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INVESTIGATION OF A FAST-ORCAFLEX COUPLING MODULE FOR INTEGRATING TURBINE AND MOORING DYNAMICS OF OFFSHORE FLOATING WIND TURBINES

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Abstract

To enable offshore floating wind turbine design, the following are required: accurate modeling of the coupled wind turbine structural dynamics, aerodynamics, platform hydrodynamics, a mooring system and control algorithms. Mooring and anchor design can appreciably affect the dynamic response of offshore wind platforms that are subject to environmental loads. From an engineering perspective, system behavior and line loads must be studied well to ensure the overall design is fit for the intended purpose.

FAST is a comprehensive simulation tool used for modeling land-based and offshore wind turbines. In the case of a floating turbine, quasi-static continuous cable theory is used to emulate mooring line behavior. Improved modeling fidelity can be achieved through the use of finite element mooring theory. This can be accomplished by the new FASTlink coupling module, which couples FAST with OrcaFlex, a commercial simulation tool used for modeling mooring line dynamics.

In this coupling, FAST is responsible for capturing the aerodynamic loads, the turbine control system, the platform global motion and flexure of the wind turbine and its tower, and OrcaFlex models the mooring line and hydrodynamic effects below the water surface. The FASTlink coupling module was developed by Orcina, the creators of OrcaFlex, in collaboration with industrial partners. This paper can be considered as a third-party assessment of the coupling which investigates the accuracy and stability of the FAST/OrcaFlex coupling operation.

Nomenclature

F_{ap}^{i}	applied force for iteration i
K_z	heave buoyant stiffness
m	total mass specified in FAST
T_0	z-axis tether tension from all fairleads
$\overline{T}^{i}$	mean z-axis tether tension from all fairleads for iteration i
$\overline{z}^{i}$	mean z-axis displacement for iteration i
AE	mooring line axial stiffness
μ	mooring line mass per length
L	mooring line unstretched length

Introduction

Although the conventions and practices used in simulating land-based wind turbines are well established, modeling tools specially tailored for floating wind turbines are still in development. To meet immediate design needs, tools can be assimilated from various disciplines to satisfy the specialized needs of the offshore wind industry.

Conveniently, the resources needed to model, design and construct such systems can be adapted from the offshore oil and gas industry. Perhaps most notable, collaboration between the wind and the offshore engineering industry is evident in Europe’s North Sea area, where wind turbines are attached atop monopiles embedded in the seabed. Fixed towers are perhaps the simplest form of offshore wind installation accessible today. The large moments created from wind loads on the turbine, however, require upgraded foundation designs, likely making this concept less cost-competitive as depths increase, effectively limiting the design to shallow waters.

Floating offshore wind turbines show promise for deep waters because they: 1) require less construction material, thereby reducing costs; and 2) can withstand greater sea loads compared to their towered counterparts [1]. Floating offshore wind systems can be supported by a variety of platform designs, including a Tension Leg Platform (TLP), a spar buoy, or a simple barge (Figure 1). Offshore industry innovations are malleable to meet immediate offshore wind development needs.

In most floating offshore systems, the mooring lines can alter the vessel response characteristics; therefore, the mooring lines should be included in the design process. The moorings are crucial additions to ensure safety and stability of the platform; however, if designed improperly, they can have catastrophic effects, especially if resonance matching between the platform and its tethers occurs. The likelihood of this happening increases as the floating system is installed in deeper waters [2,3]. For this reason, a rigorous analysis of the mooring line dynamics and its effects on the offshore wind turbine response is important.

