Solving Unit Commitment Problems with Demand Responsive Loads

Preprint

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***Abstract*—This paper focuses on using variations of the Frank-Wolfe algorithm for solving unit commitment problems with high volumes of demand responsive loads on the power grid. We present a formulation of the unit commitment problem with demand responsive loads. We then show through reformulation and relaxations of the problem that variations of the Frank-Wolfe algorithm can be used to determine the time series decisions for the demand responsive loads. We show through computational experiments on the IEEE Reliability Test System that the time series of demand responsive load decisions obtained through our approach are near optimal and describe how large-scale parallel implementations of our approach can be highly computationally efficient.**

***Keywords*—Demand Response, Unit Commitment, Frank-Wolfe Algorithm**

I. Introduction

Production cost models (PCMs) are used to simulate scheduling of power generation and transmission system operations to minimize the cost of generating electricity to meet demand. They are often used by utilities, regional transmission operators, and independent system operators to evaluate generator and/or transmission expansion plans and understand the impacts of scheduling practices under different resource realization scenarios. Simulations typically consist of a sequence of rolling-horizon least-cost optimization problems to establish generator unit commitment (UC) and/or (economic) dispatch (ED) schedules. PCMs can be used to predict electricity prices, emissions, resource adequacy, and other system and market conditions. These features, along with the sensitivity of PCMs to cost inputs, make PCMs particularly well suited for policy analysis. As an example, they can be used to understand the system reaction to different inputs changes [1], such as load rescheduling in coordination with generation forecasts using electricity prices, often refereed to as demand response (DR)[2].

Benefits of considering DR in PCMs include efficient resource use, reduced market wholesale prices by removing the need to use expensive generators during high demand periods, improved system reliability, and payment and incentives received by customers who participate in DR programs [3]. In addition, the need for flexible resources to maintain supply-demand balance can increase as power grids incorporate more variable renewable energy. Distributed and schedulable energy resources, such as electric vehicles and batteries, as well as other schedulable load have the potential to provide additional flexibility at low cost. Solving a centralized UC problem that considers DR from distributed energy agents can facilitate scheduling load and energy production from the different DR agents to improve grid reliability in the face of increased renewable energy penetration [4], [5]. Additionally, solutions can provide a useful baseline to compare results from different demand response market structures. However, expanding the scope of system scheduling problems to include distributed and demand side resources can lead to computational challenges.

This paper contributes to the literature by providing a novel approach for obtaining high quality solutions to the UC problem with flexible load scheduling, which we demonstrate through computational experiments. This is done through a reformulation of the problem which allows for variations of the Frank-Wolfe (FW) algorithm [6] to be applied. We also explain how computationally efficient implementations are possible via warm starting and parallel computation, making these approaches promising for large scale problems.

This paper is structured as follows. In section II we present the UC problem with flexible loads. In section III we present our problem reformulation, along with our solutions approaches. In section IV we present the test case used for our computational experiments. In section V we present and discuss results from our computational experiments, and in

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section VI conclusions are drawn.

II. Problem Statement

Similar to [7], we present a generic version of the UC problem, but with demand responsive agents and their respective flexible loads, which we call the UCDR problem. Here G is the set of generators, $T = \{1, \ldots, N_T\}$ is the set of time periods. p_g^t and x_g^t represent the generator set point and any internal state variables of generator g at time t respectively. Note we let $p_g = (p_g^1, \ldots, p_g^{N_T})$ and $x_g = (x_g^1, \ldots, x_g^{N_T})$ where each x_g^t might be a tuple of internal generator state variables (e.g., unit on/off). c_g^t is a linear function of p_g^t and x_g^t that computes the cost of operating generator g at time t. The variables $y_i^{t,\pm}$ are power imbalance slack variables at bus i at time t used in equation (2) to ensure feasibility. c_i^t is a linear function of $y_i^{t,\pm}$ that enforces a high penalty for power imbalance at bus i at time t. Equation (2) is the power balance constraint at a given bus and time, where Φ is the set of buses, and $\mathcal{E}$ is the set of lines with arbitrary directions assigned. f_e^t is the power flow on line e at time t, $\mathcal{E}_{in}(i)$ and $\mathcal{E}_{out}(i)$ represent directed lines into and out of bus i, respectively, G_i represents generators at bus i, and $\hat{d}_i^t$ is the associated fixed demand. The line limits are modeled by (3). We use the DC approximation to AC power flow, given by (4), where θ_i^t is the voltage angle at bus i at time t, and B_e represents the line susceptance of transmission line e [8]. We let Π_g represent the set constraints that specify the set of feasible dispatch schedules (p_g, x_g) for generator g over the horizon T.

