



User's Guide to MBC3: Multi-Blade Coordinate Transformation Code for 3-Bladed Wind Turbines

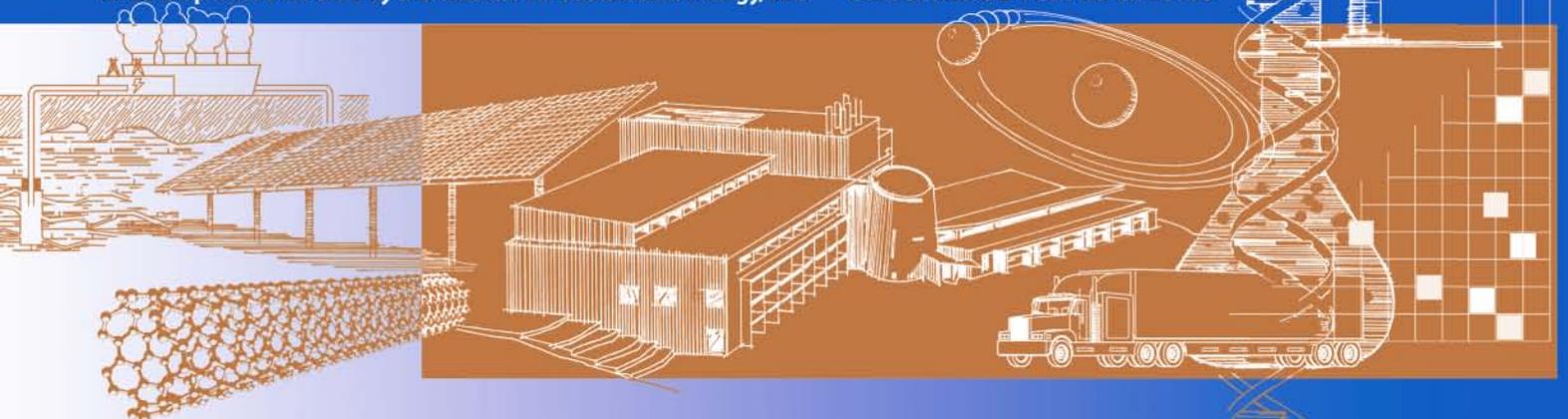
G.S. Bir

Technical Report
NREL/TP-500-44327
September 2010



NREL is operated for DOE by the Alliance for Sustainable Energy, LLC

Contract No. DE-AC36-08-GO28308



User's Guide to MBC3: Multi-Blade Coordinate Transformation Code for 3-Bladed Wind Turbines

Technical Report
NREL/TP-500-44327
September 2010

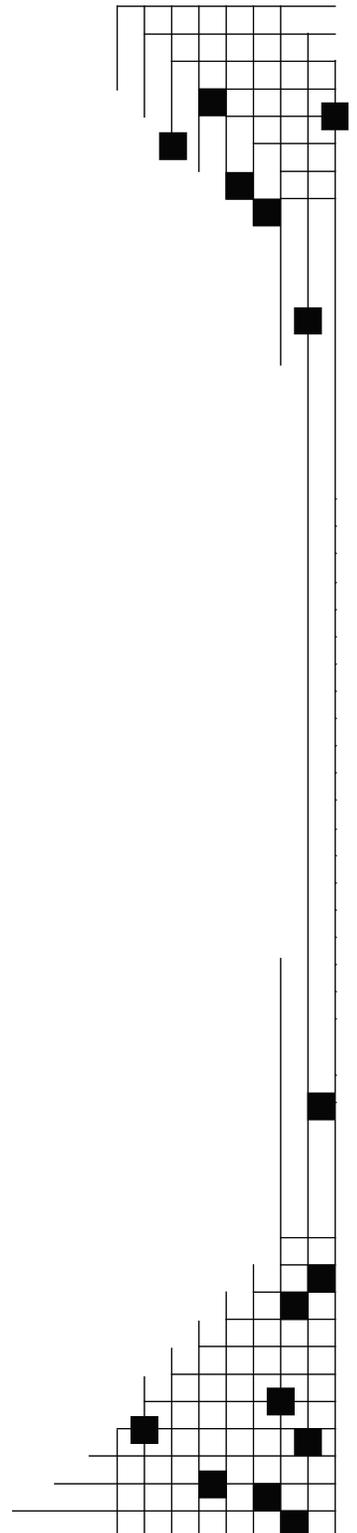
G.S. Bir

Prepared under Task No. WER9.3535

National Renewable Energy Laboratory
1617 Cole Boulevard, Golden, Colorado 80401-3393
303-275-3000 • www.nrel.gov

NREL is a national laboratory of the U.S. Department of Energy
Office of Energy Efficiency and Renewable Energy
Operated by the Alliance for Sustainable Energy, LLC

Contract No. DE-AC36-08-GO28308



NOTICE

This report was prepared as an account of work sponsored by an agency of the United States government. Neither the United States government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States government or any agency thereof.

Available electronically at <http://www.osti.gov/bridge>

Available for a processing fee to U.S. Department of Energy and its contractors, in paper, from:

U.S. Department of Energy
Office of Scientific and Technical Information
P.O. Box 62
Oak Ridge, TN 37831-0062
phone: 865.576.8401
fax: 865.576.5728
email: <mailto:reports@adonis.osti.gov>

Available for sale to the public, in paper, from:

U.S. Department of Commerce
National Technical Information Service
5285 Port Royal Road
Springfield, VA 22161
phone: 800.553.6847
fax: 703.605.6900
email: orders@ntis.fedworld.gov
online ordering: <http://www.ntis.gov/ordering.htm>



About This Guide

The dynamics of wind turbine rotor blades are conventionally expressed in rotating frames attached to the individual blades. The tower-nacelle subsystem though, sees the combined effect of all rotor blades, not the individual blades. Also, the rotor responds as a whole to excitations such as aerodynamic gusts, control inputs, and tower-nacelle motion—all of which occur in a nonrotating frame. Multi-blade coordinate transformation (MBC) helps integrate the dynamics of individual blades and express them in a fixed (nonrotating) frame. MBC involves two steps: transforming the rotating degrees of freedom and transforming the equations of motion. Reference 1 details the MBC operation. This guide summarizes the MBC concept and underlying transformations.

This guide also explains how to use MBC3, a MATLAB-based script we developed to perform multi-blade coordinate transformation of system matrices for three-bladed wind turbines. In its current form, MBC3 can be applied to system matrices generated by FAST.² The system matrices can have arbitrary—but physically consistent—sizes. These matrices can also optionally include control, disturbance, output, and feed-forward matrices. In addition, the associated state, control, and disturbance vectors can have states that are partly referred to the fixed frame and partly to the rotating frame. A key feature of MBC3 is that it can handle variable-speed operation and turbines with dissimilar blades. Depending on the analysis objective, users can generate system matrices in either the first-order (state-space) form or the second-order (physical-domain) form. MBC3 can handle both the first- and second-order system matrices. These matrices in general govern the aero-hydro-servo-elastic behavior of a wind turbine. After multi-blade coordinate transformation, MBC3 also performs eigenanalysis, and the results are presented in different formats to aid modal and stability analysis. MBC3 automatically detects whether optional matrices (e.g., control, disturbance, output, and feed-forward matrices) are present and performs multi-blade coordinate transformation accordingly.