Offshore Wind Modeling Tools

The purpose of this work is to assess the accuracy and stability of a coupling module used to combine the capabilities of two software tools to enable high-fidelity modeling of a floating offshore wind turbine. This coupling module is to be used exclusively with FAST [4] and OrcaFlex. FAST is a modeling tool used to compute the response of a land-based or offshore wind turbine. While FAST has many sophisticated features for modeling the aerodynamics, hydrodynamics, turbine structure and controller, it currently employs a simplistic quasi-static cable model to approximate the mooring forces on an offshore floating wind turbine. Quasi-static models use continuous cable theory to estimate tether forces by assuming the cable is in static equilibrium at every time step; hence, this model accounts for cable weight, but not its inertia [5,6]. Improvements can be made by replacing the quasi-static cable model with a dynamic cable model so that cable inertia, drag, and fluid added mass can be modeled. This is done by linking FAST with OrcaFlex using the appropriately named *FASTlink* coupling module.

FASTlink was assembled by Orcina, the originators of OrcaFlex, in collaboration with industrial partners. The National Renewable Energy Laboratory chose to perform an independent third-party evaluation to assess numerical stability, perform model-to-model comparisons, and verify the accuracy of the coupling module.

This paper focuses on the usability and numerical stability aspects of FASTlink by comparing rudimentary simulations in FAST with results generated using the FAST/OrcaFlex coupling tool, FASTlink. The analysis is performed using a 5-MW OC3-HyWind spar platform deployed in a water depth of 320 meters. The reader is directed to the study in Robertson and Jonkman [7] for a detailed summary of the OC3-HyWind spar, but the essential attributes of the system are listed in Table 1 below.

Table 1: OC3-HyWind spar Properties

Spar + Turbine Mass	$7.466\text{x}10^6$ kg
Fluid Displacement	8,029 m^3
Water Depth	320 m
Spar Draft	120 m
Spar Diameter	6.5 – 9.4 m
Mooring Mass per Length	77.7 kg/m
Mooring Axial Stiffness	$3.842\text{x}10^8$N/m
Mooring Line Length	902.2 m

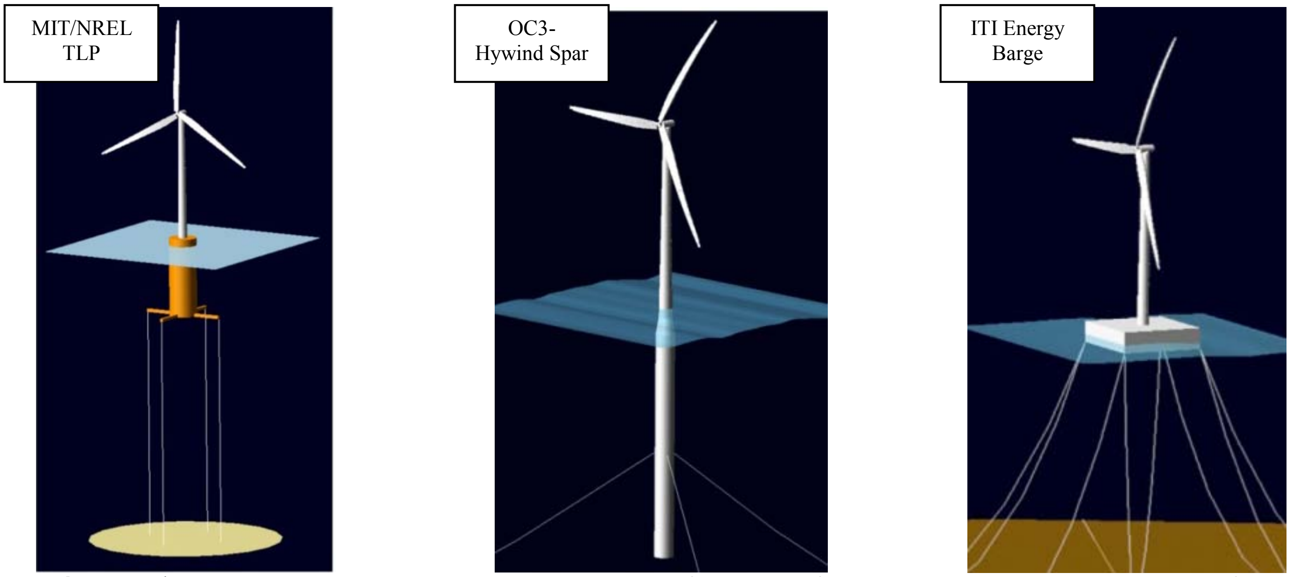

Figure 1: Images of several concepts actively being pursued for offshore wind interests. From left to right: a TLP, a spar and a floating barge.

