# Theory Manual for the Tuned Mass Damper Module in FAST v8

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This manual describes new functionality in FAST 8 that simulates the addition of tuned mass dampers (TMDs) in the nacelle for structural control. For application studies of these systems, refer to [1–6]. The TMDs are two independent, 1 DOF, linear mass spring damping elements that act in the fore-aft ($x$) and side-side ($y$) directions. We first present the theoretical background and then describe the code changes.

## 1 Theoretical Background

### 1.1 Definitions

$O$: origin point of global inertial reference frame

$P$: origin point of non-inertial reference frame fixed to nacelle where TMDs are at rest

$TMD$: origin point of a TMD

$G$: axis orientation of global reference frame

$N$: axis orientation of nacelle reference frame with unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$

$\vec{r}_{_{TMD/O_G}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{_{TMD/O_G}}$ : position of a TMD with respect to (w.r.t.) $O$ with orientation $G$

$\vec{r}_{_{TMD/P_N}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{_{TMD/P_N}}$ : position of a TMD w.r.t. $P_N$

$\vec{r}_{_{TMD_X}}$ : position vector for $TMD_X$

$\vec{r}_{TMD_Y}$: position vector for $TMD_Y$

$$\vec{r}_{P/O_G} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{P/O_G} \quad : \text{ position vector of nacelle w.r.t. } O_G$$

$R_{N/G}$: 3 x 3 rotation matrix transforming orientation $G$ to $N$

$R_{G/N} = R^T_{N/G}$: transformation from $N$ to $G$

$$\vec{\omega}_{N/O_N} = \begin{bmatrix} \dot{\theta} \\ \phi \\ \psi \end{bmatrix}_{N/O_N} \quad : \text{ angular velocity of nacelle in orientation } N\text{; defined likewise for } G$$

$\dot{\vec{\omega}}_{N/O_N} = \vec{\alpha}_{N/O_N}$: angular acceleration of nacelle

$$\vec{a}_{G/O_G} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}_{/O_G} \quad : \text{ gravitational acceleration in global coordinates}$$

$$\vec{a}_{G/O_N} = R_{N/G}\vec{a}_{G/O_G} = \begin{bmatrix} a_{G_X} \\ a_{G_Y} \\ a_{G_Z} \end{bmatrix}_{/O_N} \quad : \text{ gravity w.r.t. } O_N$$

## 1.2 Equations of motion

The position vectors of the TMDs in the two reference frames $O$ and $P$ are related by

$$\vec{r}_{TMD/O_G} = \vec{r}_{P/O_G} + \vec{r}_{TMD/P_G}$$

Expressed in orientation $N$,

$$\vec{r}_{TMD/O_N} = \vec{r}_{P/O_N} + \vec{r}_{TMD/P_N}$$
$$\Rightarrow \vec{r}_{TMD/P_N} = \vec{r}_{TMD/O_N} - \vec{r}_{P/O_N}$$

Differentiating,[1]

$$\dot{\vec{r}}_{TMD/P_N} = \dot{\vec{r}}_{TMD/O_N} - \dot{\vec{r}}_{P/O_N} - \vec{\omega}_{N/O_N} \times \vec{r}_{TMD/P_N}$$

differentiating again gives the acceleration of the TMD w.r.t. $P$ (the nacelle position), oriented with $N$:

$$\begin{aligned} \ddot{\vec{r}}_{TMD/P_N} = \; & \ddot{\vec{r}}_{TMD/O_N} - \ddot{\vec{r}}_{P/O_N} - \vec{\omega}_{N/O_N} \times (\vec{\omega}_{N/O_N} \times \vec{r}_{TMD/P_N}) \\ & -\vec{\alpha}_{N/O_N} \times \vec{r}_{TMD/P_N} - 2\vec{\omega}_{N/O_N} \times \dot{\vec{r}}_{TMD/P_N} \end{aligned} \tag{1}$$

The right-hand side contains the following terms:

[1] Note that $(Ra) \times (Rb) = R(a \times b)$.