$$\min_{p,x,y^{\pm}} \quad \sum_{g \in G, t \in T} c_g^t(p_g^t, x_g^t) + \sum_{i \in \Phi, t \in T} c_i^t(y_i^{t,+}, y_i^{t,-}) \tag{1}$$

$$\text{s. t.} \quad \sum_{g \in G_i} p_g^t + \sum_{e \in \mathcal{E}_{in}(i)} f_e^t - \sum_{e \in \mathcal{E}_{out}(i)} f_e^t \tag{2}$$

$$= \hat{d}_i^t + \sum_{a \in A} s_{a,i}^t + y_i^{t,+} - y_i^{t,-} \quad \forall\, i \in \Phi, t \in T$$

$$\underline{F}_e \leq f_e^t \leq \overline{F}_e \quad \forall e \in \mathcal{E} \tag{3}$$

$$B_e\left(\theta_i^t - \theta_j^t\right) = f_e^t \quad \forall e \in \mathcal{E}, t \in T. \tag{4}$$

$$(p_g, x_g) \in \Pi_g \quad \forall g \in G \tag{5}$$

$$(s_a, z_a) \in \Pi_a \quad \forall a \in A \tag{6}$$

This problem differs from a standard UC problem in that it contains the variables $s_{a,i}^t$ in (2) and the additional constraints in (6). We let A be a set indexing the demand responsive loads in the system. The variables $s_{a,i}^t$ represent the load at time t on bus i from the demand responsive load $a \in A$. For each demand responsive load we let s_a represent the collection of these variables. The constraint set for each demand responsive load a is denoted by Π_a, which determines its feasible set of schedules. We let z_a be the collection of any internal state variables for demand responsive load a, similar to x_g for the generators. We assume for the duration of this paper that the sets Π_g and Π_a consist of linear constraints where the variables z_a and x_g be can be continuous and integer. Thus the UCDR problems is a mixed integer linear program (MILP).

III. Solution Approaches

To address potential difficulties when solving UCDR model using traditional methods and we propose a solution method that leverages relaxations of the UCDR to obtain good solutions with less computational difficulties. Obviously the most straight forward solution approach to solve the UCDR problem is to use a traditional solution method such as branch and cut to solve to it directly. There are a few reasons that solving the UCDR might be harder than solving the standard UC problem. One reason is that if $|A|$ is large, a large number of constraints and potentially integer variables are added to the problem. A second reason is that flexible loads at every bus have the same mathematical impact as virtual bidders at every bus, which is known to cause computational issues for the UC problem when utilizing power transfer distribution factor or generation shift factor transmission formulations [9], [10]. To address these computational issues we present two methods to obtain high quality solutions to the UCDR problem.

A. The Linear Programming Relaxation

The first approach we present is straightforward. We first solve the linear programming (LP) relaxation of the UCDR problem which we denote as (R-UCDR). If z_a contains integer variables for any of the demand responsive loads, the values of the variables $s_{a,i}^t$ must be projected onto the set Π_a to ensure feasibility. We denote the projected values by $\hat{d}_{a,i}^t$. Next the $\hat{d}_{a,i}^t$ values are used to fix the values of the variables $s_{a,i}^t$ in equation (2) creating a standard UC problem to be solved. This approach makes sense when the R-UCDR is easy to solve and is a tight enough relaxation of the UCDR problem so that the values $\hat{d}_{a,i}^t$ obtained provide good estimates for the variables $s_{a,i}^t$ in the true UCDR problem. However, the R-UCDR might not be an easy problem to solve on a large network when $|A|$ is large, due to a dense constraint matrix see [9], [10]. Additionally, this method forces the variables x_g and z_g to be continuously relaxed which in turn relaxes the constraint sets Π_a and Π_g. These relaxations run the risk of reducing the quality of the $\hat{d}_{a,i}^t$ values obtained. We now present a method that has the potential to address these issues.