FAST Capabilities

FAST is an open source program developed by the National Renewable Energy Laboratory. Used by academics and in industry, FAST is a recognized tool for modeling land-based and offshore wind turbines. Among its many abilities, FAST can incorporate user defined controllers to adjust the nacelle heading and rotor blade pitch with changing wind speed and direction.

The current version of FAST features a fusion of several programs to create one comprehensive and complete model. The main program, FAST, maintains the time-domain based equations of motion, system kinematics, kinetics and control algorithms. FAST also models the natural flexure of the tower, the rotor blades and drive train. A secondary program, AeroDyn [8], provides the aerodynamic wind resistance, rotor blade lift and drag forces using quasi-steady or dynamic blade-element/momentum theory and static or dynamic stall. Similarly, the HydroDyn module solves the hydrodynamic forces on the submerged portion of the offshore wind turbine support platform [9]. This module is used in conjunction with the WAMIT [10] preprocessor to resolve the radiation, diffraction and hydrostatic restoring force coefficients. In parallel, Morison's equation gives the fluid resistance caused by viscous effects.

OrcaFlex Capabilities

OrcaFlex is a time-domain, finite element commercial software used to model the response of cables or to couple behavior between a surface vessel and its moorings. Such models are often referred to as discretized cable models because the mooring line is idealized as a system of mass components (nodes) connected to visco-elastic elements. The inherent mass, stiffness, and damping properties of the mooring ensure it responds to varying end forces and displacements. This results in excitation loads that cannot be anticipated by the quasi-static model.

The basic underlying theory for such models has existed in some form since the 1960s [11,12], and over the years, incremental refinements have led to the models being used today. OrcaFlex is commonly used in the offshore industry as a means to predict the mooring line snap loads, rapid re-tensioning, and transient motions. As an added benefit, OrcaFlex also provides a graphical user interface (GUI), whereas FAST does not; therefore, OrcaFlex is useful in the visualization process. OrcaFlex is designed to be robust for a variety of cable configurations, whether the cable is slack or taut. Similar to FAST's HydroDyn, WAMIT input files can be loaded into OrcaFlex to capture vessel reactions to waves.

FASTlink Capabilities

FASTlink is a coupling module being developed by Orcina in conjunctions with industrial partners to exploit the strengths of FAST and OrcaFlex to create a high-fidelity modeling tool for offshore wind turbines. The wind turbine, its aerodynamic loads, control system, tower and the six degree-of-freedom rigid-body platform motion is modeled in FAST. The subsea components, such as the mooring lines and support platform hydrodynamics, are modeled in OrcaFlex. Therefore, when FASTlink is being utilized, HydroDyn is no longer performing calculations. Naturally, as part of this coupling process, the quasi-static cable model in FAST is disabled. Mooring forces would then be represented in OrcaFlex.

In the FASTlink coupling module, FAST passes the vessel position and velocity vectors into OrcaFlex; the line tension and hydrodynamic added mass matrix and non-acceleration dependent hydrodynamic forces are calculated by OrcaFlex; the resulting added mass matrix and total force and moment on the platform is passed back into FAST; then the resulting platform motion is solved in FAST. There is an inconsistency between FAST and OrcaFlex regarding the way platform static equilibrium is calculated, and this will be discussed next.