$\ddot{\vec{r}}_{TMD/O_N}$: acceleration of the TMD in the *inertial* frame $O_N$

$\ddot{\vec{r}}_{P/O_N} = R_{N/G}\ddot{\vec{r}}_{P/O_G}$: acceleration of the Nacelle origin $P$ w.r.t. $O_N$

$\vec{\omega}_{N/O_N} = R_{N/G}\vec{\omega}_{N/O_G}$ : angular velocity of nacelle w.r.t. $O_N$

$\vec{\omega}_{N/O_N} \times (\vec{\omega}_{N/O_N} \times \vec{r}_{TMD/P_N})$ : Centrifugal force

$\vec{\alpha}_{N/O_N} \times \vec{r}_{TMD/P_N}$: Euler force

$2\vec{\omega}_{N/O_N} \times \dot{\vec{r}}_{TMD/P_N}$: Coriolis force

The acceleration in the inertial frame $\ddot{\vec{r}}_{TMD/O_N}$ can be replaced with a force balance

$$\ddot{\vec{r}}_{TMD/O_N} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix}_{TMD/O_N} = \frac{1}{m}\begin{bmatrix} \sum F_X \\ \sum F_Y \\ \sum F_Z \end{bmatrix}_{TMD/O_N} = \frac{1}{m}\vec{F}_{TMD/O_N}$$

Substituting the force balance into Equation 1 gives the general equation of motion for a TMD:

$$\begin{aligned} \ddot{\vec{r}}_{TMD/P_N} = \; & \tfrac{1}{m}\vec{F}_{TMD/O_N} - \ddot{\vec{r}}_{P/O_N} - \vec{\omega}_{N/O_N} \times (\vec{\omega}_{N/O_N} \times \vec{r}_{TMD/P_N}) \\ & -\vec{\alpha}_{N/O_N} \times \vec{r}_{TMD/P_N} - 2\vec{\omega}_{N/O_N} \times \dot{\vec{r}}_{TMD/P_N} \end{aligned} \tag{2}$$

We will now solve the equations of motion for $TMD_X$ and $TMD_Y$.

**TMD_X :** The external forces $\vec{F}_{TMD_X/O_N}$ are given by

$$\vec{F}_{TMD_X/O_N} = \begin{bmatrix} -c_x\dot{x}_{TMD_X/P_N} - k_x x_{TMD_X/P_N} + m_x a_{G_X/O_N} + F_{ext_x} + F_{StopFrcX} \\ F_{Y_{TMD_X/O_N}} + m_x a_{G_Y/O_N} \\ F_{Z_{TMD_X/O_N}} + m_x a_{G_Z/O_N} \end{bmatrix}$$

$TMD_X$ is fixed to frame $N$ in the $y$ and $z$ directions so that

$$r_{TMD_X/P_N} = \begin{bmatrix} x_{TMD_X/P_N} \\ 0 \\ 0 \end{bmatrix}$$

The other components of Eqn. 2 are:

$$\vec{\omega}_{N/O_N} \times (\vec{\omega}_{N/O_N} \times \vec{r}_{TMD_X/P_N}) = x_{TMD_X/P_N} \begin{bmatrix} -(\dot{\phi}^2_{N/O_N} + \dot{\psi}^2_{N/O_N}) \\ \dot{\theta}_{N/O_N}\dot{\phi}_{N/O_N} \\ \dot{\theta}_{N/O_N}\dot{\psi}_{N/O_N} \end{bmatrix}$$

$$2\vec{\omega}_{N/O_N} \times \dot{\vec{r}}_{TMD_X/P_N} = \dot{x}_{TMD_X/P_N} \begin{bmatrix} 0 \\ 2\dot{\psi}_{N/O_N} \\ -2\dot{\phi}_{N/O_N} \end{bmatrix}$$

$$\vec{\alpha}_{N/O_N} \times \vec{r}_{TMD_X/P_N} = x_{TMD_X/P_N} \begin{bmatrix} 0 \\ \ddot{\psi}_{N/O_N} \\ -\ddot{\phi}_{N/O_N} \end{bmatrix}$$