B. The Frank-Wolfe Algorithm

Variations of the FW [6] algorithm can be used to obtain high quality solutions to a relaxation tighter than the R-UCDR. These solutions can then be used in the same fashion as the solutions to the R-UCDR were used to compute solutions to the UCDR via projections and variable fixing. The traditional FW algorithm solves the following problem

$$\min_{s \in \mathcal{D}} \quad f(x) \tag{7}$$

where $\mathcal{D}$ is a convex compact set in a vector space and f is convex differentiable function mapping into the real numbers. In [11] the authors extend the FW algorithm and establish convergence in the case where f is only convex and L-Lipschitz continuous as follows:

1) initialization: $0 \leftarrow k$, $x_0 \in \mathcal{D}$, $-\infty \leftarrow l_k$, ϵ
2) solve

$$\min_{s \in \mathcal{D}} \max_{d \in T(x_k, \epsilon)} \quad f(x_k) + (s - x_k)^T d \tag{8}$$

3) update step size $\alpha \leftarrow \frac{2}{k+2}$
4) update solution $x_{k+1} \leftarrow x_k + \alpha(s - x_k)$

5) compute upper bound $u_k \leftarrow f(x_{k+1})$
6) compute lower bound l_k (see [11] for details)
7) if: $u_k - l_k < \epsilon$
    break
   else:
    return to step 2.

We will refer to this algorithm as FW-max. Here we let $\partial f(x)$ denote the subdifferential of f and the point x. Let $N(x,\epsilon)$ be a neighborhood around x and define $T(x,\epsilon)$ as

$$T(x,\epsilon) := \begin{cases} \{\nabla f(x)\} & \text{if } f \text{ is differentiable on } N(x,\epsilon) \\ \bigcup_{u\in N(x,\epsilon)} \partial f(u) & \text{otherwise.} \end{cases}$$

Equation (8) can be viewed as picking the most robust search direction s by considering the "worst case" linear approximation created from the set of all subgradients at nearby points, from the perspective of the resulting objective value.

To apply the FW-max algorithm, we must refomulate the UCDR problem to obtain an appropriate relaxation.

$$\min_{d} \quad H(d) \tag{9}$$
$$\text{s. t.} \quad d_i^t = \hat{d}_i^t + \sum_{a\in A} s_{a,i}^t \tag{10}$$
$$(s_a, z_a) \in \Pi_a \quad \forall a \in A \tag{11}$$

where $H(d)$ is defined as

$$H(d) = \min_{p,x,y^{\pm}} \sum_{g\in G, t\in T} c_g^t(p_g^t, x_g^t) + \sum_{i\in\Phi, t\in T} c_i^t(y_i^{t,+}, y_i^{t,-}) \tag{12}$$
$$\text{s. t.} \sum_{g\in G_i} p_g^t + \sum_{e\in\mathcal{E}_{in}(i)} f_e^t - \sum_{e\in\mathcal{E}_{out}(i)} f_e^t \tag{13}$$
$$= d_i^t + y_i^{t,+} - y_i^{t,-} \quad \forall\, i \in \Phi, t\in T$$
$$\underline{F}_e \le f_e^t \le \overline{F}_e \quad \forall e \in \mathcal{E} \tag{14}$$
$$B_e\left(\theta_i^t - \theta_j^t\right) = f_e^t \quad \forall e \in \mathcal{E}, t \in T \tag{15}$$
$$(p_g, x_g) \in \Pi_g \quad \forall g \in G. \tag{16}$$

Here we collect all variables d_i^t in d. This problem defined by (9) - (16), is equivalent to the UCDR problem. Additionally, when any integer variables in (16) are relaxed it is well known that H becomes a convex piece-wise linear function of d, see [12], [13]. We will denote the resulting function from this relaxation as $\hat{H}$. Using $\hat{H}$ we can now define a relaxation of the UCDR problem as follows

$$\min_{d} \quad \hat{H}(d) \tag{17}$$
$$\text{s. t.} \quad d_i^t = \hat{d}_i^t + \sum_{a\in A} s_{a,i}^t \tag{18}$$
$$s_a \in \text{Conv}(\Pi_a) \quad \forall a \in A \tag{19}$$

where Conv(Π_a) denotes the convex hull of the set of feasible s_a. We denote this problem as R-UDCR-FW, and note that since we optimize over Conv(Π_a) it is a tighter relaxation than R-UCDR. One does however need a way to optimize over the sets Conv(Π_a). Fortunately, the FW algorithm and its variations provide a way to do exactly this without an explicit representation of the convex hull sets.

By approximating the set $T(x,\epsilon)$ through sampling nearby subgradients and maximizing over the convex hull of those subgradient samples, we can apply an approximate FW-max algorithm to solve the R-UCDR-FW problem. This algorithm we refer to as FW-sample and the algorithm is as follows when applied to the R-UCDR-FW problem.