Code Coupling Considerations

One challenge with coupling software packages together is ensuring program compatibility is maintained. The purpose of FASTlink is to ensure compatibility issues are settled between FAST and OrcaFlex. A unique feature of FASTlink is that it selects the units in OrcaFlex to match FAST's units automatically. This feature prevents an obvious mismatch between the models, which often times is overlooked. There are, however, considerations the end user must account for when using FASTlink to avoid double counting of forces, continuity between reference frames and consistent time-stepping between programs. The following steps are taken to ensure uniformity between models:

- The mooring lines should be arranged in OrcaFlex relative to the same reference frame origin identified in FAST, which is level with the mean surface line.
- To avoid double counting of the gravitational forces, all mass quantities (both submerged and wind turbine components) are specified in FAST, and the vessel mass is set to zero in OrcaFlex.
- In this work, the OrcaFlex integration time-step is equal to that of FAST's to ensure consistent resolution between the coupled and uncoupled data sets.

The last point identified above is not ideal if the goal is to run the simulation quickly. Instead, it is acceptable to increase OrcaFlex's time-step by an integer multiple of FAST's.

Issues may arise in the coupling process because of how the platform weight and buoyancy in handled in the FASTlink exchange. This can lead to a system not beginning at static equilibrium. In the next section, corrective measures are prescribed to account for this particular disparity occurring between the two models.

Resolving the Static Solution with FASTlink

The platform static configuration must first be determined before the simulation is initiated. This is done to ensure the ensuing platform motions are not caused by an imbalance in the initial conditions. Although there are several ways to force the platform into a static configuration, this correction is applicable to circumstances where the total mass (platform and wind turbine combined) is specified in the FAST input file. To begin, an applied force equal to the undisplaced buoyancy is specified at the center of buoyancy in the OrcaFlex model.

$$F_{ap}^{1} = mg + T_0 \tag{1}$$

The simulation is run for a sufficient amount of time to ensure the initial start-up transients are converging toward a steady-state value. If transients still occur, the solution for F^{i}_{ap} at any simulation iteration i can be refined using:

$$F_{ap}^{i+1} = F_{ap}^{i} - K_z \overline{z}^{i} - \left(T_0 - \overline{T}^{i}\right) \tag{2}$$

Equation (2) can be repeated as necessary until the static errors are sufficiently small. This process resolves the static equilibrium for the heave direction because the applied force is inserted in the z direction only.

Preliminary Results and Stability Assessment

Several simulations with varying conditions were performed; however, for brevity, only subsets of results are reported to illustrate accuracy, stability, and usability of FASTlink. All simulations are performed over an 800-second time interval with a constant time step of Δt = 0.01 seconds in both OrcaFlex and FAST. The results being presented include:

- A single degree-of-freedom free-decay simulation to ensure accuracy of the FAST/OrcaFlex coupling by comparing platform motions to results generated with the standalone FAST program
- OC3HyWind spar response to regular waves for the purpose of illustrating multi degree-of-freedom motion and to ensure solution stability to wave loads
- The spar platform in calm water with a below-cut-off wind speed of 8 m/s. This is done to ensure wind load calculations are being carried out correctly in the simulation process.

Each simulation is performed with the rotor in a ‘parked’ arrangement and with turbine control turned off, even when wind velocity is prescribed. This ensures aerodynamic loads are computed, but control forces are not introduced into the system.

CASE I: Single Degree-of-Freedom Offset, z_0 = 0.2 meters

A total of six free-decay simulations (one for each platform degree-of-freedom) are carried out to ensure consistency between the FAST/OrcaFlex coupling tool and the standalone FAST program. Figure 2 shows the free decay response for the heave (z) direction with the platform initially displaced z_0 = 0.2 meters. The platform natural frequency is used to assess continuity between FAST (using the quasi-static cable model) and FASTlink (labeled as ‘FAST/OrcaFlex’ in the figures). This step serves as a check to ensure the time-step is properly selected, to ensure similar transient characteristics are present and assess simulation stability.

Spectral analysis is an effective means to compare time series data because subtle phase shifts in the data may skew results. Different mooring forces are calculated in FAST versus OrcaFlex, which may lead to different platform displacements. This, in turn, may lead to differing wave forces because the platform will be positioned differently in the wave field. Hence, there is no guarantee the time series will be identical between the two models.

