahead market when settling the real-time market. Therefore, equation (7) shows the total settlements from day-ahead and real-time for energy and ancillary services for an entire day given the probability-weighted LMP and RCP proposal using a STSCUC in the day-ahead market and a deterministic SCED in the real-time market.

$$Payment_{i,h} = [\sum_{s=1}^{NS} \{\pi_s * (Pg_{i,h,s}^{DA} * [K_{i,n}] * LMP_{n,h,s}^{DA} + \sum_{r=1}^R RS_{i,h,r,s}^{DA} * RCP_{h,r,s}^{DA})\} + (Pg_{i,h}^{RT} - \sum_{s=1}^{NS} \pi_s * Pg_{i,h,s}^{DA}) * [K_{i,n}] * LMP_{n,h}^{RT} + \sum_{r=1}^R (RS_{i,h,r}^{RT} - \sum_{s=1}^{NS} \pi_s * RS_{i,h,r,s}^{DA}) * RCP_{h,r}^{RT}] \quad (7)$$

The pricing schemes used in energy markets are in place to ensure proper incentives to participate in the day-ahead market so that system operators can plan with better knowledge of what will occur in real-time. One method to evaluate the pricing scheme is to see how often and to what magnitude negative profits are occurring. Negative profits can show that generating resources are not being paid sufficiently enough to recover their costs to run. This can show inefficiency in that the incentives are not enough for the generating resources to be running at the scheduled level even when they are needed at the scheduled level. It should be noted that negative profits are expected to occur to some degree due to the non-convexity of generator costs (e.g., due to NLC and SUC) and because of inter-temporal constraints (e.g., minimum run times). The generator costs used in our analysis can be seen in (8).

$$Cost_{i,h} = SUC_i * z_{i,h} + NLC_i * u_{i,h} + \{\sum_{k \in K_i} IC_{i,k} * Pg_{i,k,h}^{RT}\} + \sum_{r=1}^R RC_{i,r,h} * RS_{i,h,r} \quad (8)$$

Lastly, profit (revenue) can be calculated as seen in (9).

$$Profit_{i,h} = Payment_{i,h} - Cost_{i,h} \quad (9)$$

The number of negative profits is therefore one metric that shows the efficiency of a pricing scheme. Other analysis and behavior monitoring would be required to fully assess the appropriateness of this pricing scheme. The quality of the probabilistic forecasts would also be assessed. Lastly, market monitoring procedures would need to be evaluated to see if any gaming behavior can differ in this pricing proposal compared

to the single pricing procedure. However, these analyses and procedures are out of the scope of this paper.

## V. CASE STUDY

A modified 118-bus IEEE test system is used for simulation. This model gives a realistic representation of large systems without being too overly computational. The system contains 54 thermal generators, 3 pumped hydro storage facilities, 177 AC transmission lines, a single phase shifting transformer, and 8 HVDC lines. Generating resources ranged from quick start combustion turbines that could start in 10 to 30 minutes and had capacities of 20 to 50 MW and larger steam units with longer start times, minimum run times and capacities of up to 420 MW. All thermal units had minimum capacities of at least 25% of maximum capacity. PHS had a value designed to them for having energy stored at the end of the day (STV<sub>i</sub>) of about \$15/MWH and had 80% round-trip efficiency. An amount of wind energy equal to 25% of the load was added to this system, with a large portion in one transmission region representing a “good wind resource” where transmission was not able to always accommodate high wind output. This helped represent some of the typical situations that occur today in systems with growing wind penetrations. The wind data was taken from [17] with load data taken from the same days and locations from [18] and scaled down to the system characteristics. Lastly, reserve requirements represented 50% of the largest contingency (420 MW) for spinning, 100% of the largest contingency for non-spinning, 1% of the hourly load for regulating, and 150% of the largest contingency for replacement reserves. Spinning reserve automatically contributed to non-spinning and non-spinning automatically contributed to replacement reserve.