1) initialization: $0 \leftarrow k$, ϵ, n, N, and d_0 feasible for (10) and (11)
2) solve $\hat{H}(d_0)$ and collect dual variables π_i^t from constraints (13).
3) solve

$$\min_{\substack{s_a\in\Pi_a \\ a\in A}} \max_{\pi\in\Gamma} \hat{H}(d_k) + \sum_{i\in\Phi}\sum_{t\in T}\left(\hat{d}_i^t + \sum_{a\in A} s_{a,i}^t - d_{i,k}^t\right)\pi_i^t \tag{20}$$

4) update step size $\alpha \leftarrow \frac{2}{k+2}$
5) update solution $d_{i,k+1}^t \leftarrow d_{i,k}^t + \alpha(\hat{d}_i^t + \sum_{a\in A} s_{a,i}^t - d_{i,k}^t)$
6) compute $\hat{H}(d_{k+1})$ and collect dual variables π_i^t from constraints (13).
7) for $j = 1,\ldots,N$
    sample $d_{k+1,j} \in \mathcal{B}(d_{k+1},\epsilon)$
    solve $\hat{H}(d_{k+1,j})$
    collect dual variables $\pi_{i,j}^t$ from constraints (13)
8) $k+1 \leftarrow k$
9) if: $k > n$
    break
   else:
    return to step 3.

The dual variables π_i^t from constraints (13) constitute a subgradient of $\hat{H}$, see [13]. Here $\Gamma = \text{Conv}(\pi_1,\ldots,\pi_N)$, and $\mathcal{B}(d_{k+1},\epsilon)$ is a closed ball of radius ϵ is some norm around the point d_{k+1}. In this algorithm we sample N points nearby the current flexible load value d_{k+1} in the set $\mathcal{B}(d_{k+1},\epsilon)$ and collect N subgradients using those nearby points. The convex hull of these N subgradients, Γ, is then used as our approximation of $T(x,\epsilon)$. We note that this approach leads to many interesting research questions about how to best approximate $T(x,\epsilon)$ through sampling approaches. Here, we omit the upper and lower bound computation steps from the algorithm because it is being used as a heuristic. It's also worth highlighting that omission of step 7 in the FW-sample algorithm results in the classic FW algorithm, but using subgraidients.

Steps 3, 6, and 7 are the most computationally intensive steps in this algorithm. In step 3 we can take advantage of the fact that optimizing (20) is equivalent to optimizing

$$\sum_{a\in A} \min_{s_a\in\Pi_a} \max_{\pi\in\Gamma} \sum_{i\in\Phi}\sum_{t\in T}(s_{a,i}^t + \frac{1}{|A|}(\hat{d}_i^t - d_{i,k}^t))\pi_i^t \tag{21}$$

which can be done by optimizing

$$\min_{s_a\in\Pi_a} \max_{\pi\in\Gamma} \sum_{i\in\Phi}\sum_{t\in T}(s_{a,i}^t + \frac{1}{|A|}(\hat{d}_i^t - d_{i,k}^t))\pi_i^t \tag{22}$$

for each $a \in A$ separately. Optimizing (22) for a given a can be shown to be equivalent to optimizing

$$\min_{t, s_a} \quad t \tag{23}$$

$$\text{s. t.} \quad t \geq \sum_{i \in \Phi} \sum_{t \in T} (s^t_{a,i} + \frac{1}{|A|}(\hat{d}^t_i - d^t_{i,k}))\pi^t_{i,j} \ \forall j \in \{1, \dots, N\} \tag{24}$$

$$s_a \in \Pi_a. \tag{25}$$

Provided N is not too large the optimization problems defined by (23) - (25) for each $a \in A$ are small LPs or MILPs which can still be solved very fast. We also note that step 3 can be carried out in parallel over the set A.

The other computationally intensive steps 6 and 7, where $\hat{H}(d_k)$ is computed $N + 1$ times, involves solving the LP relaxation of a UC problem. This problem has fixed loads at each bus, so the sparsity concerns regarding the constraint matrix mentioned before from the inclusion of flexible demand responsive loads are not a concern here. Still for large systems this might be a time consuming step. This can by remedied by observing that in step 2 of the algorithm $\hat{H}(d_0)$ is computed which is equivalent to optimizing the LP relaxation of problem defined by equations (12) - (16) and thus provides a feasible solution for the dual as well by LP optimality conditions [12]. Since the update in step 5 only changes the right hand side of the constraints in this relaxation the solution to its dual LP stays feasible because only the objective function of the dual has been changed. Hence, steps 6 and 7 can be warm started with the dual feasible solution from the previous iteration and solved via the dual simplex method with a limited number of pivots. Thus it is possible that step 2 might be slow, but unlikely the computations in steps 6 and 7 will be, especially as α decreases, if these observations are taken into account. Additionally all evaluations of $\hat{H}$ in steps 6 and 7 can be batched and done in parallel. Thus by leveraging parallelism in step 3, 6, and 7 along with warm staring and duality in steps 6 and 7, computationally efficient implementations of the FW-sample algorithm for solving the R-UCDR-FW problem are possible.