In the figure below, the time series z(t) displacement plots are nearly identical, except for the exponential decay appearing in the FAST/OrcaFlex model. This is likely caused by tether damping, which is not present in FAST's default quasi-static model. The frequency response plot, however, shows exceptional agreement, and it is concluded that FASTlink operates reasonably well under the prescribed conditions.

Appearing to the right of the z-axis displacement plot is the tether tension time history and its associated spectral response. The OC3-HyWind spar has three mooring lines, and the one being represented in the plot is aligned with the platform x-axis. Observing the mooring line tension is a concise and effective way to monitor the platform response, health, and condition, because a displacement in any degree-of-freedom will influence the leash tension. As expected, the mooring line tension is heavily influenced by the heave natural frequency. This is a characteristic shared by both models. In the FAST/OrcaFlex model, several high frequency vibration components are present. This is because of the wind turbine and mooring line axial natural frequency. The mooring line natural frequency is easily identifiable from [13]:

$$f_u^n = \frac{n}{2L}\sqrt{\frac{AE}{\mu}} \tag{3}$$

where n is the n^{th} vibrations mode. The first three modes, which agree with the Power Spectral Density plot below, are f_u^n = {1.23 ; 2.46 ; 3.70} Hz.

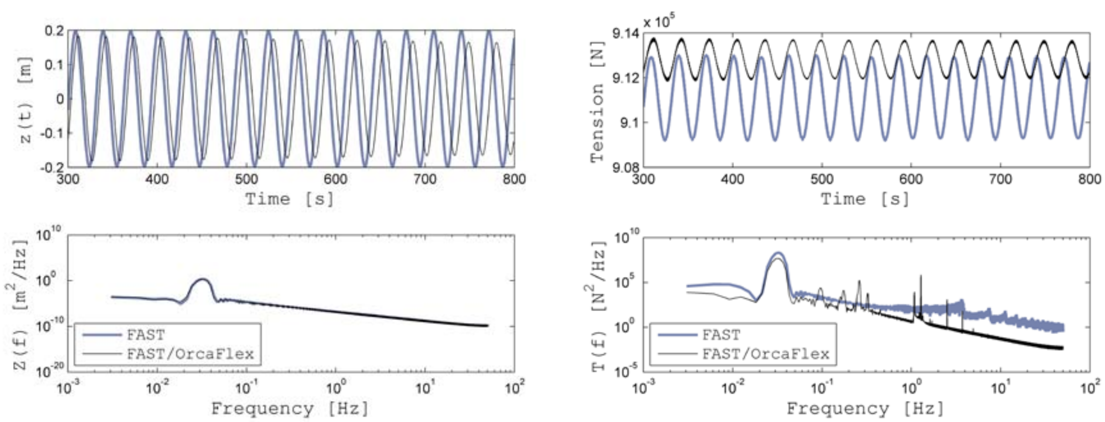

Figure 2: Simulation results for Case I: free decay with z_0 = 0.2 meters.

CASE II: Regular Waves, H = 5 meters, T = 20 seconds

A test case is performed with the OC3-HyWind platform in regular waves to evaluate FASTlink's stability to environmental loads. In Figure 3, the time series for surge (x), heave (z), and pitch (θ) directions, as well as the mooring line tension at the platform, is given. Because the waves are progressing in a direction aligned with surge, there is no motion in sway (y) and roll (φ); hence, they are not shown alongside Figure 3. The Power Spectral Density plot for each degree-of-freedom complements the time-series plots. Once analyzed in the frequency domain, the data sets reveal strong agreement between FAST and the FAST/OrcaFlex coupling tool.