Multi-blade coordinate transformation has several applications,³ but it is essential to modal and stability analyses. Commonly, wind turbine researchers first compute the periodic state-space matrix, time average it over the rotor rotational period, and then apply conventional eigenanalysis to compute modal and stability characteristics. Direct averaging, however, eliminates all periodic terms that contribute to system dynamics, producing errors. Although averaging itself is not always a bad approach, it must follow MBC. Reference 1 contains sample results that illustrate this point. The main advantages of MBC are capturing cumulative dynamics of the rotor blades and interaction of the rotor with the tower-nacelle subsystem, well-conditioning the system matrices by eliminating nonessential periodicity, and performing filtering operation.¹

This guide gives step-by-step instructions on how to load system matrices from FAST to MATLAB, how to perform multi-blade coordinate transformation on these matrices, and how to interpret the transformed matrices. The guide also describes the inputs to MBC3. The script runs quickly (usually in a few seconds). In the future, we will release two versions of MBC3, one written in MATLAB and the other in Fortran, with clearly

defined interfaces. This will allow users to directly integrate MBC3 with FAST or any other aeroelastic code.

Acknowledgments

Thanks to Sandy Butterfield for motivating the development of MBC3. Thanks also to Bonnie Jonkman for her help with writing the code certification script and to Marshall Buhl for his help with uploading the MBC files to our Web site. Thanks to Jason Jonkman for providing the GetMats.m script that loads FAST output data into the MATLAB workspace. DOE supported this work under contract number DE-AC36-08-GO28308.

Document Revision Record

Revision	Date	Description
1.00	Oct.13, 2008	First publication. Supports MBC3 v1.00.00.
1.10	May 18, 2008	Second publication. Supports MBC3 v1.01.00, which includes control-specific upgrades and generates descriptions of MBC-transformed states.

Table of Contents

1. Introduction	1
2. MBC Concept and Associated Transformation	3
3. Transformation of System Matrices	5
3.1 Second-Order System Matrices	5
3.2 First-Order System Matrices.....	10
4. Input Description	13
5. Executing MBC3	16
6. Output Description	18
7. Downloading and Verifying MBC-Specific Files	22
8. Future Plans	23
References	24

1. Introduction

The dynamics of wind turbine rotor blades are generally expressed in rotating frames attached to the individual blades. The tower-nacelle subsystem sees the combined effect of all rotor blades, not the individual blades. Also, the rotor responds as a whole to excitations such as aerodynamic gusts, control inputs, and tower-nacelle motion—all of which occur in a nonrotating frame. Multi-blade coordinate transformation (MBC) helps integrate the dynamics of individual blades and express it in a fixed (nonrotating) frame.

MBC offers several benefits. First, it properly models the dynamic interaction between the nonrotating tower-nacelle and the spinning rotor. Second, it offers physical insight into rotor dynamics and how the rotor interacts with fixed-system entities such as wind, controls, and the tower-nacelle subsystem. Third, MBC filters out all periodic terms except those that are integral multiples of ΩN , where Ω is the rotor angular speed and N is the number of rotor blades. A wind turbine system is basically a periodic system; equations governing its dynamics show periodic parameters that arise because of the periodic interaction between the rotating subsystem (rotor) and the nonrotating entities (tower, nacelle, wind, controls, and gravity). The blade equations usually contain all harmonics. MBC behaves as a filter, allowing the fixed system to feel only specific harmonics of blade motion. This filtering action also renders the system equations numerically well-conditioned, retaining only those periodic terms that are actually seen in a fixed reference frame.³

MBC is widely used in the helicopter field. Miller⁴ used it to analyze flap-motion related stability and control. Coleman and Feingold⁵ used it to analyze rotor in-plane motion (lag motion); theirs was the first successful attempt to understand the helicopter ground resonance problem, which had eluded earlier researchers. These efforts, however, applied MBC only in a heuristic fashion. Hohenemser and Yin⁶ gave the first mathematically sound basis. Later, Johnson³ delineated a systematic mathematical basis and presented a thorough exposition of MBC. Using this mathematical basis, Bir et al.⁷ developed a numerical MBC approach that could be applied to a general helicopter system governed by arbitrary degrees of freedom; they used the approach to analyze the stability and response of several helicopters.

Because MBC offers so many benefits, it is receiving attention in the wind turbine field. Bir, Wright, and Butterfield⁸ included MBC in a stability analysis scheme and predicted an instability caused by coalescence between the rotor in-plane and the tower motions. They also showed how MBC gives physical insight into the rotor in-plane motion. In the turbine field, Malcolm⁹ appears to be the first to derive a mathematical form of the turbine equations after applying the MBC. His prime motivation was to relate the inflow characteristics with the turbine response and to extract linearized models from general aeroelastic codes such as ADAMS.¹⁰ McCoy¹¹ extended this effort to obtain linear time-invariant system equations required in the standard control design approaches. Hansen¹² used MBC for improved modal dynamics to avoid stall-induced vibrations and later combined it with an eigenvalue approach to study the aeroelastic stability characteristics of a three-bladed turbine.¹³ Riziotis et al.¹⁴ applied MBC to analyze stability of two three-bladed turbines: one stall regulated and the other pitch regulated. Bir and Jonkman

used MBC in conjunction with FAST to study aeroelastic characteristics of a 5 MW turbine in both land-based and offshore configurations.¹⁵

All attempts at MBC thus far, in both the helicopter and the wind turbine fields, assume that the rotor speed is constant and the rotor blades are similar. A modern wind turbine is rarely constant speed. Also, turbines may not have identical blades, structurally or aerodynamically. We need an MBC approach that overcomes these limitations.

This report describes a new MBC scheme that is applicable to a variable-speed turbine, which may also have dissimilar blades. The scheme also covers control, disturbance, output, and feed-forward matrices, which have been ignored to date. Depending on the analysis objective, wind turbine researchers may generate system matrices in either the first-order (state-space) form or the second-order (physical-domain) form. We develop MBC relations for both these forms. In the literature, MBC is also referred to as the Fourier coordinate transformation (FCT) and as the Coleman transformation.

Although MBC has several applications, it is essential to and mostly used for modal and stability analyses and state-space-based controls design. Commonly, wind turbine researchers first compute the periodic state-space matrix, time average it over the rotor rotational period, and then apply conventional eigenanalysis to compute modal and stability characteristics. Averaging, however, eliminates all periodic terms that contribute to system dynamics and can yield erroneous results. Although averaging itself is not a bad approach, it must follow MBC.¹ Theoretically, a Floquet analysis—not averaging—must follow MBC. In normal cases, however, MBC eliminates the dominant periodic terms; the remaining periodic terms (N_b/rev , $2N_b/\text{rev}$, ..., etc., for an N_b -bladed turbine) usually have a negligible magnitude and, therefore, direct averaging of MBC-transformed matrices suffices.¹

In this guide, Section 2 summarizes the MBC concept and how it relates the blade coordinates to rotor coordinates. Section 3 shows how we transform the system matrices from rotating to nonrotating (fixed) coordinates, which is the objective of MBC. We develop transformation relations for both first- and second-order matrices. Section 4 describes the inputs required by MBC3, and Section 5 explains how MBC3 is executed. Section 6 describes the MBC3-generated outputs; these include MBC-transformed system matrices and eigenanalysis results. Section 7 lists the MATLAB scripts required for MBC; we also give our Web site for access to these scripts, along with sample I/O files. Section 8 summarizes future plans.

2. MBC Concept and Associated Transformation

The turbine equations (i.e., the coupled tower-nacelle-rotor equations) are generally derived using mixed degrees of freedom. Some degrees of freedom may be in the rotating frame and others in the nonrotating frame. This can be desirable. For example, in some simulation studies, we might be interested in studying the tower response in the ground-fixed (nonrotating) frame and the blades' response in their respective rotating frames. Often, however, we are interested in understanding the coupled behavior of the tower-nacelle-rotor system. In such cases, we want to express the full system behavior in a fixed frame. MBC helps us achieve this through a rotating-frame to nonrotating-frame coordinate transformation.