Therefore $\ddot{x}_{TMD_X/P_N}$ is governed by the equations

$$\begin{aligned} \ddot{x}_{TMD_X/P_N} = \; & (\dot{\phi}^2_{N/O_N} + \dot{\psi}^2_{N/O_N} - \tfrac{k_x}{m_x})x_{TMD_X/P_N} - (\tfrac{c_x}{m_x})\dot{x}_{TMD_X/P_N} - \ddot{x}_{P/O_N} + a_{G_X/O_N} \\ & + \tfrac{1}{m_x}(F_{ext_X} + F_{StopFrc_X}) \end{aligned} \tag{3}$$

The forces $F_{Y_{TMD_X/O_N}}$ and $F_{Z_{TMD_X/O_N}}$ are solved noting $\ddot{y}_{TMD_X/P_N} = \ddot{z}_{TMD_X/P_N} = 0$:

$$F_{Y_{TMD_X/O_N}} = m_x \left( -a_{G_Y/O_N} + \ddot{y}_{P/O_N} + (\ddot{\psi}_{N/O_N} + \dot{\theta}_{N/O_N}\dot{\phi}_{N/O_N})x_{TMD_X/P_N} + 2\dot{\psi}_{N/O_N}\dot{x}_{TMD_X/P_N} \right) \tag{4}$$

$$F_{Z_{TMD_X/O_N}} = m_x \left( -a_{G_Z/O_N} + \ddot{z}_{P/O_N} - (\ddot{\phi}_{N/O_N} - \dot{\theta}_{N/O_N}\dot{\psi}_{N/O_N})x_{TMD_X/P_N} - 2\dot{\phi}_{N/O_N}\dot{x}_{TMD_X/P_N} \right) \tag{5}$$

**TMD_Y:** The external forces $\vec{F}_{TMD_Y/P_N}$ on $TMD_Y$ are given by

$$\vec{F}_{TMD_Y/P_N} = \begin{bmatrix} F_{X_{TMD_Y/O_N}} + m_y a_{G_X/O_N} \\ -c_y \dot{y}_{TMD_Y/P_N} - k_y y_{TMD_Y/P_N} + m_y a_{G_Y/O_N} + F_{ext_y} + F_{StopFrc_Y} \\ F_{Z_{TMD_Y/O_N}} + m_y a_{G_Z/O_N} \end{bmatrix}$$

$TMD_Y$ is fixed to frame $N$ in the $x$ and $z$ directions so that

$$r_{TMDYX/P_N} = \begin{bmatrix} 0 \\ y_{TMD_Y/P_N} \\ 0 \end{bmatrix}$$

The other components of Eqn. 2 are:

$$\vec{\omega}_{N/O_N} \times (\vec{\omega}_{N/O_N} \times \vec{r}_{TMD_Y/P_N}) = y_{TMD_Y/P_N} \begin{bmatrix} \dot{\theta}_{N/O_N}\dot{\phi}_{N/O_N} \\ -(\dot{\theta}^2_{N/O_N} + \dot{\psi}^2_{N/O_N}) \\ \dot{\phi}_{N/O_N}\dot{\psi}_{N/O_N} \end{bmatrix}$$

$$2\vec{\omega}_{N/O_N} \times \dot{\vec{r}}_{TMD_Y/P_N} = \dot{y}_{TMD_Y/P_N} \begin{bmatrix} -2\dot{\psi}_{N/O_N} \\ 0 \\ 2\dot{\theta}_{N/O_N} \end{bmatrix}$$

$$\vec{\alpha}_{N/O_N} \times \vec{r}_{TMD_Y/P_N} = y_{TMD_Y/P_N} \begin{bmatrix} -\ddot{\psi}_{N/O_N} \\ 0 \\ \ddot{\theta}_{N/O_N} \end{bmatrix}$$