Although the surge time histories look uncoordinated between FAST and FAST/OrcaFlex, the surge Power Spectral Density plot, X(f), illustrates that both data sets are in agreement. Equally satisfactory data are reported for the remaining degrees-of-freedom. In reference to the tension Power Spectral Density plot T(f), large differences between the two models occur at frequencies coinciding with the wave-band frequencies; however, it is stressed that the purpose of this exercise is not to investigate why difference occurs, but rather to show that the coupling tool reports comparable results to its FAST counterpart. Differences between the two models can be attributable to:

- Longitudinal and transverse cable excitations;
- Ringing or springing;
- Mathieu instabilities;
- Resonance matching between the wind turbine and tethers;
- Internal cable damping, cross-flow cable drag and fluid added mass.

Naturally, differences in mooring line tension are expected because the two cable models are fundamentally different. This model-to-model comparison projects the appearance that FASTlink is operating as designed, the results given by its solution are reasonable, and the coupling module has a high degree of usability.

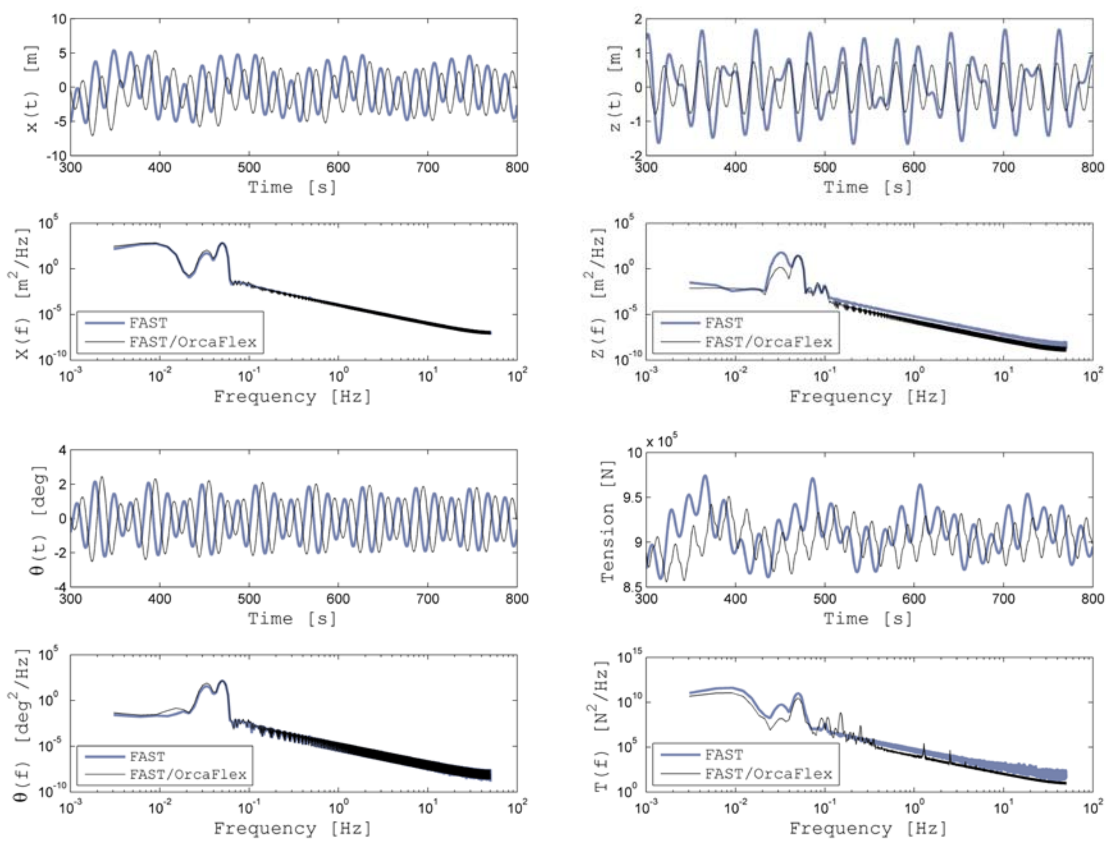

Figure 3: Simulation results for Case II: Regular Waves – H = 5 meters, T = 20 seconds.