Consider a rotor with N_b blades spaced equally around the rotor azimuth. The azimuth location of b th blade is given by

$$\psi_b = \psi + (b-1) \frac{2\pi}{N_b} \quad (1)$$

where ψ is the azimuth of the first (reference) blade (see Figure 1 in which $N_b = 3$). We assume that $\psi = 0$ implies the first blade is vertically up.

Let q_b be a particular rotating degree of freedom for the b th blade. The MBC is a linear transformation that relates the rotating degrees of freedom to new degrees of freedom, defined as follows³:

$$\begin{aligned} q_0 &= \frac{1}{N_b} \sum_{b=1}^{N_b} q_b \\ q_{nc} &= \frac{2}{N_b} \sum_{b=1}^{N_b} q_b \cos n\psi_b \\ q_{ns} &= \frac{2}{N_b} \sum_{b=1}^{N_b} q_b \sin n\psi_b \\ q_{N/2} &= \frac{1}{N_b} \sum_{b=1}^{N_b} q_b (-1)^b \end{aligned} \quad (2)$$

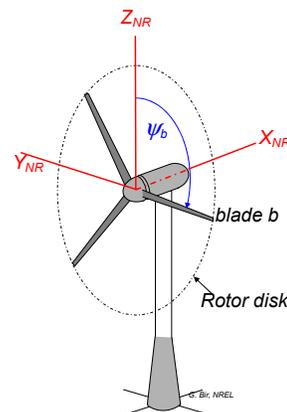


Figure 1. Rotor reference frame

Mathematically, the new degrees of freedom are the projections of the blade degrees of freedom onto the nonrotating (fixed) frame, which is the frame (X_{NR}, Y_{NR}, Z_{NR}) shown in Figure 1. The axis X_{NR} is directed along the shaft axis, and Z_{NR} lies in the vertical plane. In the literature, $q_0, q_{nc}, q_{ns}, q_{N_b/2}$ are called nonrotating degrees of freedom; we call these rotor coordinates because they express the cumulative behavior of all rotor blades (not individual blades) in the fixed frame. The physical interpretation of each rotor coordinate depends on the degree of freedom to which it refers. For example, if q_b is a flap degree of

freedom, then q_0 is the rotor coning, q_{1c} is the rotor tip-path-plane fore-aft tilt about an axis that is horizontal and normal to the rotor shaft, and q_{1s} is the rotor tip-path-plane side-side tilt about an axis that is vertical and normal to the rotor shaft. If q_b is a lag degree of freedom, then q_0 is the rotor collective lag, q_{1c} is the horizontal displacement of the rotor center-of-mass in the rotor plane, and q_{1s} is the vertical displacement of the rotor center-of-mass in the rotor plane.⁸ Other degrees of freedom can be interpreted using Equation 2 and drawing supporting sketches. The rotor modes corresponding to q_{nc} and q_{ns} ($n > 1$) and $q_{Nb/2}$ are called reactionless modes because they do not cause any transference of moments or forces from the rotor to the hub (fixed frame). The value of n goes from 1 to $(N_b - 1)/2$ if N_b is odd, and from 1 to $(N_b - 2)/2$ if N_b is even. The $q_{Nb/2}$ mode, called the differential mode, exists only if the number of blades is even.³

Most of the wind turbines in the world have three blades. Therefore, we assume $N_b = 3$ in this report. The formulation we developed here for $N_b = 3$, however, is general and can be readily extended for more blades. Setting $N_b = 3$ in Equation 2, we obtain

$$\begin{aligned}
 q_0 &= \frac{1}{3} \sum_{b=1}^3 q_b \\
 q_c = q_{1c} &= \frac{2}{3} \sum_{b=1}^3 q_b \cos \psi_b \\
 q_s = q_{1s} &= \frac{2}{3} \sum_{b=1}^3 q_b \sin \psi_b
 \end{aligned} \tag{3}$$

We call q_{1c} the cosine-cyclic mode and q_{1s} the sine-cyclic mode. These two cyclic modes, together with the coning mode, q_0 , lead to coupling of the rotor with the rest of the turbine. Equation 3 determines the rotor coordinates given the blade coordinates. The inverse transformation, which yields the blade coordinate given the rotor coordinates, is

$$q_b = q_0 + q_c \cos \psi_b + q_s \sin \psi_b; \quad b = 1, 2, 3. \tag{4}$$

3. Transformation of System Matrices

Most aeroelastic codes generate linear equations in the second-order form and, if required (for controls, for example), transform these to a first-order form. We derive MBC relations for both the second- and first-order systems.

3.1 Second-Order System Matrices

Second-order system equations can be written as

$$M\ddot{X} + C\dot{X} + KX = Fu + F_d w \quad (5a)$$

and the associated output equation as

$$Y = C_v \dot{X} + C_d X + Du + D_d w \quad (5b)$$

where M , C , K , F , and F_d are the mass, damping, stiffness, control, and disturbance matrices, respectively. The u and w are, respectively, the control and disturbance vectors. The size of the M , C , and K matrices is $n \times n$, where n is the number of system degrees of freedom. The sizes of the u , w , and Y vectors are n_c , n_w , and n_o , respectively, where n_c is the number of control variables, n_w is the number of disturbance states, and n_o is the number of outputs. Note that the C matrix contains gyroscopic terms in addition to the structural damping terms, even though we designate it as a damping matrix. Similarly, K contains centrifugal terms in addition to the structural and aerodynamic stiffness terms. Also, each of these matrices contains direct and cross-coupling terms. For example, M contains direct blade inertias as well as blade-tower coupling inertias. The formulation we develop does not, however, require the user to explicitly partition any of the matrices into submatrices delineating direct and cross-coupling terms. The partitioning is implicit in the definition of the coordinates vector X (also called the physical vector in the literature). Assume that X is expressed as

$$X = \begin{Bmatrix} X_F \\ q_1^1 \\ q_2^1 \\ q_3^1 \\ \vdots \\ q_1^j \\ q_2^j \\ q_3^j \\ \vdots \\ q_1^m \\ q_2^m \\ q_3^m \end{Bmatrix} \quad (6)$$

where X_F is a $nF \times 1$ column vector representing the nF fixed-frame-referenced degrees of freedom, q_b^j is the j^{th} rotating degree of freedom for the b^{th} blade, and m is the number of rotating degrees of freedom for each blade. The length n of vector X , then equals $nF + 3m$, the total number of degrees of freedom for the full system. Most often, aeroelastic codes such as FAST assume that the physical vector has the form of Equation 6. If not, we can always perform simple rows and columns exchange to transform system equations (Equation 5) such that the physical vector X assumes the form of Equation 6.