IV. Test Case

To test the effectiveness the FW and FW-sample algorithms for obtaining solutions to the UCDR problem, we conducted a numerical study on the IEEE Reliability Test System (RTS-GMLC) [14]. To represent flexible loads on the system we allowed for a certain percentage of the load at each bus to be a flexible self scheduling load. Thus there is one flexible load a at each bus i on the system. We only require that for the flexible load at bus i that $\sum_{t \in T} s^t_{a,i} = \delta \sum_{t \in T} \hat{d}^t_i$ where $\hat{d}^t_i$ is the original UC load at bus i at time t in the RTS-GMLC system and δ is a parameter determining the percentage of load that is self scheduling. In other words some percentage of the load, defined by δ, at each bus can redistributed over the day as long as it is all served. For our experiments we consider cases where δ is set uniformly across all buses, and consider the values $\delta = [0, 0.05, \dots, 0.45, 0.5]$.

Our implementations of the FW and FW-sample algorithms were done in the Julia language and used the PowerSystems.jl [15] and PowerSimulations.jl [16] ecosystem, an open-source power grid modeling toolbox. These packages were used for the construction of all power grid optimization models in our work. The FICO Xpress Optimizer version 8.8.0. was used for all computational experiments performed, and all MILPs were solved to a 0.01% gap.

To establish a baseline to compare methods against we solved the true UCDR problem for each δ. We then tested the linear programming relaxation approach by solving the R-UCDR for the same range of δ, fixing the resulting loads in the UC problem. We ran the FW algorithm and FW-sample algorithm each for 200 iterations on the full range of δ. All runs of the FW-sample algorithm were done using 10 samples taken from $\mathcal{B}_\infty(d_{k+1}, \epsilon)$ where $\epsilon = \sqrt{\alpha}$, in accordance with [11]. 200 iterations were used because that was the number of iterations that allowed the FW-sample algorithm to achieve good results for most δ values. We also ran the FW algorithm for 1000 iterations to see if it could match the FW-sample algorithm's performance with more iterations.

V. Results

Savings observed from rescheduling flexible loads can be seen in Fig 1. We observe from the UCDR line, the true optimal solution, that the percentage of the UC cost saved increased as more of the load at each bus is flexible, with approximately a 25% reduction in cost when 50% of the load at each bus is flexible. However, there is a reduction in the relative benefit as the percentage of flexible load increases.

In Fig 2 we show the solutions obtained to the R-UCDR problem via the FW and FW-sample algorithm in comparison with the global solution to the R-UCDR. The numbers in FW-200, FW-1000, and FW-sample-200 are the number of iterations the algorithm was run. We see that the FW-sample-200 performs similarly to the global R-UCDR solution for all δ values considered up to 0.4. The FW-sample-200 matches or out performs the FW-200 for all δ. The same is true is when the FW-sample-200 is compared with FW-1000 with the exception of the case when $\delta = 0.45$. In the case when $\delta = 0.45$ we found that with 200 iterations the FW-sample algorithm sometimes matched the performance of FW-1000, and almost always did given 500 iterations. It is unclear why the in this case the FW-sample needed more iterations, but in general the FW and FW-sample began to struggle when the percentage of flexible load approached 50%.

Fig 1 also shows how the flexible load schedules computed in the experiments displayed in Fig 2 performed when used in the full UC problem. With the exception of the R-UCDR when $\delta = 0.45$ the UC savings seem to be correlated with the objective function values observed in Fig 2. The outlier in the case of the R-UCDR with $\delta = 0.45$ seems to suggest an extreme sensitivity in the UC problem to the flexible load profiles. This could be caused by the power imbalance slack variables $y^{t,\pm}_i$ being used, which are penalized at a cost of 10^6 \$/MWh.