Therefore $\ddot{y}_{TMD_Y/P_N}$ is governed by the equations

$$\begin{aligned} \ddot{y}_{TMD_Y/P_N} = \; & (\dot{\theta}^2_{N/O_N} + \dot{\psi}^2_{N/O_N} - \tfrac{k_y}{m_y})y_{TMD_Y/P_N} - (\tfrac{c_y}{m_y})\dot{y}_{TMD_Y/P_N} - \ddot{y}_{P/O_N} + a_{G_Y/O_N} \\ & + \tfrac{1}{m_y}(F_{ext_Y} + F_{StopFrc_Y}) \end{aligned} \tag{6}$$

The forces $F_{X_{TMD_Y/O_N}}$ and $F_{Z_{TMD_Y/O_N}}$ are solved noting $\ddot{x}_{TMD_Y/P_N} = \ddot{z}_{TMD_Y/P_N} = 0$:

$$F_{X_{TMD_Y/O_N}} = m_y \left( -a_{G_X/O_N} + \ddot{x}_{P/O_N} - (\ddot{\psi}_{N/O_N} - \dot{\theta}_{N/O_N}\dot{\phi}_{N/O_N}) y_{TMD_Y/P_N} - 2\dot{\psi}_{N/O_N}\dot{y}_{TMD_Y/P_N} \right) \quad (7)$$

$$F_{Z_{TMD_Y/O_N}} = m_y \left( -a_{G_Z/O_N} + \ddot{z}_{P/O_N} + (\ddot{\theta}_{N/O_N} + \dot{\phi}_{N/O_N}\dot{\psi}_{N/O_N}) y_{TMD_Y/P_N} + 2\dot{\theta}_{N/O_N}\dot{y}_{TMD_Y/P_N} \right) \quad (8)$$

## 1.3 State Equations

**Inputs:** The inputs are the nacelle linear acceleration and angular position, velocity and acceleration:

$$\vec{u} = \begin{bmatrix} \ddot{\vec{r}}_{P/O_G} \\ \vec{R}_{N/G} \\ \vec{\omega}_{N/O_G} \\ \vec{\alpha}_{P/O_G} \end{bmatrix} \Rightarrow \begin{bmatrix} \ddot{\vec{r}}_{P/O_N} \\ \vec{\omega}_{N/O_N} \\ \vec{\alpha}_{N/O_N} \end{bmatrix} = \begin{bmatrix} \vec{R}_{N/G}\ddot{\vec{r}}_{P/O_G} \\ \vec{R}_{N/G}\vec{\omega}_{N/O_G} \\ \vec{R}_{N/G}\vec{\alpha}_{P/O_G} \end{bmatrix}$$

**States:** The states are the position and velocity of the TMDs along their respective DOFS in the Nacelle reference frame:

$$\vec{R}_{TMD/P_N} = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}_{TMD/P_N} = \begin{bmatrix} x_{TMD_X/P_N} \\ \dot{x}_{TMD_X/P_N} \\ y_{TMD_Y/P_N} \\ \dot{y}_{TMD_Y/P_N} \end{bmatrix}$$

The equations of motion can be re-written as a system of non-linear first-order equations of the form

$$\dot{\vec{R}}_{TMD} = A\vec{R}_{TMD} + B$$

where

$$A(\vec{u}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ (\dot{\phi}^2_{P/O_N} + \dot{\psi}^2_{P/O_N} - \frac{k_x}{m_x}) & -(\frac{c_x}{m_x}) & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & (\dot{\theta}^2_{P/O_N} + \dot{\psi}^2_{P/O_N} - \frac{k_y}{m_y}) & -(\frac{c_y}{m_y}) \end{bmatrix}$$

and

$$B(\vec{u}) = \begin{bmatrix} 0 \\ -\ddot{x}_{P/O_N} + a_{G_X/O_N} + \frac{1}{m_x}(F_{ext_X} + F_{StopFrc_X}) \\ 0 \\ -\ddot{y}_{P/O_N} + a_{G_Y/O_N} + \frac{1}{m_y}(F_{ext_Y} + F_{StopFrc_Y}) \end{bmatrix}$$

The inputs are coupled to the state variables, resulting in A and B as $f(\vec{u})$.