Two methods transform Equation 5 to the nonrotating frame: the operational method and the substitution method.³ In the operational method, we apply the following summation operators to the rotating-frame equations of motion:

$$\begin{aligned} & \frac{1}{3} \sum_{b=1}^3 (rot_eqns) \\ & \frac{2}{3} \sum_{b=1}^3 (rot_eqns) \cos \psi_b \\ & \frac{2}{3} \sum_{b=1}^3 (rot_eqns) \sin \psi_b \end{aligned} \quad (7)$$

where *rot_eqns* refers to the rotating-frame equations of motion. Note the similarity of summation operators in Equations 3 and 7. To accomplish these operations, we first need the *rot_eqns* to be available in an analytical form, explicitly showing all periodic terms. The periodic terms involve trigonometric functions of $\sin(k \Psi_b)$ and $\cos(k \Psi_b)$ terms, where k in general can have any integer value, depending on the system. First, we must express all products of trigonometric functions as sums of trigonometric functions. Next, we must perform cumbersome operations to ensure that the value of each n in these trigonometric functions satisfies the requirements of Equation 3. Finally, we use transformation (Equation 3) to convert the *rot_eqns* to the nonrotating frame. The operational method is thus quite tedious and applicable only if we have equations of motion available in an analytical form.

All aeroelastic codes generate equations of motion in a numerical form and are therefore not amenable to the operational method. We must instead use the substitutional method. In this method, we substitute the rotational degrees of freedom with the rotor coordinate using Equation 4. If we do this, the three rotational degrees of freedom (q_1^j, q_2^j, q_3^j), corresponding to the j^{th} degree of freedom for each of the three blades, can be transformed to nonrotating coordinates (q_0^j, q_c^j, q_s^j) using the following relation:

$$\begin{Bmatrix} q_1^j \\ q_2^j \\ q_3^j \end{Bmatrix} = \tilde{t} \begin{Bmatrix} q_0^j \\ q_c^j \\ q_s^j \end{Bmatrix} \quad (8)$$

where

$$\tilde{t} = \begin{bmatrix} 1 & \cos \psi_1 & \sin \psi_1 \\ 1 & \cos \psi_2 & \sin \psi_2 \\ 1 & \cos \psi_3 & \sin \psi_3 \end{bmatrix} \quad (9)$$

Using Equation 8, the full-system, rotating-frame, degrees-of-freedom vector, X , is expressed in terms of the nonrotating-frame, degrees-of-freedom vector, X_{NR} , as follows

$$X = T_1 X_{NR} \quad (10)$$

where T_1 is the block diagonal matrix:

$$T_1 = \begin{bmatrix} I_{nF \times nF} & & & & \\ & \tilde{t} & & & \\ & & \tilde{t} & & \\ & & & \ddots & \\ & & & & \tilde{t} \end{bmatrix}_{(nF+3m) \times (nF+3m)} \quad (11)$$

and

$$X_{NR} = \begin{Bmatrix} X_F \\ q_0^1 \\ q_c^1 \\ q_s^1 \\ \vdots \\ q_0^j \\ q_c^j \\ q_s^j \\ \vdots \\ q_0^m \\ q_c^m \\ q_s^m \end{Bmatrix} \quad (12)$$

Taking the first and second time derivatives of the two sides of Equation 10, we obtain

$$\begin{aligned} \dot{X} &= T_1 \dot{X}_{NR} + \Omega T_2 X_{NR} \\ \ddot{X} &= T_1 \ddot{X}_{NR} + 2\Omega T_2 \dot{X}_{NR} + (\Omega^2 T_3 + \dot{\Omega} T_2) X_{NR} \end{aligned} \quad (13)$$

where Ω is the rotor angular velocity and $\dot{\Omega}$ is the rotor angular acceleration. The T_2 and T_3 transformation matrices are given by

$$T_2 = \begin{bmatrix} \mathbf{0}_{nF \times nF} & & & & \\ & \tilde{t}_2 & & & \\ & & \tilde{t}_2 & & \\ & & & \ddots & \\ & & & & \tilde{t}_2 \end{bmatrix}_{(nF+3m) \times (nF+3m)} \quad (14)$$

$$T_3 = \begin{bmatrix} \mathbf{0}_{nF \times nF} & & & & \\ & \tilde{t}_3 & & & \\ & & \tilde{t}_3 & & \\ & & & \ddots & \\ & & & & \tilde{t}_3 \end{bmatrix}_{(nF+3m) \times (nF+3m)} \quad (15)$$

where

$$\tilde{t}_2 = \begin{bmatrix} 0 & -\sin \psi_1 & \cos \psi_1 \\ 0 & -\sin \psi_2 & \cos \psi_2 \\ 0 & -\sin \psi_3 & \cos \psi_3 \end{bmatrix} \quad \text{and} \quad \tilde{t}_3 = \begin{bmatrix} 0 & -\cos \psi_1 & -\sin \psi_1 \\ 0 & -\cos \psi_2 & -\sin \psi_2 \\ 0 & -\cos \psi_3 & -\sin \psi_3 \end{bmatrix} \quad (16)$$

Substituting for X and its time derivatives from Equations 10 and 13 in Equations 5a and 5b, we obtain the system equations of motion in the nonrotating frame as

$$M_{NR}\ddot{X}_{NR} + C_{NR}\dot{X}_{NR} + K_{NR}X_{NR} = F_{NR}u_{NR} + F_{dNR}w \quad (17)$$

and

$$Y_{NR} = C_{vNR}\dot{X}_{NR} + C_{dNR}X_{NR} + D_{NR}u_{NR} + D_{dNR}w \quad (18)$$

The subscript NR identifies the associated quantity to be in the nonrotating frame. The various matrices in Equations 17 and 18 are as follows:

$$\begin{aligned} M_{NR} &= MT_1 \\ C_{NR} &= 2\Omega MT_2 + CT_1 \\ K_{NR} &= \Omega^2 MT_3 + \dot{\Omega} MT_2 + \Omega CT_2 + KT_1 \\ F_{NR} &= FT_{1c} \\ F_{dNR} &= F_d \\ C_{vNR} &= T_{1o}^{-1} C_v T_1 \\ C_{dNR} &= T_{1o}^{-1} (\Omega C_v T_2 + C_d T_1) \\ D_{NR} &= T_{1o}^{-1} DT_{1c} \\ D_{dNR} &= T_{1o}^{-1} D_d \end{aligned} \quad (19)$$

Note that the variable-speed operation affects only the stiffness matrix. Also, the disturbance matrix stays unchanged because the disturbance w is already in the nonrotating frame. If F_c and m_c are the number of controls in the fixed and rotating frames, respectively, then the control vectors u and u_{NR} are related as follows:

$$u = T_{1c} u_{NR} \quad (20)$$

where

$$T_{1c} = \begin{bmatrix} I_{F_c \times F_c} & & & \\ & \tilde{t} & & \\ & & \tilde{t} & \\ & & & \ddots \\ & & & & \tilde{t} \end{bmatrix}_{(F_c+3m_c) \times (F_c+3m_c)} \quad (21)$$

Similarly,

$$Y = T_{1o} Y_{NR} \quad (22)$$

where

$$T_{1o} = \begin{bmatrix} I_{F_o \times F_o} & & & \\ & \tilde{t} & & \\ & & \tilde{t} & \\ & & & \ddots \\ & & & & \tilde{t} \end{bmatrix}_{(F_o+3m_o) \times (F_o+3m_o)} \quad (23)$$

and where F_o and m_o are the number of outputs in the fixed and rotating frames, respectively. Note that $F_c+3 \times m_c$ equals n_c , the total number of control variables. Similarly, $F_o+3 \times m_o$ equals n_o , the total number of disturbance states. In summary, after applying MBC, the rotating-frame, second-order equations (Equations 5a and 5b) are transformed to the nonrotating-frame equations (Equations 17 and 18).