The simulations used the STSCUC model as described in Section III for the day-ahead market and followed with a deterministic real-time SCUC/SCED model for the real-time market. The real-time model could commit resources with start times less than one hour, but all other commitments whether on or off were held fixed from the day-ahead STSCUC. The wind power in the STSCUC was represented as probabilistic forecasts and generally was given as normally distributed around a mean given some hourly standard deviation. Note that a normal distribution is not the best representation of wind power forecasts, [19], [20], but is used here for illustrative purposes. Ultimately, on real systems the probabilistic forecasts will depend on the weather pattern on the system. Five scenarios were used as input to the STSCUC and load was treated deterministically. In the subsequent real-time SCUC/SCED run, a random number was drawn as the actual wind power output and was drawn from the original normal probability distribution function.

Simulations were modeled for a high wind day in April and a high load day in August. Payments were calculated as in (6) for the single-scenario pricing and (7) for the probability-weighted pricing schemes. Costs and revenues were calculated as in (8) and (9). LMP and RCP values were calculated from each scenario and the five scenarios represented probabilities {0.1, 0.25, 0.5, 0.25, and 0.1} based on the probability distribution function and taking a weighted average of numbers that fell in each bin. This method converted the

continuous pdf to be used as a discrete pmf which can be represented in the STSCUC. LMPs (averaged for all buses on network) from the April day are shown in Fig. 1 for each of the scenarios. The single price LMP/RCP pricing scheme took prices and schedules from the third median scenario. A relative duality gap of 0.5% was chosen for the simulations. Table I shows negative profits based on LMP only and Table II shows negative profits based on LMP and RCP.

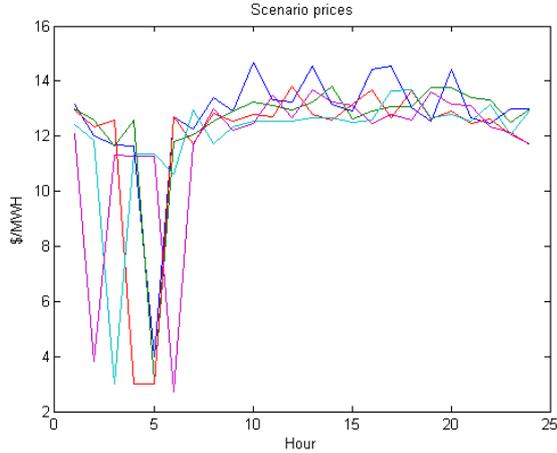


Fig. 1. LMPs for each scenario of April day.

TABLE I

TOTAL UNIT-HOURS WITH NEGATIVE PROFITS FOR DAY (LMP ONLY)

	Unit-hours with negative profit		Total \$ of negative profit	
	LMP based on single scenario	Probability weighted LMP	LMP based on single scenario	Probability weighted LMP
August	71	66	-\$8,939	-\$5,857
April	114	122	-\$7,203	-\$6,542

TABLE II

TOTAL UNIT-HOURS WITH NEGATIVE PROFITS FOR DAY (LMP AND RCP)

	Unit-hours with negative profit		Total \$ of negative profit	
	LMP based on single scenario	Probability weighted LMP	LMP based on single scenario	Probability weighted LMP/RCP
August	77	43	-\$9,252	-\$4,180
April	73	42	-\$6,357	-\$4,724

The first observation is that the probability-weighted LMP pricing scheme provides a lesser amount of negative profits and thus less uplift. The April case in Table I did show more unit-hours with negative profit in the probability-weighted pricing scheme, however. It is important to note, as is shown in Table III for the summer case, that the actual comparison of prices between the two schemes shows no large premium and

that on average the prices are very close to one another. In fact, the total revenue from the single median scenario pricing scheme for the April scenario was larger than the probability-weighted pricing scheme (\$918,835 compared to \$913,705). This can support the fact that the probability-weighted pricing scheme is not necessarily increasing prices and profits but distributing them more efficiently.