### 1.4 Outputs

The output vector $\vec{Y}$ is

$$\vec{Y} = \begin{bmatrix} \vec{F}_{P_G} \\ \vec{M}_{P_G} \end{bmatrix}$$

The output includes reaction forces corresponding to $F_{Y_{TMD_X/O_N}}$, $F_{Z_{TMD_X/O_N}}$, $F_{X_{TMD_Y/O_N}}$, and $F_{Z_{TMD_Y/O_N}}$ from Eqns. 4, 5, 7, and 8. The resulting forces $\vec{F}_{P_G}$ and moments $\vec{M}_{P_G}$ acting on the nacelle are

$$\vec{F}_{P_G} = R^T_{N/G} \begin{bmatrix} k_x x_{TMD/P_N} + c_x \dot{x}_{TMD/P_N} - F_{StopFrcX} - F_{ext_x} - F_{X_{TMD_Y/O_N}} \\ k_y y_{TMD/P_N} + c_y \dot{y}_{TMD/P_N} - F_{StopFrcY} - F_{ext_y} - F_{Y_{TMD_X/O_N}} \\ -F_{Z_{TMD_X/O_N}} - F_{Z_{TMD_Y/O_N}} \end{bmatrix}$$

and

$$\vec{M}_{P_G} = R^T_{N/G} \begin{bmatrix} M_X \\ M_Y \\ M_Z \end{bmatrix}_{N/N} = R^T_{N/G} \begin{bmatrix} -(F_{Z_{TMD_Y/O_N}}) y_{TMD/P_N} \\ (F_{Z_{TMD_X/O_N}}) x_{TMD/P_N} \\ (-F_{Y_{TMD_X/O_N}}) x_{TMD/P_N} + (F_{X_{TMD_Y/O_N}}) y_{TMD/P_N} \end{bmatrix}$$

**Stop Forces** The extra forces $F_{StopFrcX}$ and $F_{StopFrcY}$ are added to output forces in the case that the movement of TMD_X or TMD_Y exceeds the maximum track length for the mass. Otherwise, they equal zero. The track length has limits on the upwind (UW) and downwind (DW) ends in the $x$ direction (X_UWSP and X_DWSP), and the positive and negative lateral ends in the $y$ direction (Y_PLSP and Y_NLSP). If we define a general maximum and minimum displacements as $x_{max}$ and $x_{min}$, respectively, the stop forces have the form

$$F_{StopFrc} = -\begin{cases} k_S \Delta x & : (x > x_{max} \wedge \dot{x} <= 0) \vee (x < x_{min} \wedge \dot{x} >= 0) \\ k_S \Delta x + c_S \dot{x} & : (x > x_{max} \wedge \dot{x} > 0) \vee (x < x_{min} \wedge \dot{x} < 0) \\ 0 & \text{:otherwise} \end{cases}$$

where $\Delta x$ is the distance the mass has traveled beyond the stop position and $k_S$ and $c_S$ are large stiffness and damping constants.

## 2 Code Modifications

The TMD function is submodule called in ServoDyn. In addition to references in ServoDyn.f90 and ServoDyn.txt, new files that contain the TMD module are listed below.