To show how the behavior of the FW-sample algorithm differs from the FW algorithm, in Fig 3 we show the progression of the objective value for the two algorithms in comparison with the R-UCDR objective value. We omit the first 10 iterations since they exhibit extremely large oscillations. This figure shows that on average the FW-sample algorithm takes steps that lead to larger reductions in the objective value. They also show that many more iterations of the FW algorithm are needed to match the performance of the FW-sample algorithm.

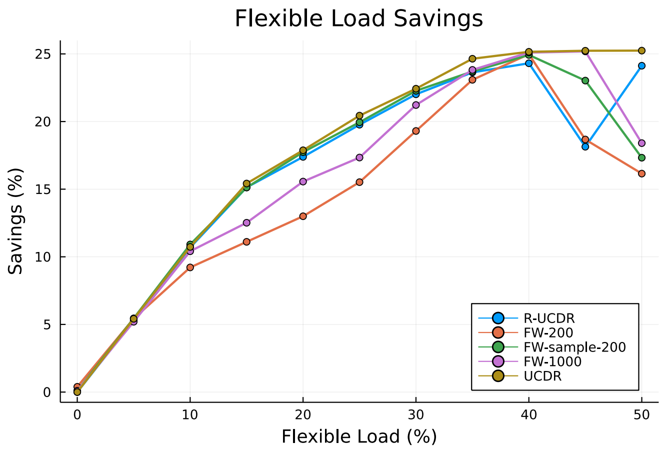

Fig. 1. Percent of cost saved on the RTS-GMLC system in a 24 hour UCDR problem with different percentages of flexible load.

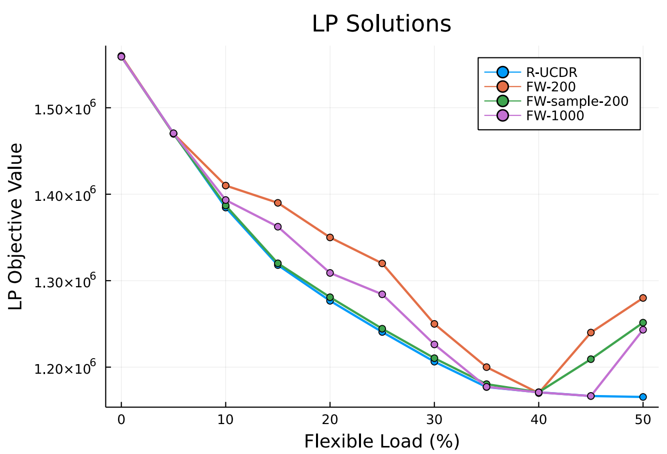

Fig. 2. Solutions to the R-UCDR for RTS-GMLC system over 24 hours with different percentages of flexible load.

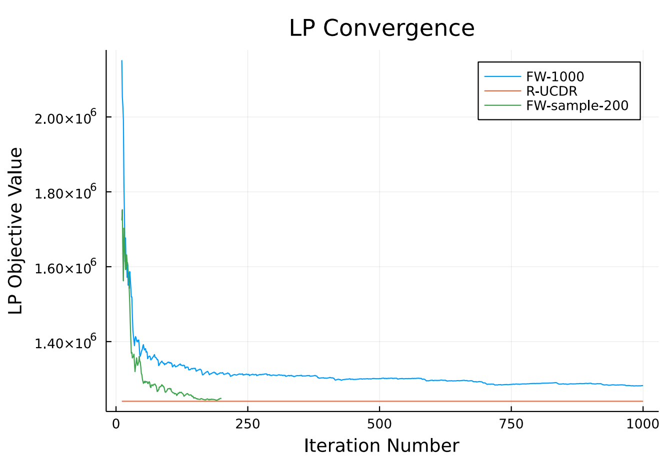

Fig. 3. Convergence of the FW-sample over 200 iterations compared to the FW with 1000 iterations when $\delta = 0.25$, compared to the R-UCDR solution.

VI. Conclusion

We have shown that through reformulation of the UCDR problem that the FW and FW-sample algorithms can be applied to the R-UCDR problem. We have seen that the FW-sample algorithm with a small number of samples can be used to obtain high quality solutions to the UCDR problem. With this approach, power system scheduling problems can represent the centralized control of large numbers of flexible distributed energy resources, enabling DER value analysis on large, realistic datasets. Future directions of research in this area include, exploring larger scale test cases and a more complex set of demand responsive loads, exploring sampling strategies for the FW-sample algorithm, profiling parallel implementations, and better understanding the behavior of the algorithm to determine the cases where it is most applicable.

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