TABLE III

COMPARISON OF HOURLY LMPs FOR REFERENCE BUS FOR APRIL DAY

	Scenario 3 (\$/MWh)	Probability Weighted (\$/MWh)
0	17.55	22.03
1	16.01	16.07
2	15.61	15.28
3	14.27	14.48
4	14.31	14.26
5	13.53	12.07
6	13.93	14.63
7	15.96	15.72
8	16.39	16.19
9	16.69	16.80
10	17.62	17.64
11	19.13	18.77
12	18.89	18.90
13	19.18	19.08
14	19.16	19.64
15	24.28	22.08
16	17.05	18.83
17	19.12	18.73
18	19.26	19.12
19	15.30	16.89
20	15.21	16.89
21	17.20	17.22
22	14.69	15.65
23	14.49	15.15

In the examples of Table I and Table II, the standard deviation of the probabilistic forecasts were 6% of the nameplate capacity of the wind, with 5 selected hours at 10% to represent a more variable period of the day. To analyze how these pricing methods behave in relation to uncertainty, we now increase the standard deviation to 12% at most hours and 15% at the 5 higher varying hours. Results are shown in Table IV and Table V.

TABLE IV

TOTAL UNITS WITH NEGATIVE PROFITS FOR DAY WITH HIGHER STANDARD DEVIATION (LMP ONLY)

	Unit-hours with negative profit		Total \$ of negative profit	
	LMP based on single scenario	Probability weighted LMP	LMP based on single scenario	Probability weighted LMP
August	132	100	-\$15,562	-\$10,164
April	99	103	-\$9,763	-\$8,150

TABLE V

TOTAL UNITS WITH NEGATIVE PROFITS FOR DAY WITH HIGHER STANDARD DEVIATION (LMP AND RCP)

	Unit-hours with negative profit		Total \$ of negative profit	
	LMP based on single scenario	Probability weighted LMP	LMP based on single scenario	Probability weighted LMP/RCP
August	126	71	-\$17,623	-\$8,326
April	76	33	-\$10,035	-\$5,546

A few observations can be discovered from these results. Generally, it appears that the probability-weighted LMP has less total negative profits. It can also be observed that adding RCP to the equation makes the probability-weighted pricing appear more efficient. In fact, some units had increases in negative profits for the single-price pricing scheme when adding the RCP to the payments. Since RCPs cannot be negative, this had to do with the units in particular that were being adjusted. The reserve schedules from the final real-time run must have been less than in the median scenario of the STSCUC for certain units. Lastly, it appears that the higher the uncertainty (higher standard deviation), the more efficient this probability-weighted pricing scheme becomes.

## VI. CONCLUSION

The benefits that stochastic SCUC can give in terms of reliability and economics can be substantial in power systems with high penetrations of variable generation. Currently, no systems we are aware of have begun using STSCUC in actual day-ahead market operations. Difficulties in solving within feasible times due to computational intensity and the issues involved with integration into existing market structures are two reasons why this has not yet occurred. This paper attempts to facilitate the interaction between stochastic SCUC and day-ahead energy markets by introducing a probability-weighted LMP and RCP pricing scheme for paying generators.

Testing the proposal on a small but practical system showed great benefits in terms of lesser negative profits occurring when compared to a pricing scheme that only used schedules and prices from one scenario. The benefits appear to increase when more uncertainty is present in the system and when probability-weighted RCP is used in addition to probability-weighted LMP. This might be one of the decisions when deciding to implement this procedure in a market that has decided to utilize STSCUC. If the market had highly variable wind power that was difficult to predict and therefore probabilistic wind power forecasts had high uncertainties, the probability-weighted pricing scheme would ensure generators would recover costs more often and would reduce make-whole side payments resulting from inefficiencies.

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