### 2.1 New Files

- TMD.f90 : TMD module
- TMD.txt : registry file

  include files, inputs, states, parameters, and outputs shown in Tables 1 and 2

- TMD_Types.f90 : automatically generated

| InitInput | Input u | Parameter p | State x | Output y |
|---|---|---|---|---|
| InputFile | $\ddot{\vec{r}}_{P/O_G}$ | $m_x$ | $\vec{tmd}_x$ | Mesh |
| Gravity | $\vec{R}_{N/O_G}$ | $c_x$ | | |
| $\vec{r}_{N/O_G}$ | $\vec{\omega}_{N/O_G}$ | $k_x$ | | |
| | $\vec{\alpha}_{P/O_G}$ | $m_y$ | | |
| | | $c_y$ | | |
| | | $k_y$ | | |
| | | $K_S = [k_{SX} \quad k_{SY}]$ | | |
| | | $C_S = [c_{SX} \quad c_{SY}]$ | | |
| | | $P_{SP} = [X_{DWSP} \quad Y_{PLSP}]$ | | |
| | | $P_{SP} = [X_{UWSP} \quad Y_{NLSP}]$ | | |
| | | $Fext$ | | |
| | | $Gravity$ | | |
| | | TMDX_DOF | | |
| | | TMDY_DOF | | |
| | | $X_{DSP}$ | | |
| | | $Y_{DSP}$ | | |

Table 1: Summary of field definitions in the TMD registry. Note that state vector $\vec{tmd}_x$ corresponds to $\vec{R}_{TMD/P_N}$, and that the outputs $\vec{F}_{P_G}$ and $\vec{M}_{P_G}$ are contained in the MeshType object (y.Mesh). $X_{DSP}$ and $Y_{DSP}$ are initial displacements of the TMDs.

### 2.2 Variables

The input, parameter, state and output definitions are summarized in Table 1. The inputs from file are listed in Table 2.

## 3 Acknowledgements

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## References

[1] Matthew A. Lackner and Mario A. Rotea. Passive structural control of offshore wind turbines. Wind energy, 14(3):373–388, 2011.

[2] Matthew A. Lackner and Mario A. Rotea. Structural control of floating wind turbines. Mechatronics, 21(4):704–719, 2011.

[3] Hazim Namik, M. A. Rotea, and Matthew Lackner. Active structural control with actuator dynamics on a floating wind turbine. In Proceedings of the 51st AIAA Aerospace Sciences Meeting, pages 7–10, 2013.

| Field Name | Field Type | Description |
|---|---|---|
| TMD_CMODE | int | Control Mode (1:passive, 2:active) |
| TMD_X_DOF | logical | DOF on or off |
| TMD_Y_DOF | logical | DOF on or off |
| TMD_X_DSP | real | TMD_X initial displacement |
| TMD_Y_DSP | real | TMD_Y initial displacement |
| TMD_X_M | real | TMD mass |
| TMD_X_K | real | TMD stiffness |
| TMD_X_C | real | TMD damping |
| TMD_Y_M | real | TMD mass |
| TMD_Y_K | real | TMD stiffness |
| TMD_Y_C | real | TMD damping |
| TMD_X_DWSP | real | DW stop position (maximum X mass displacement) |
| TMD_X_UWSP | real | UW stop position (minimum X mass displacement) |
| TMD_X_K_SX | real | stop spring stiffness |
| TMD_X_C_SX | real | stop spring damping |
| TMD_Y_PLSP | real | positive lateral stop position (maximum Y mass displacement) |
| TMD_Y_NLSP | real | negative lateral stop position (minimum Y mass displacement) |
| TMD_Y_K_S | real | stop spring stiffness |
| TMD_Y_C_S | real | stop spring damping |
| TMD_P_X | real | x origin of P in nacelle coordinate system |
| TMD_P_Y | real | y origin of P in nacelle coordinate system |
| TMD_P_Z | real | z origin of P in nacelle coordinate system |

Table 2: Data read in from TMDInputFile.

[4] G. Stewart and M. A. Lackner. Optimization of a passive tuned mass damper for reducing loads in offshore wind turbines. IEEE Transactions on Control Systems Technology, 21(4):1090–1104, 2013.

[5] Gordon M. Stewart and Matthew A. Lackner. The effect of actuator dynamics on active structural control of offshore wind turbines. Engineering Structures, 33(5):1807–1816, 2011.

[6] Gordon M. Stewart and Matthew A. Lackner. The impact of passive tuned mass dampers and windwave misalignment on offshore wind turbine loads. Engineering Structures, 73:54–61, 2014.