CASE III: Below Rated Wind, 8 m/s

The final case is initiated to ensure the aerodynamic calculations between FAST and the FAST/OrcaFlex coupling modules are consistent under wind excitation. The platform is initially at its equilibrium position with zero translational and rotational displacement. A uniform unsheared wind of 8 m/s is prescribed in a direction aligned with the platform x-axis, with no waves or currents acting on the platform. Similar to Case II, model symmetry and the wind direction confine the platform motion to primarily the surge (x), heave (z), and pitch (θ) directions. The results from this simulation are given in Figure 4. Once again, the model-to-model comparison suggests each program is executing its assigned calculations correctly and the coupling process between FAST and OrcaFlex appears to be operating properly.

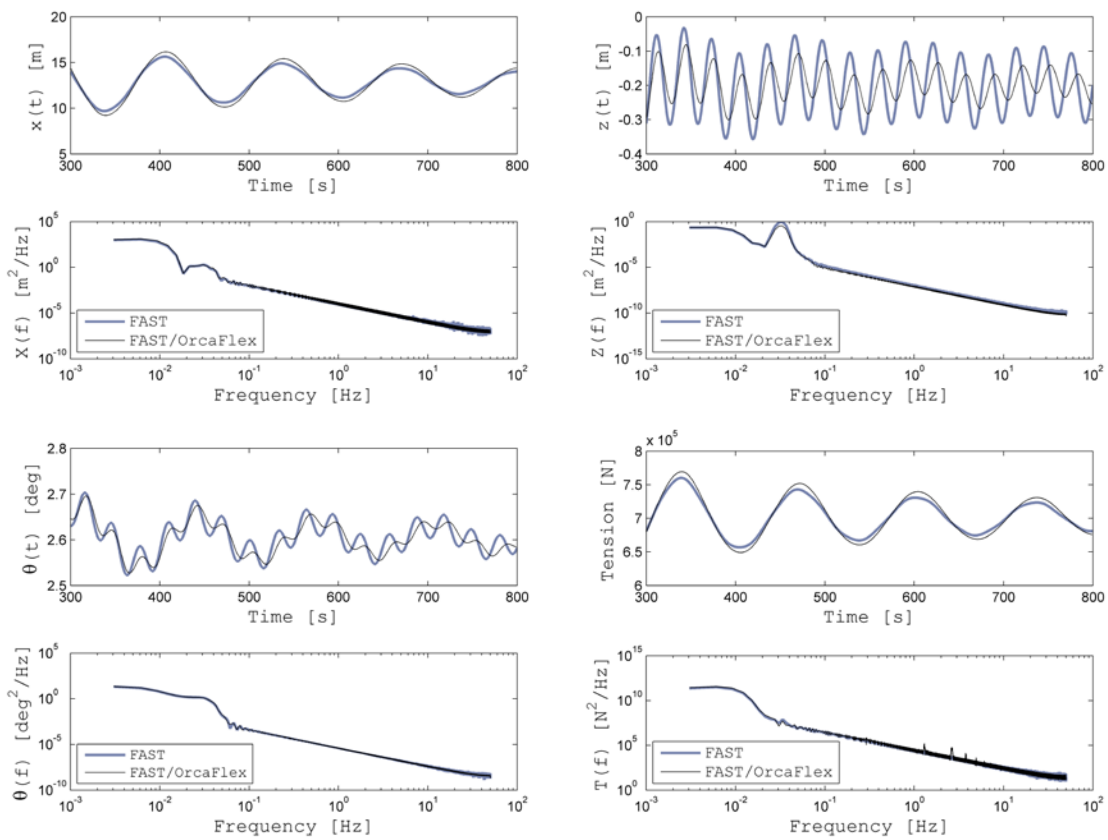

Figure 4: Case III: Uniform, unsheared wind. The turbine is operating at a below-rated capacity with the wind speed set to 8 m/s.