3.2 First-Order System Matrices

First-order system equations are generally expressed as

$$\dot{z} = Az + Bu + B_d w \quad (24)$$

where z is the state-space vector. The A , B , and B_d are, respectively, the system dynamics, control, and disturbance matrices. The u and w are the control and disturbance vectors, which are the same vectors that appear in Equation 5a. The associated output equation is

$$Y = Cz + Du + D_d w \quad (25)$$

The size of the A matrix is $N \times N$, where N is the number of system states. The sizes of the u , w , and Y vectors are n_c , n_w , and n_o , respectively, where n_c is the number of control variables, n_w is the number of disturbance states, and n_o is the number of outputs. From

this information, we can readily infer the sizes of the remaining matrices (listed in Table 1). The state-space vector, z , is related to the physical vector, X , as follows:

$$z = \begin{Bmatrix} X \\ \dot{X} \end{Bmatrix} \quad (26)$$

Using Equations 26, 8–16, and 20–23, Equations 24 and 25 are transformed to the nonrotating frame as follows:

$$\dot{z}_{NR} = A_{NR}z_{NR} + B_{NR}u_{NR} + B_{dNR}w \quad (27)$$

and

$$Y_{NR} = C_{NR}z_{NR} + D_{NR}u_{NR} + D_{dNR}w \quad (28)$$

The subscript NR identifies the associated quantity as being in the nonrotating frame. The various matrices in Equations 27 and 28 are as follows:

$$A_{NR} = \begin{bmatrix} T_1^{-1} & 0 \\ 0 & T_1^{-1} \end{bmatrix} \left\{ A \begin{bmatrix} T_1 & 0 \\ \Omega T_2 & T_1 \end{bmatrix} - \begin{bmatrix} \Omega T_2 & 0 \\ \Omega^2 T_3 + \dot{\Omega} T_2 & 2\Omega T_2 \end{bmatrix} \right\} \quad (29)$$

$$B_{NR} = \begin{bmatrix} T_1^{-1} & 0 \\ 0 & T_1^{-1} \end{bmatrix} B T_{1C} \quad (30)$$

$$C_{NR} = T_{10}^{-1} [C_d T_1 + \Omega C_v T_2 \quad C_v T_1]; \quad C = [C_d \quad C_v] \quad (31)$$

$$D_{NR} = T_{10}^{-1} D T_{1C} \quad (32)$$

$$D_{dNR} = T_{10}^{-1} D_d \quad (33)$$

In summary, after applying MBC, the rotating-frame first-order equations (Equations 24 and 25) are transformed to the nonrotating-frame equations (Equations 27 and 28). Also, note that the dynamics matrix, A , is influenced by the variable rotor speed.

4. Input Description

MBC3 is a postprocessor for FAST; its inputs come from a FAST output file. Typically, for a given wind turbine model, the user first runs FAST with the option to generate state-space matrices. These matrices and other ancillary data are stored in a FAST output file named **.lin*, where *** represents the name (without extension) of the input file the user supplies to FAST. For example, if *test01.fst* is the name of the user input file to FAST with an option to generate a linear model, then FAST would generate an output file named *test01.lin* (FAST automatically appends the suffix *.lin*). This file contains system matrices at each of the N_ψ azimuth steps; the user specifies N_ψ via the parameter *NAzimStep* in the FAST input file. The FAST output file also contains the vector Ψ , which is the vector of reference-blade azimuth positions at the N_ψ steps. The ψ is the reference azimuth position defined in Equation 1. The user loads the **.lin* file into MATLAB using the *GetMats* command (see Section 5). The loaded file includes data items that MBC3 and CCE (discussed in the Section 5) use as inputs. Table 1 lists these inputs and their descriptions.

Table 1. Inputs to MBC3 and CCE

Input Parameter	Description	See Equation	Symbol	Size
<i>NActvDOF</i>	Number of degrees of freedom	5a	n	scalar
<i>N</i>	Number of states	24	N	scalar
<i>NInputs</i>	Number of control inputs	34	N_c	scalar
<i>NDisturbs</i>	Number of disturbance states	24	n_w	scalar
<i>NumOuts</i>	Number of outputs	35	N_o	scalar
<i>NAzimStep</i>	Number of azimuth steps at which system matrices are input	—	N_ψ	scalar
<i>Azimuth</i>	Vector of azimuth positions, in degrees, of the reference blade (blade 1) with respect to the Z_{NR} axis shown in Figure 1. System matrices are input at each of these azimuth positions.	—	Ψ	N_ψ
<i>Omega</i>	Vector of rotor speeds at specified azimuths (rad/s)	19	Ω	N_ψ
<i>OmegaDot</i>	Vector of rotor accelerations at specified azimuths (rad/s ²)	19	$\dot{\Omega}$	N_ψ
<i>AMat</i>	First-order system matrices at N_ψ azimuth locations	24	A	$N \times N \times N_\psi$
<i>BMat</i>	First-order control matrices at N_ψ azimuth locations	24	B	$N \times N_c \times N_\psi$
<i>BdMat</i>	First-order disturbance matrices at N_ψ azimuth locations	24	B_d	$N \times N_w \times N_\psi$
<i>CMat</i>	First-order output matrices associated with states at N_ψ azimuth locations	25	C	$N_o \times N \times N_\psi$
<i>DMat</i>	Output matrices associated with controls at N_ψ azimuth locations (same for first- and second-order systems)	5b & 25	D	$N_o \times N_c \times N_\psi$
<i>DdMat</i>	Output matrices associated with disturbances at N_ψ azimuth locations (same for first- and second-order systems)	5b & 25	D_d	$N_o \times N_w \times N_\psi$

<i>MassMat</i>	Second-order system mass matrices at N_ψ azimuth locations	5a	M	$n \times n \times N_\psi$
<i>DampMat</i>	Second-order system damping matrices at N_ψ azimuth locations	5a	C	$n \times n \times N_\psi$
<i>StffMat</i>	Second-order system stiffness matrices at N_ψ azimuth locations	5a	K	$n \times n \times N_\psi$
<i>FMat</i>	Second-order control matrices at N_ψ azimuth locations	5a	F	$n \times N_c \times N_\psi$
<i>FdMat</i>	Second-order system disturbance matrices	5a	F_d	$n \times N_w \times N_\psi$
<i>VelCMat</i>	Second-order output matrices associated with velocities at N_ψ azimuth locations	5b	C_v	$N_o \times n \times N_\psi$
<i>DspCMat</i>	Second-order output matrices associated with displacements at N_ψ azimuth locations	5b	C_d	$N_o \times n \times N_\psi$
<i>RotTriplet-IndicesStates</i>	The i^{th} row in this matrix identifies the indices of the i^{th} rotating-frame degrees of freedom associated with blades 1, 2, and 3	—	—	$m \times 3$
<i>RotTripletIndicesCntrlInpt</i>	The i^{th} row in this matrix identifies the indices of the i^{th} rotating-frame control associated with blades 1, 2, and 3	—	—	$m_c \times 3$
<i>RotTripletIndicesOutput</i>	The i^{th} row in this matrix identifies the indices of the i^{th} rotating-frame output associated with blades 1, 2, and 3	—	—	$m_o \times 3$
<i>xdop</i>	Matrix of operating-point state-derivative vectors at N_ψ azimuth locations	—	—	$N \times N_\psi$
<i>xop</i>	Matrix of operating-point state vectors at N_ψ azimuth locations	—	—	$N \times N_\psi$
<i>qd2op</i>	Matrix of operating-point acceleration vectors at N_ψ azimuth locations	—	—	$n \times N_\psi$
<i>qdop</i>	Matrix of operating-point velocity vectors at N_ψ azimuth locations	—	—	$n \times N_\psi$
<i>qop</i>	Matrix of operating-point displacement vectors at N_ψ azimuth locations	—	—	$n \times N_\psi$
<i>yop</i>	Matrix of operating-point output vectors at N_ψ azimuth locations	—	—	$N_o \times N_\psi$
<i>DescStates</i>	Description of degrees-of-freedom vector	—	—	$n \times 1$