Other Simulations Considered

Although the authors have presented a limited number of test cases in this manuscript, other starting conditions were examined to ensure consistency between models and ensure functionality of the FAST/OrcaFlex coupling tool. Similar to Case I, a free-decay numerical experiment was performed on the remaining degrees-of-freedom. This test revealed that the FAST/OrcaFlex tool obtains platform natural frequencies comparable to the standalone FAST program. This test was essential to make certain the platform and/or wind turbine weight was not counted twice in the simulation.

A test combining the attributes of Case II and Case III was also performed. In this case, a steady wind of 8 m/s and a regular sea defined by H = 5 meters and T = 20 seconds were introduced into the model concurrently. This simulation was performed with the seas propagating in the same direction as the platform x direction, as well as one with the sea offset by 20° relative to the x direction. Three simulations which integrated wind turbine control algorithms were also performed to further verify stability of the coupling module. These simulations introduced irregular sea conditions using the JONSWAP spectrum; the sea was defined by a significant wave height of H_s = 6 meters and an average period of T_{avg} = 20 seconds. The wind conditions were generated using a normal turbulence model, and the three mean wind speeds considered were U_m = 10, 12 and 18 m/s. An analysis of the simulation result supports the same conclusions presented in this paper.

While only results for the OC3-HyWind spar are presented in this manuscript, the authors have also performed limited analysis on the MIT/NREL TLP, and our preliminary findings reveal the MIT/NREL TLP tends to be less stable compared to the OC3-HyWind spar. The source of instability is likely attributable to the mooring implementation rather than the platform design itself. Instability issues are more likely to occur with taut leg systems (such as the MIT/NREL TLP) as opposed to systems using slack mooring lines (such as the OC3-HyWind spar) since the cable can alternate between slack and taut conditions. This swing in operation mode can introduce large jumps in cable tension, even if the platform displacement is small. These instability issues are often cured by simply decreasing the time step resolution. A further investigation is needed to identify failure modes, track the source of instability and offer remedies to correct this issue.

Remarks and Conclusions

In this work, the stability, usability, and functionality of FASTlink are assessed. The results reported with the FAST/OrcaFlex coupling tool are consistent with those obtained while using FAST with its default quasi-static cable model. The FAST/OrcaFlex coupling module reports promising results, which warrants a further study to identify the mechanisms that cause the mooring lines to influence the wind turbine response.

As expected, the simulation run-time is extended when using the FASTlink coupling module; however, high-fidelity modeling almost always requires simulation speed to be sacrificed. Earlier, it was noted that that simulation speed can be increased by simply increasing the simulation time-step. In this work, the time-step used in FAST equals that of OrcaFlex's. However, recalling that FAST calculates the tower and rotor blade aerodynamics and deflection, the size of FAST's time-step must be balanced to ensure the tower/blade dynamics resolution is acceptable between integration steps. On the same note, OrcaFlex's time step must be small to ensure the cable does not go unstable. Increasing the integration time step may not be a viable option for some systems, such as a TLP.

The method used to select time-steps between FAST and OrcaFlex is up to the user, and no clear rules are stipulated. One area needing further investigation involves defining a method to select time-step length. In terms of stability, the time-step has a profound effect on the solution, and in general, a smaller time-step implies greater simulation stability.

In terms of usability, OrcaFlex's GUI allows easy management of platform hydrodynamic properties and cable configuration. Once the simulation is run with FASTlink, the simulation file can be opened in OrcaFlex, allowing the user to gain an appreciation of the wind turbine motions through three-dimensional rendering, although the turbine dynamics are not animated in OrcaFlex.

In terms of applicability, the FASTlink tool appears suitable for a multitude of offshore wind turbine designs, including semi-submersibles and TLPs, whether the mooring lines are taut or slack. Our analysis of FASTlink suggests that the results are reasonable, the solution method is stable, and the overall coupling is meaningful. By combining the best attributes of FAST and OrcaFlex, a high-fidelity offshore wind turbine model is garnered.

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