The first column in Table 1 lists the names of the inputs required by MBC3. The user does not explicitly input any of the parameters; these are loaded automatically into MATLAB via the command *GetMats*. The second column gives brief descriptions of the input parameters. Entries in the third and fourth columns refer to equation numbers and symbols used in this guide for the various parameters. A dash in these columns implies that corresponding parameter is not used in the mathematical derivation and, therefore, has no associated symbol or equation number. The last column lists the sizes of these parameters. The MBC3 code automatically identifies the sizes of the various matrices by and checks to ensure compatibility with inputs n and N . As described in Section 4, n equals $nF + 3m$, where nF is the degrees of freedom referenced to the fixed frame and m is the number of rotating degrees of freedom for each blade. N_c is the total number of control parameters:

$$N_c = F_c + 3m_c \quad (34)$$

where F_c is number of controls referenced to the fixed frame and m_c is the number of controls referenced to each rotating-blade frame. N_o is the total number of outputs:

$$N_o = F_o + 3m_o \quad (35)$$

where F_o is number of outputs referenced to the fixed frame and m_o is the number of outputs referenced to each rotating-blade frame.

The matrix yop is generated by FAST at user request; j^{th} column in this matrix represents the output vector at the j^{th} azimuth location. This column vector is transformed as follows:

$$Y_{NR} = T_{1o}^{-1} Y_{op} \quad (36)$$

Note that all inputs to MBC3 (first column of Table 1) must come from the FAST-generated *.lin output file. In general, however, some inputs to MBC3 may not be available in the FAST output file. For example, a user can execute FAST with options to generate only first-order matrices and exclude disturbances. In this case, second-order matrices as well as disturbance-related matrices (BdMat and DdMat) will not appear in the FAST output file. MBC3 automatically detects which optional matrices (e.g., control, disturbance, output, and feed-forward matrices) are present and which are not and performs multi-blade coordinate transformation accordingly.

In addition, note that FAST always generates the first-order system matrix, $AMat$, even if the user requests only a second-order system analysis. Therefore, the *GetMats* command always loads this matrix into the MATLAB workspace.

5. Executing MBC3

To execute MBC3, follow the steps outlined in this section.

Step 1: Open MATLAB (ensure that paths have been set to directories where MBC3-related scripts and FAST output files reside). Issue this command:

GetMats

The following message will appear on the workspace screen, followed by a prompt sign:

ENTER:

Name of FAST linearization output file to process
(omit the .lin extension when entering the name)

<-----

<

At the prompt sign, type the name of the FAST output file without the extension *.lin*. For example, if *test01.lin* is the name of the FAST output file, type

test01

To confirm that the inputs listed in Table 1 have been loaded successfully into MATLAB, type *who* and check the parameters that appear in the workspace.

Step 2: Issue this command

mbc3

If the MBC execution is successful, the following message will appear on the workspace screen:

Multi-Blade Coordinate transformation completed

Type *who* to confirm that the MBC-related outputs, listed in Table 2 and described in the Section 6, have been generated.

Next, if you are interested in azimuth-averaged system matrices and the system eigenanalysis results, type

cce

Note that azimuth-averaging of system matrices zeroes out all periodic terms, which is equivalent to retaining just the constant-coefficient terms in the system equations. As a result, the literature often refers to this operation as the constant-coefficient equations

(CCE) approximation. After you have issued the *cce* command and the eigenanalysis of the constant-coefficient approximation is successful, the following message will appear:

cce.m completed

Note that *cce* performs eigenanalysis only on the first-order system matrix, *A*. Type *who* to confirm if the CCE-related outputs, listed in Table 3, are generated.

After you have loaded FAST-generated variables into MATLAB (using the *GetMats* command), you have three options to issue further commands: execute only *mbc3*, execute only *cce*, or execute both *mbc3* and *cce* (the sequence is immaterial). If you choose the last option, outputs generated from each command, together with the original inputs, will appear in the MATLAB workspace. Section 6 describes the outputs generated by *mbc3* and *cce* commands.

6. Output Description

The command *mbc3* generates two sets of outputs: MBC-transformed matrices, and eigen-results associated with the azimuth-averaged MBC-transformed system matrices. Table 2 lists these outputs.

Table 2. MBC3-Generated Outputs

Output Parameter	Description	Symbol	See Equation	Size
<i>MBC_A</i>	MBC-transformed first-order system matrices at N_ψ azimuth locations	A_{NR}	27	$N \times N \times N_\psi$
<i>MBC_B</i>	MBC-transformed first-order control matrices at N_ψ azimuth locations	B_{NR}	27	$N \times N_c \times N_\psi$
<i>MBC_Bd</i>	MBC-transformed first-order disturbance matrices at N_ψ azimuth locations	B_{dNR}	27	$N \times N_w \times N_\psi$
<i>MBC_C</i>	MBC-transformed output matrices, associated with states, at N_ψ azimuth locations	C_{NR}	28	$N_o \times N \times N_\psi$
<i>MBC_D</i>	MBC-transformed output matrices, associated with controls, at N_ψ azimuth locations	D_{NR}	28	$N_o \times N_c \times N_\psi$
<i>MBC_Dd</i>	MBC-transformed output matrices, associated with disturbances, at N_ψ azimuth locations	D_{dNR}	28	$N_o \times N_w \times N_\psi$
<i>MBC_AvgA</i>	MBC-transformed first-order system matrix (azimuth-averaged)	—	—	$N \times N$
<i>MBC_AvgB</i>	MBC-transformed first-order control matrix (azimuth-averaged)	—	—	$N \times N_c$
<i>MBC_AvgBd</i>	MBC-transformed first-order disturbance matrix (azimuth-averaged)	—	—	$N \times N_w$
<i>MBC_AvgC</i>	MBC-transformed first-order output matrix associated with states (azimuth-averaged)	—	—	$N_o \times N$
<i>MBC_AvgD</i>	MBC-transformed first-order output matrix associated with controls (azimuth-averaged)	—	—	$N_o \times N_c$
<i>MBC_AvgDd</i>	MBC-transformed first-order output matrix associated with disturbances (azimuth-averaged)	—	—	$N_o \times N_w$
<i>MBC_M</i>	MBC-transformed mass matrix at N_ψ azimuth locations	M_{NR}	17	$n \times n \times N_\psi$
<i>MBC_Dmp</i>	MBC-transformed	C_{NR}	17	$n \times n \times N_\psi$

<i>MBC_K</i>	damping/gyroscopic matrix at N_ψ azimuth locations MBC-transformed stiffness matrix at N_ψ azimuth locations	K_{NR}	17	$n \times n \times N_\psi$
<i>MBC_F</i>	MBC-transformed second-order control matrices at N_ψ azimuth locations	F_{NR}	17	$n \times N_c \times N_\psi$
<i>MBC_Fd</i>	MBC-transformed second-order disturbance matrices at N_ψ azimuth locations	F_{dNR}	17	$n \times N_w \times N_\psi$
<i>MBC_Vc</i>	MBC-transformed second-order output matrices associated with velocities at N_ψ azimuth locations	C_{vNR}	18	$N_o \times n \times N_\psi$
<i>MBC_Dc</i>	MBC-transformed second-order output matrices associated with displacements at N_ψ azimuth locations	C_{dNR}	18	$N_o \times n \times N_\psi$
<i>MBC_Yop</i>	MBC-transformed matrix of operating-point output vectors at N_ψ azimuth locations	—	—	$N_o \times N_\psi$
<i>MBC_AvgYop</i>	MBC-transformed operating-point output vector (azimuth-averaged)	—	—	$N_o \times 1$
<i>MBC_DampedFrequency</i>	MBC-transformed and azimuth-averaged system damped frequencies in rad/s	—	—	$M \times 1$
<i>MBC_DampedFrequencyHz</i>	MBC-transformed and azimuth-averaged system damped frequencies in Hz	—	—	$M \times 1$
<i>MBC_DampingRatio</i>	MBC-transformed and azimuth-averaged system damping ratios	—	—	$M \times 1$
<i>MBC_DecrementRate</i>	MBC-transformed and azimuth-averaged system decrement rates	—	—	$M \times 1$
<i>MBC_EigenVects</i>	MBC-transformed and azimuth-averaged system eigenvectors	—	—	$N \times N$
<i>MBC_Evals</i>	MBC-transformed and azimuth-averaged system eigenvalues	—	—	$N \times 1$
<i>MBC_ModeShapeMagnitude</i>	MBC-transformed and azimuth-averaged system modal amplitudes	—	—	$n \times M$
<i>MBC_ModeShapePhaseDeg</i>	MBC-transformed and azimuth-averaged modal phases (deg)	—	—	$n \times M$
<i>MBC_NaturalFrequency</i>	MBC-transformed and azimuth-averaged natural frequencies in rad/s	—	—	$M \times 1$
<i>MBC_NaturalFrequencyHz</i>	MBC-transformed and azimuth-averaged natural frequencies in Hz	—	—	$M \times 1$
<i>MBC_reduced_size_eigenVals</i>	MBC-transformed and	—	—	$M \times 1$

	azimuth-averaged system eigenvalues (showing complex conjugate eigenvalues only with non-negative imaginary parts)			
<i>MBC_reduced_size_eigenVects</i>	MBC-transformed and azimuth-averaged system eigenvectors associated with <i>MBC_reduced_size_eigenVals</i>	—	—	$n \times M$
<i>MBC_States_Desc</i>	Description of MBC-transformed states	—	—	$n \times 1$

Symbols in the fifth column identify sizes of the MBC3-generated outputs. Section 4 defines all symbols except M . Before we define M , note that eigenvalues of a real system matrix appear as real or as complex conjugate values. A pair of complex conjugate eigenvalues refers to the same mode. Only one of these conjugate eigenvalues, say the eigenvalue with the positive imaginary part, suffices to yield modal frequency and damping; the other eigenvalue, the one with the negative imaginary part, does not give any additional information. Accordingly, we output eigen-results corresponding to only those eigenvalues that are either real or have positive imaginary parts.

Now, we are ready to define M ; it equals $N - m_n$, where m_n is the number of eigenvalues with negative imaginary parts. Thus, elements of vectors *MBC_DampedFrequency*, *MBC_DampedFrequencyHz*, *MBC_DampingRatio*, *MBC_DecrementRate*, *MBC_NaturalFrequency*, *MBC_NaturalFrequencyHz*, and *MBC_eigenVals* correspond to those eigenvalues whose imaginary parts are either zero or positive.

The command *cce*, as described in Section 5, generates azimuth-averaged matrices for first- and second-order systems and also obtains an eigensolution of the system matrix A . Table 3 lists these outputs.

Table 3. CCE-Generated Outputs

Output Parameter	Description	Size
<i>AvgAMat</i>	Azimuth-averaged system matrix	$N \times N$
<i>AvgBMat</i>	Azimuth-averaged first-order control matrix	$N \times N_c$
<i>AvgBdMat</i>	Azimuth-averaged first-order disturbance matrix	$N \times N_w$
<i>AvgCMat</i>	Azimuth-averaged first-order output matrix associated with states	$N_o \times N$
<i>AvgDMat</i>	Azimuth-averaged first-order output matrix associated with controls	$N_o \times N_c$
<i>AvgDdMat</i>	Azimuth-averaged output matrix associated with disturbances	$N_o \times N_w$
<i>Avgxdop</i>	Azimuth-averaged vector of the time-derivatives of the states	$N \times 1$
<i>Avgxop</i>	Azimuth-averaged vector of the system states	$N \times 1$
<i>AvgMassMat</i>	Azimuth-averaged mass matrix	$n \times n$
<i>AvgDampMat</i>	Azimuth-averaged damping matrix	$n \times n$
<i>AvgStffMat</i>	Azimuth-averaged stiffness matrix	$n \times n$
<i>AvgFMat</i>	Azimuth-averaged second-order control matrix	$n \times N_c$
<i>AvgFdMat</i>	Azimuth-averaged second-order disturbance matrix	$n \times N_w$
<i>AvgVelCMat</i>	Azimuth-averaged second-order output matrix associated	$N_o \times n$

	with velocities	
<i>AvgDspCMat</i>	Azimuth-averaged second-order output matrix associated with displacements	$N_0 \times n$
<i>Avgqd2op</i>	Azimuth-averaged vector of the operating-point accelerations	$n \times 1$
<i>Avgqdop</i>	Azimuth-averaged vector of the operating-point velocities	$n \times 1$
<i>Avgqop</i>	Azimuth-averaged vector of the operating-point displacements	$n \times 1$
<i>Avgyop</i>	Azimuth-averaged vector of the operating-point outputs	$N_0 \times 1$
<i>AvgEigenvectors_all</i>	All eigenvectors of azimuth-averaged system matrix	$N \times N$
<i>AvgEvals_all</i>	All eigenvalues of azimuth-averaged system matrix	$N \times 1$
<i>AvgEigenVects</i>	Reduced-size eigenvectors of azimuth-averaged system matrix	$n \times M$
<i>AvgEigenVals</i>	Reduced-size eigenvalues of azimuth-averaged system matrix	$M \times 1$
<i>AvgDampedFrequency</i>	Damped frequencies of azimuth-averaged system matrix (rad/s)	$M \times 1$
<i>AvgDampedFrequencyHz</i>	Damped frequencies of azimuth-averaged system matrix (Hz)	$M \times 1$
<i>AvgNaturalFrequency</i>	Natural frequencies of azimuth-averaged system matrix (rad/s)	$M \times 1$
<i>AvgNaturalFrequencyHz</i>	Natural frequencies of azimuth-averaged system matrix (Hz)	$M \times 1$
<i>AvgDecrementRate</i>	Decrement rates of azimuth-averaged system matrix	$M \times 1$
<i>AvgDampingRatio</i>	Damping ratios of azimuth-averaged system matrix	$M \times 1$
<i>AvgModeShapeMagnitude</i>	Modal amplitudes of azimuth-averaged system matrix	$n \times M$
<i>AvgModeShapePhaseDeg</i>	Modal phases of azimuth-averaged system matrix (deg)	$n \times M$

The decrement rates listed in Tables 2 and 3 are simply the negatives of the real parts of the associated eigenvalues. We recommend using these rates when presenting stability analysis results. Alternately, damping ratios can be used. Problems can arise, however, if an eigenvalue has a zero imaginary part (i.e., it represents a rigid-body mode). In this case, the damping ratio would be either infinity or indeterminate.

7. Downloading and Verifying MBC-Specific Files

The MATLAB scripts required to carry out the MBC operation are *mbc3.m*, *eianalysis.m*, *get_new_seq.m*, and *row_col_xtion.m*. The *mbc3.m* is the main script and calls the other three. The script *GetMats.m* is required to load select contents of the FAST-generated linear-data output file to the MATLAB workspace. The *cce.m* script is not required for MBC. We include it, however, to allow interested users to obtain eigen-results of an azimuth-averaged system matrix. The *cce.m* script calls *eianalysis.m* to perform eigenanalysis. Visit our Web site at <http://wind.nrel.gov/designcodes/postprocessors/> to download these scripts, along with sample I/O files and this guide.

If the download is successful, you will see the main folder named *MBC* and two subfolders named *Source* and *CertTest*. The main folder contains the user's manual. The *Source* subfolder contains the MATLAB scripts required for MBC and CCE operations. The *CertTest* subfolder contains a test input file named *test01.lin* and a subfolder named *TestFiles*. The file *test01.lin* was generated by FAST with options to output first-order state matrices, including control, disturbance, and output matrices. You can open this file to examine these matrices and supporting information describing state vectors, number of azimuth steps, and so on (for a description of the contents of this file, see the FAST user's manual, which is also available on our Web site). Currently, we have only one test input file, but we plan to add more in the future. The subfolder *TestFiles* contains the output file *Filetest01.mat*, which is a MATLAB binary file generated using the sequence of commands *GetMats*, *mbc3*, and *cce* described in Section 5. You can load this file into MATLAB and examine its contents if you like (Tables 1–3 describe these contents).

To verify that you have successfully loaded the required files, set the path name to the *CertTest* subfolder in your MATLAB workspace. Next, issue the command *CertTest*. This command performs MBC and CCE operations and compares the resulting outputs with those in the *Filetest01.mat* output file. The results of this comparison are displayed in the MATLAB workspace. *No difference* displayed for all variables implies successful verification. Now you can use FAST to generate state matrices for a turbine of your interest and follow the steps described in Section 5 to perform MBC or CCE or both.

8. Future Plans

Though the MBC formulation we developed assumes a three-bladed turbine, it is general enough to be readily extended to a turbine with more than three blades. In the future, depending on user demand, we will update both the theory basis and the scripts to accommodate rotors with three or more blades. We will exclude two-bladed rotors because, even though an MBC-like transformation is theoretically possible for a two-bladed rotor (yielding coning and teeter rotor modes), such a transformation does not result in a filtering action similar to the one that the conventional MBC provides.

As mentioned earlier, the current MBC scripts work only with FAST. The future versions will accept data (e.g., state matrices) from any rotary-system aeroelastic code. We will also write a FORTRAN-based MBC utility and integrate it into FAST. This will allow a user to invoke MBC operation directly from the FAST input file.

Finally, we will update the theory basis to explain the filtering operation of MBC and include additional I/O files to demonstrate this operation. We will also include results to show how MBC can be applied to rotors with dissimilar blades.

References

1. Bir, G.S. "Multi-Blade Coordinate Transformation and Its Application to Wind Turbine Analysis." *Proceedings of the AIAA/ASME Wind Energy Symposium*, Reno, NV, January 2008.
2. Jonkman, J.M.; Buhl Jr., M.L. *FAST User's Guide*. NREL/EL-500-29798. Golden, CO: National Renewable Energy Laboratory, 2005.
3. Johnson, W. *Helicopter Theory*. Princeton, NJ: Princeton University Press, 1980.
4. Miller, R.H. "Helicopter Control and Stability in Hovering Flight." *JAS*, Vol. 15, August 1948.
5. Coleman, R.P.; Feingold, A.M. *Theory of Self-Excited Mechanical Oscillations of Helicopter Rotors with Hinged Blades*. NACA Technical Report TR 1351, 1958.
6. Hohenemser, K.H.; Yin, S.K. "Some Applications of the Method of Multiblade Coordinates," *J. Amer. Helicopter Soc.*, Vol. 17, No. 3, 1972.
7. Bir, G., et al. *University of Maryland Advanced Rotor Code (UMARC) Theory Manual*. Technical Report UM-AERO 94-18. College Park, MD: Center for Rotorcraft Education and Research, University of Maryland, July 1994.
8. Bir, G.S.; Wright, A.D.; Butterfield, S. "Stability Analysis of Variable-Speed Wind Turbines." *Proceedings of the 1997 ASME Wind Energy Symposium, Reno, January 6–9, 1997*.
9. Malcolm, D.J. "Modal Response of 3-Bladed Wind Turbines." *J. Solar Eng.*, Vol. 124, No. 4, November 2002, pp. 372–377.
10. Elliott, A.S.; McConville, J.B. "Application of a General-Purpose Mechanical Systems Analysis Code to Rotorcraft Dynamics Problems." Presented at the American Helicopter Society National Specialists' Meeting on Rotorcraft Dynamics, 1989.
11. McCoy, T.J. "Wind Turbine ADAMS Model Linearization Including Rotational and Aerodynamic Effects." *2004 ASME Wind Energy Symposium*, January 2004, pp. 224–233.
12. Hansen, M.H. "Improved Modal Dynamics of Wind Turbines to Avoid Stall-Induced Vibrations." *Wind Energy*, Vol. 6, Issue 2, 2003, pp.179–195.
13. Hansen, M. H., "Stability Analysis of Three-Bladed Turbines Using an Eigenvalue Approach," *2004 ASME Wind Energy Symposium*, January 2004, pp. 192–202.
14. Riziotis, V.A.; Voutsinas, S.G.; Politis, E.S.; Chaviaropoulos, P.K. "Aeroelastic Stability of Wind Turbines: The Problem, the Methods, and the Issues," *Wind Energy*, Vol. 7, Issue 4, 2004, pp.373–392.
15. Bir, G.; Jonkman, J., "Aeroelastic Instabilities of Large Offshore and Onshore Wind Turbines," *J. Physics: Conference Series*, 2007.
16. Stol, K.; Moll, H.; Bir, G.; Namik, H. "A Comparison of Multi-Blade Coordinate Transformation and Direct Periodic Techniques for Wind Turbine Control Design." *47th AIAA Aerospace Sciences Meeting, Jan. 5–8 2009, Orlando, Florida, AIAA 2009-0439*.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Executive Services and Communications Directorate (0704-0188). Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ORGANIZATION.

1. REPORT DATE (DD-MM-YYYY) September 2010		2. REPORT TYPE Technical Report		3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE User's Guide to MBC3: Multi-Blade Coordinate Transformation Code for 3-Bladed Wind Turbine				5a. CONTRACT NUMBER DE-AC36-08-GO28308	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) G.S. Bir				5d. PROJECT NUMBER NREL/TP-500-44327	
				5e. TASK NUMBER WER9.3535	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) National Renewable Energy Laboratory 1617 Cole Blvd. Golden, CO 80401-3393				8. PERFORMING ORGANIZATION REPORT NUMBER NREL/TP-500-44327	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S) NREL	
				11. SPONSORING/MONITORING AGENCY REPORT NUMBER	
12. DISTRIBUTION AVAILABILITY STATEMENT National Technical Information Service U.S. Department of Commerce 5285 Port Royal Road Springfield, VA 22161					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT (Maximum 200 Words) This guide explains how to use MBC3, a MATLAB-based script NREL developed to perform multi-blade coordinate transformation of system matrices for three-bladed wind turbines. In its current form, MBC3 can be applied to system matrices generated by FAST.2.					
15. SUBJECT TERMS wind turbines; MBC3; MATLAB-based script; multi-blade coordinate transformation; FAST					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UL	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT Unclassified	b. ABSTRACT Unclassified	c. THIS PAGE Unclassified			19b. TELEPHONE NUMBER (Include area code)

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18